Conference Proceedings of Science and Technology, 2(1), 2019, 20–21

Conference Proceeding of 2th International Conference on Mathematical Advences and Applications (ICOMAA-2019).

A Convergence Result for a Three-Step Iterative Algorithm

ISSN: 2651-544X

http://dergipark.gov.tr/cpost

Harun Polat[1], *Faik Gürsoy*[2,*]

[1] *Department of Mathematics, Faculty of Science and Arts, Muş Alparslan University, Muş, Turkey, ORCID:0000-0003-3955-9197*
[2] *Department of Mathematics, Faculty of Science and Arts, Adiyaman University, Adiyaman, Turkey, ORCID:0000-0002-7118-9088*
** Corresponding Author E-mail: faikgursoy02@hotmail.com*

Abstract: We prove under some mild conditions that iterative algorithm (1.7) of [1] converges strongly to the fixed point of a member in the class of weak contraction mappings.

Keywords: Fixed point, Iterative algorithm, Strong convergence.

1 Introduction

Let C be a nonempty closed convex subset of a Banach space X and $T : C \to C$ a mapping. An element x in C is said to be a fixed point of T if $Tx = x$.

Iterative approximation of fixed points has become a useful tool for solving many problems which arise in various branches of science and engineering.

Recently, Karakaya et al. [1] introduced a three-step iterative algorithm as follows:

$$\begin{cases} x_1 \in C, \\ x_{n+1} = Ty_n, \\ y_n = (1-\alpha_n) z_n + \alpha_n T z_n, \\ z_n = Tx_n, n \in \mathbb{N}, \end{cases} \tag{1}$$

where $\{\alpha_n\}_{n=1}^{\infty}$ is a real sequence in $[0, 1]$.

Definition 1. *([2]) Let (M, d) be a metric space. A mapping $T : M \to M$ is said to be weak-contraction if there exist $\delta \in [0, 1)$ and $L \geq 0$ such that*

$$d(Tx, Ty) \leq \delta d(x, y) + Ld(y, Tx), \text{ for all } x, y \in M.$$

Theorem 1. *([2]) Let (M, d) be a complete metric space and $T : M \to M$ a weak-contraction for which there exist $\delta \in [0, 1)$ and $L_1 \geq 0$ such that*

$$d(Tx, Ty) \leq \delta d(x, y) + L_1 d(x, Tx), \text{ for all } x, y \in M. \tag{2}$$

Then, T has a unique fixed point.

Karakaya et al. [1] showed that iterative algorithm (1) strongly converges to the fixed points of weak-contraction mappings. More precisely, they proved the following result.

Theorem 2. *([1]) Let C be a nonempty closed convex subset of a Banach space X and $T : C \to C$ a weak-contraction satisfying condition (2). Let $\{x_n\}_{n=1}^{\infty}$ be an iterative sequence generated by (1) with real sequence $\{\alpha_n\}_{n=1}^{\infty} \subseteq [0, 1]$ satisfying $\sum_{n=1}^{\infty} \alpha_n = \infty$. Then, $\{x_n\}_{n=1}^{\infty}$ converges to a unique fixed point p^* of T.*

2 Main result

Theorem 3. *Let C be a nonempty closed convex subset of a Banach space X and $T : C \to C$ with $p^* = Tp^*$ a weak-contraction satisfying condition (2). Let $\{x_n\}_{n=1}^{\infty}$ be an iterative sequence generated by (1) with real sequence $\{\alpha_n\}_{n=1}^{\infty} \subseteq [0, 1]$. Then, the sequence $\{x_n\}_{n=1}^{\infty}$ converges to a unique fixed point p^* of T.*

Proof: The following inequality was obtained in ([1], Theorem 2.1):

$$\left\|x_{n+1}-p^*\right\| \leq \left\|x_1-p^*\right\| \delta^{2n} \prod_{i=1}^{n} \left[1-\alpha_i\left(1-\delta\right)\right], \text{ for all } n \in \mathbb{N}. \tag{3}$$

As $\delta \in [0,1)$ and $\{\alpha_n\}_{n=1}^{\infty} \subseteq [0,1]$ implies $1-\alpha_n\left(1-\delta\right) < 1$ for all $n \in \mathbb{N}$, so inequality (3) becomes

$$\left\|x_{n+1}-p^*\right\| \leq \delta^{2n} \left\|x_1-p^*\right\|, \text{ for all } n \in \mathbb{N}. \tag{4}$$

Taking limit on both sides of inequality (4), we have $\lim_{n\to\infty} \|x_n - p^*\| = 0$. □

3 Conclusion

Theorem 2 was proven under the condition $\sum_{n=1}^{\infty} \alpha_n = \infty$. In Theorem 3, we remove this condition. Therefore, Theorem 3 is an improvement of Theorem 2.

4 References

[1] V. Karakaya, Y. Atalan, K. Doğan, N. El Houda Bouzara, *Some fixed point results for a new three steps iteration process in Banach spaces*, Fixed Point Theory, **18** (2017), 625-640.

[2] V. Berinde, *On the approximation of fixed points of weak contractive mappings*, Carpathian J. Math., **19** (2003), 7-22.