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# Graphics Constructor 3.0

**Haydar Bulgak[1*], Diliaver Eminov[2] and Murat Uçar[3]**

[1] *Selcuk University Faculty of Science, Department of Mathematics, Konya-Turkey*
[2] *CR2 Ltd, Dublin-Ireland*
[3] *Selcuk University, Graduate School of Natural Sciences, Konya-Turkey*
[*] *Corresponding author E-mail: hbulgak@selcuk.edu.tr*

**Abstract**

This paper presents the *Graphics Constructor 3.0* application. It is an extention version of the *Graphics Constructor 2.0* which were elaborated in Selcuk University.

***Keywords:*** *Computer Dialogue application, Spline functions.*
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## 1. Introduction

The Research Center of Applied Mathematics of Selçuk University developed an application called *Graphics Constructor 2.0 (GC2.0)*. The application allows to draw an approximate graph of a function defined by a number of separate fixed points,*" knots"*. The graph can be calculated as Lagrange interpolation polynomial, a linear, a special quadratic and a cubic Hermite spline functions.
The application calculates several values, namely *"Integral"*, *"Curve"*, *"Area"* and *"Volume"*.
Let $y = f(x), \quad a < x < b$ be a continuous real function then the following values

$$I = \int_a^b f(x)\,\mathrm{d}x, \qquad L = \int_a^b \sqrt{1+(f'(x))^2}\,\mathrm{d}x, \qquad V = \int_a^b f^2(x)\,\mathrm{d}x, \qquad S = 2\pi\int_a^b |f(x)|\sqrt{1+(f'(x))^2}\,\mathrm{d}x$$

are known as the integral *"Integral"*, length of the arc of the graph *"Curve"*, volume and area of the solid of revolution of the graph rotated around the X-axis *"Area"* and *"Volume"*.
The *"Integral"* is computed for the all spline interpolation functions. But another three values are computed only for the first degree spline functions.
In the paper the exact formulas for these values in case of the special quadratic spline functions are given. The exact formula for compute *"Volume"* for the Hermite cubic spline is also given. A trapezoidal rule is suggested for computing *"Curve"* and *"Area"* values for Hermite cubic spline functions. All the results are displayed in the *Graphics Constructor 3.0 (GC3.0)*. The application shows the values as *"Curve"*$(L)$, *"Volume"*$(V)$ and *"Area"*$(S)$.
*GC3.0* is implemented in C++ using MFC and compiled in Visual Studio 2013. The user interface of the *GC3.0* is very similar to the interface of the *GC2.0* with as new natural parameter "$0 < M < 65$", which is used as a control value for the third degree spline function application.
Please refer to the full description of the *GC2.0* which is given in [1]. A number applications of the *GC2.0* one can find in the textbook [2]. Let's recall some results of the *GC2.0* application. The application works with table 1. A unique linear spline function exists

| | | | | |
|---|---|---|---|---|
| $X$ | $x_0$ | $x_1$ | $\cdots$ | $x_n$ |
| $Y$ | $y_0$ | $y_1$ | $\cdots$ | $y_n$ |
| $F$ | $f_0$ | $f_1$ | $\cdots$ | $f_n$ |

**Table 1**

$$P_1(x) = a_j x + b_j, \quad x_j < x < x_{j+1}, \quad j = 0, 1, \cdots, n-1$$

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***Email addresses*: hbulgak@selcuk.edu.tr(Haydar Bulgak), diliaver.eminov@cr2.com (Diliaver Eminov), murfly.mur@hotmail.com (Murat Uçar)**

which is satisfied to table 1. The reals $a_j, b_j$ are found by solving equations

$$\begin{cases} P_1(x_j) = & y_j, \\ P_1(x_{j+1}) = & y_{j+1}. \end{cases}$$

A unique special quadratic spline function exists

$$P_2(x) = a_j x^2 + b_j x + c_j, \quad x_j < x < x_{j+1}, \quad j = 0, 1, \cdots, n-1$$

which is satisfied to the table 1. The reals $a_j, b_j, c_j$ are found by solving the equations

$$\begin{cases} P_2(x_0) = & y_0, \\ P_2(x_1) = & y_1, \\ P_2'(x_0+0) = & f_0, \end{cases} \quad \begin{cases} P_2(x_j) = & y_j, \\ P_2(x_{j+1}) = & y_{j+1}, \\ P_2'(x_j+0) = & P_2'(x_j-0). \end{cases}$$

A unique Hermite cubic spline function exists

$$P_3(x) = a_j x^3 + b_j x^2 + c_j x + d_j, \quad x_j < x < x_{j+1}, \quad j = 0, 1, \cdots, n-1$$

which is satisfied to the table 1. The reals $a_j, b_j, c_j, d_j$ are satisfied to the systems

$$\begin{cases} P_3(x_j) = & y_j, \\ P_3'(x_j) = & f_j, \\ P_3(x_{j+1}) = & y_{j+1}, \\ P_3'(x_{j+1}) = & f_{j+1}. \end{cases}$$

## 2. Special Quadratic Spline Functions

It is clear that

$$L_j = \int_{x_j}^{x_{j+1}} \sqrt{1 + \left(P_2'(x)\right)^2}\,\mathrm{d}x = \int_{x_j}^{x_{j+1}} \sqrt{1 + \left(2a_j x + b_j\right)^2}\,\mathrm{d}x,$$

guarantee that $L = L_0 + L_1 + \cdots + L_{n-1}$.
The following algorithm can be used for computing $L_j$.

**Algorithm 1**

`if` $|a_j| < 10^{-14}$ `then` $L_j = (x_{j+1} - x_j)\sqrt{1 + b_j^2}$ `else`

$$W_j(x) = \frac{1}{4a_j}\Bigg((2a_j x + b_j)\sqrt{1 + (2a_j x + b_j)^2} + \ln\left(\sqrt{1 + (2a_j x + b_j)^2} + 2a_j x + b_j\right)\Bigg);$$

$L_j = W_j(x_{j+1}) - W_j(x_j);$ `end if`.
It is clear that

$$V_j = \pi \int_{x_j}^{x_{j+1}} P_2^2(x)\,\mathrm{d}x = \pi \int_{x_j}^{x_{j+1}} \left(a_j x^2 + b_j x + c_j\right)^2 \mathrm{d}x,$$

guarantee that $V = V_0 + V_1 + \cdots + V_{n-1}$.
The following formulas can be used for computing $V_j$.

$$W(x) = \frac{1}{5}a_j^2 x^5 + \frac{1}{2}a_j b_j x^4 + \frac{1}{3}(2a_j c_j + b_j^2)x^3 + c_j b_j x^2 + c_j^2 x, \quad x_j < x < x_{j+1};$$
$$V_j = \pi\big(W(x_{j+1}) - W(x_j)\big).$$

It is clear that

$$\begin{aligned} S_j &= 2\pi \int_{x_j}^{x_{j+1}} |P_2(x)|\sqrt{1 + \left(P_2'(x)\right)^2}\,\mathrm{d}x \\ &= 2\pi \int_{x_j}^{x_{j+1}} |2a_j x + b_j x + c_j|\sqrt{1 + \left(2a_j x + b_j\right)^2}\,\mathrm{d}x, \end{aligned}$$

guarantee that $S = S_0 + S_1 + \cdots + S_{n-1}$.

The following Algorithm 2 can be used for computing $S_j$. This algorithm uses the following functions:

$$
\begin{aligned}
G(u) = &\quad u\sqrt{1+u^2}, \qquad H(u) = \ln\left(u+\sqrt{1+u^2}\right),\\
g(u) = &\quad u(2u^2+1)\sqrt{1+u^2},\\
I_j(\alpha,\beta) = &\quad \frac{4a_jc_j-b_j^2}{16a_j^2}\Big(G(2a_j\beta+b_j)-G(2a_j\alpha+b_j)\Big)\\
&+\frac{4(4a_jc_j-b_j^2)-1}{64a_j^2}\Big(H(2a_j\beta+b_j)-H(2a_j\alpha+b_j)\Big)\\
&+\frac{1}{64a_j^2}\Big(g(2a_j\beta+b_j)-g(2a_j\alpha+b_j)\Big).
\end{aligned}
$$

**Algorithm 2**

`if` $|a_j| < 10^{-14}$ `then` Applying linear spline function for the points $(x_j,y_j)$, $(x_{j+1},y_{j+1})$ one can compute $S_j$ by the *GC2.0* algorithm.
`end if.`
$z = b_j^2 - 4a_jc_j$
`if` $z > 0$ `then`
$s = \sqrt{z}$
`if` $a_j > 0$ `then` $t_j = \dfrac{-b_j - s}{2a_j}$; $T_j = \dfrac{-b_j + s}{2a_j}$; `else`
$T_j = \dfrac{-b_j - s}{2a_j}$; $t_j = \dfrac{-b_j + s}{2a_j}$; `end if.`
`if` $t_j < x_j < T_j < x_{j+1}$ `then`
$S_j = 2\pi\big(\big|I_j(x_j,T_j)\big| + \big|I_j(T_j,x_{j+1})\big|\big)$ `end if.`
`if` $x_j < t_j$ `and` $T_j < x_{j+1}$ `then`
$S_j = 2\pi\big(\big|I_j(x_j,t_j)\big| + \big|I_j(t_j,T_j)\big| + \big|I_j(T_j,x_{j+1})\big|\big)$ `end if.`
`if` $x_j < t_j < x_{j+1} < T_j$ `then`
$S_j = 2\pi\big(\big|I_j(x_j,t_j)\big| + \big|I_j(t_j,x_{j+1})\big|\big)$ `end if.`
`if` $x_j = t_j$ `and` $T_j < x_{j+1}$ `then`
$S_j = 2\pi\big(\big|I_j(x_j,T_j)\big| + \big|I_j(T_j,x_{j+1})\big|\big)$ `end if.`
`if` $x_j < t_j$ `and` $T_j = x_{j+1}$ `then`
$S_j = 2\pi\big(\big|I_j(x_j,t_j)\big| + \big|I_j(t_j,x_{j+1})\big|\big)$ `end if.`
`end if.`
$S_j = 2\pi\big|I_j(x_j,x_{j+1})\big|$.

## 3. Hermite Cubic Spline Functions

It is clear that

$$V_j = \pi\int_{x_j}^{x_{j+1}} P_3^2(x)\,\mathrm{d}x = \pi\int_{x_j}^{x_{j+1}} \left(a_jx^3+b_jx^2+c_jx+d_j\right)^2\mathrm{d}x,$$

guarantee that $V = V_0 + V_1 + \cdots + V_{n-1}$. The following formulas can be used for computing $V_j$.

$$
\begin{aligned}
W(x) = &\quad \frac{1}{7}a_j^2x^7 + \frac{1}{3}a_jb_jx^6 + \frac{1}{5}(2a_jc_j+b_j^2)x^5\\
&+\frac{1}{2}(a_jd_j+c_jb_j)x^4 + \frac{1}{3}(2b_jd_j+c_j^2)x^3\\
&+c_jd_jx^2 + d_j^2x, \qquad\qquad x_j < x < x_{j+1};\\
V_j = &\quad \pi\big(W(x_{j+1}) - W(x_j)\big).
\end{aligned}
$$

It is clear that

$$L_j = \int_{x_j}^{x_{j+1}} \sqrt{1+\left(P_3'(x)\right)^2}\,\mathrm{d}x, \qquad S_j = 2\pi\int_{x_j}^{x_{j+1}} |P_3(x)|\sqrt{1+\left(P_3'(x)\right)^2}\,\mathrm{d}x,$$

guarantee that $L = L_0 + L_1 + \cdots + L_{n-1}$ and $S = S_0 + S_1 + \cdots + S_{n-1}$.
The *Algorithm 3* can be used for computing $L_j(M)$ and $S_j(M)$ which approximate $L_j$ and $S_j$ respectively. Here, a natural number $0 < M < 65$ is a control parameter.

**Algorithm 3**

`step 1.` $h_j = \dfrac{x_{j+1} - x_j}{M}$;

`step 2.  for` $k=0,1,\cdots M \quad g_{k,j}=P_3(x_j+kh_j)$;

The approximations *"Curve"*$L_j(M)$ and *"Area"*$S_j(M)$ can be computed for the first degree spline function which is defined by the virtual table 2.

| *Vectors* | 0 | 1 | 2 | $\cdots$ | $M$ |
|---|---|---|---|---|---|
| $X$ | $x_j$ | $x_j+h_j$ | $x_j+2h_j$ | $\cdots$ | $x_{j+1}$ |
| $Y$ | $g_{0,j}$ | $g_{1,j}$ | $g_{2,j}$ | $\cdots$ | $g_{M,j}$ |

**Table 2**

Hence

$$\begin{aligned} L(M) &= L_0(M)+L_1(M)+\cdots+L_{n-1}(M), \\ S(M) &= S_0(M)+S_1(M)+\cdots+S_{n-1}(M) \end{aligned}$$

are the approximations of the requaried values.

## 4. Tests

Using *GC3.0* application we computed the following two tests for special quadratic spline functions.

| *Vectors* | 1 | 2 | 3 |
|---|---|---|---|
| $X$ | 1 | 3 | 4 |
| $Y$ | 2 | 0 | 7 |
| $F$ | 1 | | |

**Table 3**

| *Vectors* | 1 | 2 |
|---|---|---|
| $X$ | 0 | 3 |
| $Y$ | 2 | 2 |
| $F$ | $-3$ | |

**Table 4**

**Test 1**

Acording the table 3
*"Curve"*:11.0085 , *"Area"*:185.096 and *"Volume"*:45.2389

**Test 2**

Acording the table 4
*"Curve"*:5.65264 , *"Area"*:28.4417 and *"Volume"*:6.59734
Using *GC3.0* we computed the following two tests for Hermite cubic spline functions with $M=10$ control parameter.

| *Vectors* | 1 | 2 | 3 |
|---|---|---|---|
| $X$ | 0 | 1 | 2 |
| $Y$ | 1 | 2 | 1 |
| $F$ | 0 | 3 | $-6$ |

**Table 5**

| *Vectors* | 1 | 2 | 3 |
|---|---|---|---|
| $X$ | 1 | 3 | 4 |
| $Y$ | 1 | 5 | 2 |
| $F$ | 1 | $-1$ | 3 |

**Table 6**

**Test 3**

Acording the table 5
*"Curve"*:4.016171 , *"Area"*:46.5648 and *"Volume"*:21.6321

**Test 4**

Acording the table 6
*"Curve"*:8.23543 , *"Area"*:166.63 and *"Volume"*:117.585

## Conclusion

The presented algorithms are realised in application *GC3.0* which is available in the address: *www.selcuk.edu.tr/fen/matematik/Web/Sayfa/Ayrinti/65078/tr*.
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