*Konuralp Journal of Mathematics*, 7 (2) (2019) 395-398

**Konuralp Journal of Mathematics**

Journal Homepage: www.dergipark.gov.tr/konuralpjournalmath
e-ISSN: 2147-625X

# The First Theorem on UP-Biisomorphism Between UP-Bialgebras

**Daniel A. Romano**[1*]

[1]*International Mathematical Virtual Institute, Banja Luka, Bosnia and Herzegovina*
[*]*Corresponding author E-mail: bato49@hotmail.com*

**Abstract**

The concept of UP-bialgebras was introduced and analyzed by Mosrijai and Iampan at the beginning of 2019. In this article we analyzed a UP-biisomorphism between UP-bialgebras.

***Keywords:*** *UP-algebra, UP-ideal, UP-homomorphism, UP-bialgebra, UP-biideal, UP-bihomomorphism, UP-biisomorphism.*
***2010 Mathematics Subject Classification:*** *03G25*

## 1. Introduction

The concept of UP-algebras developed by Iampan in [1]. Examining the substructures in this algebra are done in articles [2, 4]. Some forms of the isomorphism theorem between UP-algebras can be found in [2, 4, 7, 8].

The concept of bialgebraic structures was discussed by Vasantha Kandasamy in 2003 [3]. The concept of UP-bialgebras with the associated substructures and their mutual connections can be found in [5]. In this article we expose a form of the isomorphism theorem between UP-bialgebras.

## 2. Preliminaries and extension of some results

In this section, we will present the necessary previous concepts of UP-algebras, their substructures and UP-homomorphisms taken from texts [1, 2, 4, 5]. We will also expose their mutual relationships in the form of proclaims necessary for our intention.

### 2.1. UP-algebras

In this subsection we will describe some elements of UP-algebras and their substructures necessary for our intentions in this text.

**Definition 2.1** ([1]). *An algebra $A = (A, \cdot, 0)$ of type $(2,0)$ is called a UP-algebra where $A$ is a nonempty set, $'\cdot'$ is a binary operation on $A$, and $0$ is a fixed element of $A$ (i.e. a nullary operation) if it satisfies the following axioms:*
*(UP-1)* $(\forall x, y \in A)((y\cdot z)\cdot((x\cdot y)\cdot(x\cdot z)) = 0)$,
*(UP-2)* $(\forall x \in A)(0\cdot x = x)$,
*(UP-3)* $(\forall x \in A)(x\cdot 0 = 0)$, *and*
*(UP-4)* $(\forall x, y \in A)((x\cdot y = 0 \wedge y\cdot x = 0) \Longrightarrow x = y)$.

On a UP-algebra $A = (A, \cdot, 0)$, we define the UP-ordering $\leqslant$ on $A$ as follows:

$(\forall x, y \in A)(x \leqslant y \Longleftrightarrow x\cdot y = 0)$.

**Definition 2.2** ([1]). *A nonempty subset $S$ of a UP-algebra $(A, \cdot, 0)$ is called*
*(1) a UP-subalgebra of $A$ if* $(\forall x, y \in A)(x\cdot y \in S)$.
*(2) a UP-ideal of $A$ if*
*(i)* $0 \in S$*; and*
*(ii)* $(\forall x, y, z \in A)((x\cdot(y\cdot z) \in S \wedge y \in S) \Longrightarrow x\cdot z \in S)$.

***Email addresses***: **bato49@hotmail.com (Daniel A. Romano)**

The set $\{0\}$ is a trivial UP-subalgebra (trivial UP-ideal) of $A$.

In the article [8], Theorem 3.3, it has been shown that the conditions (i) and (ii) in the preceding definition are equivalent to the following conditions
(iii) $(\forall x,y \in A)((x \cdot y \in S \wedge x \in S) \Longrightarrow y \in S)$,
(iv) $(\forall x,y \in A)(y \in S \Longrightarrow x \cdot y \in S)$.

**Definition 2.3** ([1]). *Let $(A,\cdot,0_A)$ and $(B,\cdot^{'},0_B)$ be two UP-algebras. A mapping $f : A \longrightarrow B$ is called a UP-homomorphism if*

$$(\forall x,y \in A)(f(x \cdot y) = f(x) \cdot^{'} f(y)).$$

*A UP-homomorphism $f : A \longrightarrow B$ is called*
(3) *a UP-epimorphism if $f$ is surjective,*
(4) *a UP-monomorphism if $f$ is injective, and*
(5) *a UP-isomorphism if $f$ is bijective.*

Let $f$ be a mapping form UP-algebra $A$ to UP-algebra $B$, and let $C$ and $D$ be nonempty subsets of $A$ and of $B$, respectively. The set $\{f(x)|x \in C\}$ which denoted by $f(C)$ is called the image of $C$ under $f$. In particular, $f(A)$ which denoted by $Im(f)$ is called the image of $f$. The dually set $\{x \in A|f(x) \in D\}$ which denoted by $f^{-1}(D)$ is called the inverse image of $D$ under $f$. Especially, the set $f^{-1}(\{0_B\})$ which written by $Ker(f)$ is called the kernel of $f$.

**Proposition 2.4** ([1]). *Let $(A,\cdot,0_A)$ and $(B,\cdot^{'},0_B)$ be UP-algebras and let $f : A \longrightarrow B$ be a UP-homomorphism. Then the following statements hold:*
(6) $f(0_A) = 0_B$*;*
(7) $(\forall x,y \in A)(x \leqslant_A y \Longrightarrow f(x) \leqslant_B f(y))$*;*
(8) *if $C$ is a UP-subalgebra of $A$, then the image $f(C)$ is a UP-subalgebra of $B$. In particular, $Im(f)$ is a UP-subalgebra of $B$;*
(9) *if $D$ is a UP-subalgebra of $B$, then the inverse image $f^{-1}(D)$ is a UP-subalgebra of $A$. In particular, $Ker(f)$ is a UP-subalgebra of $A$;*
(10) *if $D$ is a UP-ideal of $B$, then the inverse image $f^{-1}(D)$ is a UP-ideal of $A$. In particular, $Ker(f)$ is a UP-ideal of $A$;*
(11) *if $C$ is a UP-ideal of $A$ such that $Ker(f) \subseteq C$, then the image $f(C)$ is a UP-ideal of $Im(f)$; and*
(12) $Ker(f) = \{0_A\}$ *if and only if $f$ is an injective mapping.*

Let $A$ be a UP-algebra and $J$ a UP-ideal of $A$. If we define the binary relation $\sim_J$ on $A$ as follows:
$(\forall x,y \in A)(x \sim_J y \Longleftrightarrow (x \cdot y \in J \wedge y \cdot x \in J))$,
then $\sim_J$ is a congruence on $A$ suc that $J = [0]_{\sim_J}$ by Proposition 3.5 and assertion (1) of Theorem 3.6 in [1]. In addition, the family $A/\sim_J$ is a UP-algebra by the assertion (4) of Theorem 3.7 in [1]. This UP-algebra constructed by the UP-algebra $A$ through the congruence $\sim_J$ will be written as $A/J$. Specially, we will write $A/Ker(f)$ instead of $A/\sim_{Ker(f)}$.

## 2.2. UP-bialgebras

The concept of UP-bialgebras and some their substructures were introduced and analyzed by Mosrijai and Iampan in the recently published work [5] In this subsection, taking into account their determinations, we describe the concept of UP-bialgebras and their substructures. So, in this subsection, we will describe the concept of UP-bialgebras and the notions of UP-bisubalgebras and UP-biideals of UP-bialgebras, and will expose some results related to substructures of such algebras.

**Definition 2.5** ([5], Definition 3.1). *An algebra $A = (A,\cdot,*,0)$ of type $(2,2,0)$ is called a UP-bialgebra where $A$ is a nonempty set, $\cdot$ and $*$ two are binary internal operations on $A$, and $0$ is a fixed element of $A$ if there exist two distinct proper subsets $A_1$ and $A_2$ of $A$ with respect to $\cdot$ and $*$, respectively, such that*
*(UPB-1)* $A = A_1 \cup A_2$*;*
*(UPB-2)* $(A_1,\cdot,0)$ *is a UP-algebra, and*
*(UPB-3)* $(A_2,*,0)$ *is a UP-algebra.*
*We will denote the UP-bialgebra by $A = A_1 \uplus A_2$. In case of $A_1 \cap A_2 = \{0\}$, we call $A$ zero disjoint.*

On a UP-bialgebra $A = A_1 \uplus A_2$ with two binary operations $\cdot$ and $*$, we define a binary relation $\leqslant$ on $A$ as follows ([5]):

$$(\forall x,y \in A)(x \leqslant y \text{under} \cdot \text{by} x \cdot y = 0)$$

$$(\forall \in x,y \in A)(x \leqslant y \text{under} * \text{by} x * y = 0).$$

**Definition 2.6** ([5], Definition 3.7). *A nonempty subset $S$ of a UP-bialgebra $A = A_1 \uplus A_2$ is called a UP-bisubalgebra of $A$ if there exist subsets $S_1$ of $A_1$ and $S_2$ of $A_2$ with respect to $\cdot$ and $*$, respectively, such that*
(13) $S_1 \neq S_2$ *and* $S = S_1 \cup S_2$*;*
(14) $(S_1,\cdot,0)$ *is a UP-subalgebra of* $(A_1,\cdot,0)$*, and*
(15) $(S_2,*,0)$ *is a UP-subalgebra of* $(A_2,*,0)$*.*
*In case of $S_1 \cap A_2 = \{0\} = A_1 \cap S_2$, we call $S$ zero disjoint.*

**Definition 2.7** ([5], Definition 3.7). *A nonempty subset $S$ of a UPB-algebra $A = A_1 \uplus A_2$ is called a UPB-ideal of $A$ if there exist subsets $S_1$ of $A_1$ and $S_2$ of $A_2$ with respect to $\cdot$ and $*$, respectively, such that*
(16) $S_1 \neq S_2$ *and* $S = S_1 \cup S_2$*;*
(17) $(S_1,\cdot,0)$ *is a UP-ideal of* $(A_1,\cdot,0)$*, and*
(18) $(S_2,*,0)$ *is a UP-ideal of* $(A_2,*,0)$*.*
*In case of $S_1 \cap A_2 = \{0\} = A_1 \cap S_2$, we call $S$ zero disjoint.*

**Proposition 2.8** ([5], Theorem 3.15)**.** *Let $S$ be a nonempty subset of a UP-bialgebra $A = A_1 \uplus A_2$ which satisfies the following conditions:*
(19) *$(S \cap A_1, \cdot, 0)$ is a UP-subalgebra (resp., UP-ideal) of $(A_1, \cdot, 0)$ and*
(20) *$(S \cap A_2, *, 0)$ is a UP-subalgebra (resp., UP-ideal) of $(A_2, *, 0)$.*
*Then $S$ is a UP-bisubalgebra (resp., UP-biideal) of $A$.*

The reverse proposition of the previous proposition is also valid with one additional condition.

**Proposition 2.9** ([5], Theorem 3.16)**.** *Let $S$ be a nonempty subset of a UP-bialgebra $A = A_1 \uplus A_2$. Then $S$ is a zero disjoint UP-bisubalgebra (resp., zero disjoint UP-biideal) of $A$ if and only if it satisfies the following conditions:*
(21) *$(S \cap A_1, \cdot, 0)$ is a UP-subalgebra (resp., UP-ideal) of $(A_1, \cdot, 0)$, and*
(22) *$(S \cap A_2, *, 0)$ is a UP-subalgebra (resp., UP-ideal) of $(A_2, *, 0)$.*

The important consequence of this proposition is the following assertion:

**Assertion 2.10.** *If $S \supset \{0\}$ is a UP-subalgebra (resp., UP-ideal) of UP-algebra $A_1$ (of UP-algebra $A_2$, respectively), such that $\{0\} \neq S$, then on $S$ can be seen as a zero disjoint UP-bisubgebra (resp., UP-biideal) of UP-bialgebra $A = A_1 \uplus A_2$.*

*Proof.* The conditions (21) and (22) in the preceding proposition are easily and directly verified in this special case. Indeed, Let $S \supset \{0\}$ is a UP-subalgebra (resp., UP-ideal) of UP-algebra $A_1$. If we put $S = S_1$ and $S_2 = \{0\}$, we have that $S = S_1 \cup S_2$ satisfies conditions (21) and (22) in the previous proposition. Therefore, $S$ is a zero disjoint UP-bisubalgebra (resp., UP-biideal) of UP-bialgebra because $S_1 \cap S_2 = \{0\}$. The second part of the claim in case when $S \supset \{0\}$ is an UP-subalgebra (resp., UP-ideal) of UP-algebra $A_2$, is proven analogously to the previous proof. □

**Assertion 2.11.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra and let $S = S_1 \uplus S_2$ be a UP-biideal of $A$. Then*
(23) *$(\forall x, y \in A_1)((x \cdot y \in S_1 \wedge x \in S_1) \Longrightarrow y \in S_1)$;*
(24) *$(\forall x, y \in A_1)(y \in S_1 \Longrightarrow x \cdot y \in S_1)$;*
(25) *$(\forall x, y \in A_2)((x * y \in S_2 \wedge x \in S_2) \Longrightarrow y \in S_2)$;*
(26) *$(\forall x, y \in A_2)(y \in S_2 \Longrightarrow x * y \in S_2)$.*

*Proof.* Let $S$ be UP-biideal of a UP-bialgebra $A = A_1 \uplus A_2$. Then there exist subsets $S_1$ of $A_1$ and $S_2$ of $A_2$ with respect to $\cdot$ and $*$, respectively, such that (16), (17) and (18) hold. The assertions (23) and (24), and the assertions (25) and (26) follow directly from Theorem 3.1 in [8]. □

**Assertion 2.12.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra and let $S$ be a nonempty subset of $A$ such that $0 \in S$ and $S \cap A_1$ and $S \cap A_2$ fulfills the following requirements*
(27) *$(\forall x, y \in A_1)((x \cdot y \in S \cap A_1 \wedge x \in S \cap A_1) \Longrightarrow y \in S \cap A_1)$;*
(28) *$(\forall x, y \in A_1)(y \in S \cap A_1 \Longrightarrow x \cdot y \in S \cap A_1)$;*
(29) *$(\forall x, y \in A_2)((x * y \in S \cap A_2 \wedge x \in S \cap A_2) \Longrightarrow y \in S \cap A_2)$;*
(30) *$(\forall x, y \in A_2)(y \in S \cap A_2 \Longrightarrow x * y \in S \cap A_2)$.*
*Then $S$ is a UP-biideal of $A$.*

*Proof.* In accordance with Theorem 3.3. in [8], if the set $S \cap A_1$ satisfies the conditions (27) and (28), then it is UP-ideal of UP-algebra $A_1$. Analogously, if a set $S$ satisfies the conditions (29) and (30), then it is the UP-ideal of UP-algebra $A_2$, according to Theorem 3.3 in [8]. So, the set $S$ is a UP-biideal of UP-bialgebra $A$ by Proposition 2.9. □

## 3. The main results

Let $f : A \longrightarrow B$ be a function from a set $A$ to a set $B$ and $C \subseteq A$. Then the restriction of $f$ to $C$ is the function $f_{[C]} : C \longrightarrow B$.

**Definition 3.1** ([5], Definition 4.1)**.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$. A mapping $f$ form $A$ to $B$ is called a UP-bihomomorphism if it satisfies the following properties:*
(31) *$f_{[A_1]} : A_1 \longrightarrow B_1$ is a UP-homomorphism, and*
(32) *$f_{[A_2]} : A_2 \longrightarrow B_2$ is a UP-homomorphism.*
*A UP-bihomomorphism $f : A \longrightarrow B$ is called*
*- a UP-biepimorphism if $f_{[A_1]}$ and $f_{[A_2]}$ are UP-epimorphisms,*
*- a UP-bimonomorphism if $f_{[A_1]}$ and $f_{[A_2]}$ are UP-monomorphisms,*
*and*
*- a UP-biisomorphism if $f_{[A_1]}$ and $f_{[A_2]}$ are UP-isomorphisms.*

**Proposition 3.2** ([5], Theorem 4.3)**.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then the following statements hold:*
(33) *$f(0_A) = 0_B$,*
(34) *if $x \leqslant y$ under $\cdot$ (resp., $x \leqslant y$ under $*$), then $f_{[A_1]}(x) \leqslant f_{[A_1]}(y)$ (resp., $f_{[A_2]}(x) \leqslant f_{[A_2]}(y)$) for all $x, y \in A$, and*
(35) *$Ker(f) = \{0_A\}$ if and only if $f$ is an injective mapping.*

In light of the Assertion 2.1, we have reformulated Theorem 4.4 and Theorem 4.6 in the article [5] in the following way:

**Proposition 3.3.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then the following statements hold:*
(36) *if $S$ is a UP-bisubalgebra of $A$, then the image $f(S)$ is a UP-bisubalgebra of $B$; and*
(37) *if $S = S_1 \cup S_2$ is a UP-biideal of $A$, and $S_1$ and $S_2$ are subsets of $A_1$ and of $A_2$, respectively, with $Ker(f) \subseteq S_1 \cap S_2$, then the image $f(S)$ is a UP-biideal of $B$.*

**Proposition 3.4.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then the following statements hold:*
(38) *if $D$ is a UP-bisubalgebra of $B$, then the inverse image $f^{-1}(D)$ is a UP-bisubalgebra of $A$;*
(39) *if $D$ is a UP-biideal of $B$, then the inverse image $f^{-1}(D)$ is a UP-biideal of $A$.*

In this section, we formulate and prove the first isomorphism theorem between UP-bialgebras. To this direction, we need the following lemmas.

**Lemma 3.5.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then the set $Ker(f) = Ker(f_{[A_1]}) \cup Ker(f_{[A_2]})$ is a UP-biideal of $A$*

*Proof.* According to (10) and Proposition 2.9. □

**Lemma 3.6.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then the set $A/Ker(f) = A_1/Ker(f_{A_1}) \cup A_2/Ker(f_{A_2})$ is a UP-bialgebra with two binary operation $'\odot'$ and $'\circledast'$ and there exists the unique UP-biepimorphism $\pi : A \longrightarrow A/Ker(f)$.*

*Proof.* Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. By Definition 3.1 the restrictions $f_{[A_1]}$ and $f_{[A_2]}$ are UP-homomorphisms between UP-algebras. Thus there exists the unique UP-epimorphism $\pi_1 : A_1 \longrightarrow A_1/Ker(f_{[A_1]})$ and there exists the unique UP-epimorphism $\pi_2 : A_2 \longrightarrow A_2/Ker(f_{[A_2]})$ by Theorem 2.1 in [2] (or by Theorem 2.4 in [4]). Then, the mapping $\pi : A \longrightarrow A/Ker(f)$ defined by $\pi_{[A_1]} = \pi_1$ and $\pi_{[A_2]} = \pi_2$ is the unique UPB-epimorphism by Definition 3.1.
It is easy and directly verified that the set $A/Ker(f)$ is a UP-bialgebra with two binary operations $\odot$ and $\circledast$ defined by

$(\forall [x]_{\pi_1}, [y]_{\pi_1} \in A_1/Ker(f_{[A_1]}))([x]_{\pi_1} \odot [y]_{\pi_1} = [x \cdot y]_{\pi_1})$ and

$(\forall [x]_{\pi_2}, [y]_{\pi_2} \in A_2/Ker(f_{[A_2]}))([x]_{\pi_2} \circledast [y]_{\pi_2} = [x * y]_{\pi_2})$

since $A_1/Ket(f_{[A_1]})$ and $A_2/Ker(f_{[A_2]})$ are UP-algebras according to the claim (4) of Theorem 3.7 in the article [1]. □

**Theorem 3.7.** *Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. Then there exists the unique UP-bihomomorphism $g : A/Ker(f) \longrightarrow B$ such that $f = g \circ \pi$. In addition, for the UP-bisubalgebra $f(A)$ of $B$ holds $A/Ket(f) \cong f(A)$.*

*Proof.* Let $A = A_1 \uplus A_2$ be a UP-bialgebra with two binary operations $\cdot$ and $*$, and let $B = B_1 \uplus B_2$ be a UP-bialgebra with two binary operations $\cdot'$ and $*'$ and let $f : A \longrightarrow B$ be a UP-bihomomorphism. First, the restrictions $f_{[A_1]}$ and $f_{[A_2]}$ are UP-homomorphisms. Then, by Theorem 2.2 in [2] (or by Theorem 2.5 in [4]), there exists the unique UP-homomorphism $g_1 : A_1/Ker(f_{[A_1]}) \longrightarrow B_1$ such that $f_{[A_1]} = g_1 \circ \pi_1$ and there exists the unique UP-homomorphism $g_2 : A_2/Ker(f_{[A_2]}) \longrightarrow B_2$ such that $f_{[A_2]} = g_2 \circ \pi_2$. The mapping $g : A/Ker(f) \longrightarrow B$ defined by $g_{[A_1/Ker(f_{[A_1]})]} = g_1$ and $g_{[A_2/Ker(f_{[A_2]})]} = g_2$ is a UPB-homomorphism by Definition 3.1. Therefore, the following $f = g \circ \pi$ holds. □

## Final Observation

The concept of UP-algebras introduced and first results on them given by Iampan 2017 [1]. This author took part in analyzing the properties of UP-algebras and their substructures, also [6, 7, 8]. Algebraic bi-structures was analyzed by Vasantha Kandasamy in 2003 [3]. The concept of UP-bialgebras introduced and the first results were given by Mosrijai and Iampan at the beginning of 2019 [5]. Using by the concept of UP-bihomorphisms, introduced in [5], in this text, Section 3, the theorem (Theorem 3.1) is considered, which can be viewed as the First isomorphism theorem between the UP-bialgebras. Before this result, we have previously formulated and proved the two necessary lemmas. In order for ideas, concepts and evidence presented in this article to be consistent, in Section 2 we formulated and proved three assertions that relate to the properties of the ideals in the UP-bialgebra.

Of course, there remains an open possibility of formulating and trying to prove another theorems on isomorphisms between the UP-bialgebra.

## Acknowledgement

The author thanks the referees for the useful suggestions whose incorporation significantly improved the consistency of this text.

# References

[1] A. Iampan. A new branch of the logical algebra: UP-algebras. *J. Algebra Relat Top.*, **5**(1)(2017), 35-–54.
[2] A. Iampan. The UP-isomorphism theorems for UP-algebras. *Discuss. Math. General Al. Appl.*, **39**(1)(2019), 113–123.
[3] W. B. V. Kandasamy. *Bialgebraic structures and Smarandache bialgebraic structures*. India: American Research Press, 2003.
[4] P. Mosrijai, A. Satirad and A. Iampan. The new UP-isomorphism theorems for UP-algebras in the meaning of the congruence determined by a UP-homomorphism, *Fund. J. Math. Appl.*, **1**(1)(2018), 12–17.
[5] P. Mosrijai and A. Iampan. A new branch of bialgebraic structures: UP-bialgebras. *J. Taibah University Sci.*, **13**(1)(2019), 450-–459.
[6] D. A. Romano. Proper UP-filters in UP-algebra. *Universal J. Math. Appl.*, **1**(2)(2018), 98–100.
[7] D. A. Romano. Some properties of proper UP-filters of UP-algebras. *Fund. J. Math. Appl.*, **1**(2)(2018), 109–111.
[8] D. A. Romano. Notes on UP-ideals in UP-algebras. *Comm. Adv. Math. Sci.*, **1**(1)(2018), 35–38.