

Web Application for Step Size Strategies

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Abstract

In this study, it has been designed an interactive web interface which provides the online use of step size strategies to obtain the numerical solutions of Cauchy problems. This web interface has been created by using Django web framework of Python programming language.

Keywords: Step size strategy; numerical solution; web interface; python; django

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1. Introduction

The solution of Cauchy problems may not be expressed with elementary functions or its calculation may be very difficult. In such cases, the numerical methods are used, which help for finding the numerical solution. While calculating the numerical solution, it is calculated the numerical solution with the errors resulting from the numerical method. Also, the calculation errors (rounding and chopping errors) are inevitable in numerical integration. Therefore, selection of the step size in numerical integration is crucial while calculating the numerical solutions. The step size in numerical integration should be selected small in the the region which the solution is changing rapidly in order to converge the computed solution to the analytical solution. However, the selection of small step sizes in numerical integration causes the calculation errors to increase. Hence, the computed solution may diverge from the analytical solution. Selection of the step sizes should be made according to be able to minimize the number of iterations. Because the more the number of iterations reduces, the amount of accumulating errors reduces. The number of iterations is reduced if big step sizes are chosen instead of small step sizes in the region which the solution is changing slowly. Moreover, it is rather difficult to decide on a suitable initial step size, even if a constant step size selection is to be made. So, instead of using a constant step size, it is better to use variable step sizes that are appropriate to the behavior of the problem. It is often not possible to obtain a numerical solution manually without the help of a computer. The numerical methods are iterative methods and also can be easily programmed with any programming language. There are many programming languages in the market such as C, C++, Java, Python, and R. Especially in recent years, an increment has seen in the popularity of the Python programming language. Because Python is a language that is easy to learn in compared to many languages. Since Python has simple and elegance syntax, it is rather easy both write a program and read a program which written by another one. Python has very powerful scientific calculation libraries such as NumPy, SciPy, SymPy, and Matplotlib. Python allows system programming, graphical user interface programming, database programming, numerical and scientific programming, and much more. In addition to these, with Python can be done web programming too. There are some interactive websites which allows calculating the numerical solution of Cauchy problems in the literature [4, 14, 17, 25, 27, 29]. These websites use constant step size in the numerical integration of Cauchy problems. But this situation also has some disadvantages.

Let us consider the Cauchy problem,

$$x'(t) = x^2(t), x(3) = 1 \quad (1.1)$$

on $D = \{(t, x) : |t - 3| \leq 2, |x - 1| \leq 5\}$. The exact solution of Cauchy problem (1.1) is $x(t) = \frac{1}{4-t}$. The solution $x(t)$ is undefined at $t = 4$. The graphic of exact solution of Cauchy problem (1.1) is as shown in Figure 1.1. Let us calculate the numerical solution values of Cauchy problems (1.1) on the web site given in [14].

Table 1: The exact and the numerical solution values of Cauchy problem (1.1)

i	t_i	$x(t_i)$	x_i
0	3	1	1
1	3.1	1.111111	1.1
2	3.2	1.25	1.221
3	3.3	1.428571	1.3700841
4	3.4	1.666666	1.55779714
6	3.5	2.0	1.80047033
7	3.6	2.5	2.12463968
8	3.7	3.333333	2.57604906
9	3.8	5.0	3.23965193
10	3.9	14.285714	4.28918640
11	4	∞	6.12889840
12	4.1	-10	9.88523796
13	4.2	-5	19.6570309

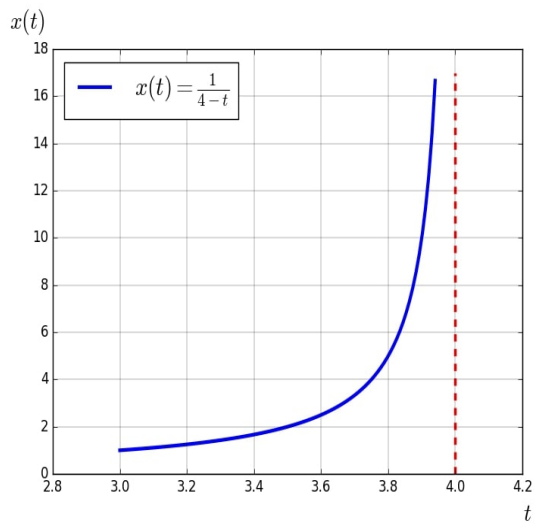


Figure 1.1: The exact solution of Cauchy problem (1.1)

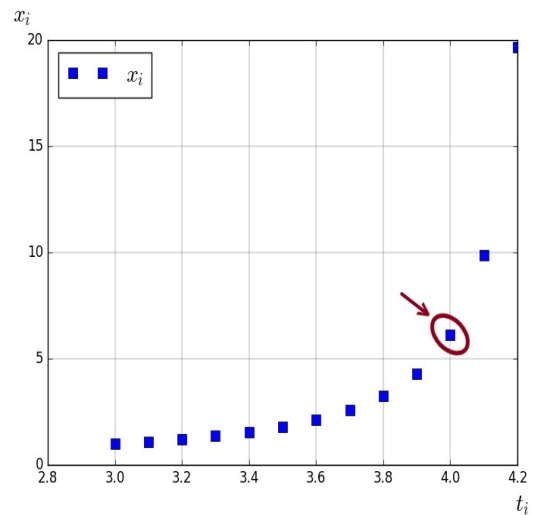


Figure 1.2: The numerical solution of Cauchy problem (1.1) for $h=0.1$

If the numerical solution of Cauchy problem (1.1) is obtained with Euler's method for $h = 0.1$ in [14] and graphic is plotted for the solution values obtained then the graphic in Figure 1.2 is obtained. It seems as if the solution of Cauchy problem (1.1) exists at $t = 4$ point at graphic given in Figure 1.2. Although it is known that the solution of Cauchy problem (1.1) does not exist at $t = 4$ point, it is seen that the solution of Cauchy problem (1.1) exist at $t = 4$ point at graphic given in Figure 1.2.

The step size strategies given in [5, 6, 7, 8, 9, 10, 11, 12] have been produced to overcome aforementioned problems. In this study, an interactive web interface is designed for the step size strategies given in [5, 6, 7, 8, 9, 10, 11, 12]. In Section 2, preliminaries on the solution of Cauchy problems have been given. In Section 3, step size strategies on the web interface have been given. In Section 4, features of python programming language have been given. In Section 5, it has been introduced the web interface with outlines. In Section 6, it has been given numerical examples which the step size strategies are applied on the web interface.

2. Background on the Solutions of Cauchy Problems

Consider the Cauchy problem

$$X'(t) = F(t, X), X(t_0) = X_0 \quad (2.1)$$

on the region

$$D = \{(t, x) : |t - t_0| \leq a, |x_j - x_{j0}| \leq b_j\} \quad (2.2)$$

where $F(t, X) \in \mathbb{R}^N$, $X(t) = x_j(t)$, $X_0 = (x_{j0})$, $x_{j0} = x_j(t_0)$, $X(t)$, X_0 , $b = (b_j) \in \mathbb{R}^N$. For $N = 1$, $F(t, X)$, x_j , b_j is $f(t, x)$, x , b respectively. The Picard-Lindelöf theorem is an existence and uniqueness theorem which gives a set of conditions under which a Cauchy problem has a solution. We let give Picard-Lindelöf theorem for the Cauchy problem given by the equation (2.1) on the region D as in (2.2).

Theorem 2.1 (Picard-Lindelöf existence and uniqueness theorem). *For $N = 1$, if $F(t, X)$ is a continuous function that satisfies the Lipschitz condition on the rectangle D as in (2.2) in a surrounding of (t_0, x_0) , then the Cauchy problem (2.1) has a unique solution in the interval $|t - t_0| < h$, where $h = \min\{a, \frac{b}{M}\}$ and $\max_{(\eta, \zeta) \in D} |F(\eta, \zeta)| \leq M$ [2].*

In this study, Euler's method, which is a well-known numerical method, has been used for a better understanding of the step size strategies. Let us give Euler's method now.

Definition 2.2 (Euler's Method). *Euler's method for Cauchy problem (2.1) is defined by*

$$\begin{aligned} Y_i &= Y_{i-1} + hF(t_{i-1}, Y_{i-1}), \\ Y_0 &= X_0, \end{aligned}$$

where $Y_i = (y_{ij}) \in \mathbb{R}^N$ and $h = t_i - t_{i-1}$.

We let give concept of local error which is given for Cauchy problem (2.1) [3].

Definition 2.3. *Let the $Z(t)$ be a solution of the Cauchy problem (2.1) over the interval $[t_{i-1}, t_i]$,*

$$Z'(t) = F(t, Z), Z(t_{i-1}) = Y_{i-1}, Y_0 = X_0. \quad (2.3)$$

Then the local error LE_i over the interval $[t_{i-1}, t_i]$ is defined as the difference between the numerical solution and the exact solution $Z(t_i)$ in the i step at the end point t_i of the interval considered. That means $LE_i = Y_i - Z(t_i)$. Here $Y_i = (y_{ij}) \in \mathbb{R}^N$. When $N = 1$, local error is $LE_i = y_i - z(t_i)$ [22].

3. Step Size Strategies

There are many studies about the variable step size strategies in literature (for example, [18, 19, 24, 30]). We have designed a web interface for different step size strategies in studies [5, 6, 7, 8, 9, 10, 11, 12]. We let give that strategies now. The following first three step size strategy for the Cauchy problem (2.1) on the region D as in (2.2) in situation which is $N = 1$ has been given.

3.1. Step Size Strategy Based on Error Analysis

Assume that solution of (2.1) is unique and exist on the region D as in (2.2). It should be taken local error into consideration for obtaining the numerical solution to be used instead of the exact solution of Cauchy problem (2.1). So, the step size in the i -th step has been obtained

$$h_i = \sqrt{\frac{2\delta_L}{M_i}}$$

in [6, 7, 8, 9, 10] such as local error of the numerical method used is smaller than required error level δ_L each step of the integration, where $\max_{\tau \in [t_{i-1}, t_i]} |z''(\tau)| \leq M_i$.

3.2. Step Size Strategy Based on Picard Theorem

Assume that solution of (2.1) is unique and exist on the region D as in (2.2). It has been calculated step size in the i -th step

$$h_i = \min\left\{a, \frac{b}{M_i}\right\}$$

by applying successively the Picard-Lindelöf theorem on the region $D_i = \{(t, y) : |t - t_i| \leq h_i, |y - y_i| \leq b\}$, each subregion of D , where $\max_{(\eta, \zeta) \in D_i} |f(\eta, \zeta)| \leq M_i$. y_i is the numerical solution obtained in the i -th step [5, 7].

3.3. Step Size Strategy Based on Picard Theorem and Error Analysis

In a region where the existence and uniqueness of the solution of Cauchy problem (2.1) is unknown, the using of step size strategy based on Picard theorem causes the situation in Figure 1.2 because of the calculation errors. To overcome this problem, step size strategy based on Picard theorem and error analysis has been developed by combining step size strategy based on Picard theorem and step size strategy based on error analysis. The step size in i -th step has been obtained

$$h_i = \min\left\{a, \frac{b_{0i-1}}{M_i}\right\}$$

in [5, 7]. Here $z(t)$ is the solution of Cauchy problem (2.1), y_i is the numerical solution obtained in the i -th step, b_{i-1} is a bound for local error $|z - y_{i-1}|$, $b_{0i-1} = \min\{b_{0i-2}, b_{i-1}\}$, $D_{i-1} = \{(t, x) : |t - t_{i-1}| \leq a, |z - y_{i-1}| \leq b_{0i-1}\}$ and M_i is a bound for $f(t, z)$ on the rectangle D_{i-1} .

The following the step size strategies have been given for $N \neq 1$.

3.4. Step Size Strategy for The Numerical Integration of Systems of Linear Differential Equations

This step size strategy is a strategy produced by using step size strategy based on error analysis which is given for first order differential equations for numerical integration in case of $F(t, X) = AX$, where $A = (a_{ij}) \in \mathbb{R}^{N \times N}$, $X \in \mathbb{R}^N$. So, the step size in the i -th step has been obtained

$$h_i = \frac{1}{\alpha \sqrt[4]{N^5}} \sqrt{\left(\frac{2\delta_L}{\beta_{i-1}}\right)}$$

in [8, 9, 10, 11] such as local error of the numerical method used is smaller than required error level δ_L each step of the integration, where

$$\alpha = \max_{1 \leq i, j \leq N} |a_{ij}|, \quad \max_{1 \leq i, j \leq N} \left(\sup_{\tau_i \in [t_{i-1}, t_i]} |z_j(\tau_i)| \right) \leq \beta_{i-1}.$$

3.5. Step Size Strategy for The Numerical Integration of Systems of Non-Linear Differential Equations

We let assume that the solution of the Cauchy problem (2.1) in case of $F(t, X) = AX + \varphi(t, X)$ is unique and exists on the region D , where $\varphi \in C^1([t_0 - a, t_0 + a] \times \mathbb{R}^N)$, $A = (a_{ij}) \in \mathbb{R}^{N \times N}$, $X \in \mathbb{R}^N$. The step size in the i -th step has been obtained

$$h_i = N^{-0.25} \sqrt{\frac{2\delta_L}{N^2 \alpha^2 \beta_{i-1} + N \alpha \gamma_{i-1} + \zeta_{i-1}}}$$

in [9, 10], where

$$\begin{aligned} \alpha &= \max_{1 \leq i, j \leq N} |a_{ij}|, \\ \max_{1 \leq i, j \leq N} \left(\sup_{\tau_i \in [t_{i-1}, t_i]} |z_j(\tau_i)| \right) &\leq \beta_{i-1}, \\ \max_{1 \leq j \leq N} \left(\sup_{\tau_i \in [t_{i-1}, t_i]} |\varphi_j(\tau_i, z(\tau_i))| \right) &\leq \gamma_{i-1}, \\ \max_{1 \leq j \leq N} \left(\sup_{\tau_i \in [t_{i-1}, t_i]} \left| \frac{d\varphi_j}{dt}(\tau_i, z(\tau_i)) \right| \right) &\leq \zeta_{i-1}. \end{aligned}$$

4. About Python Programming Language

Python is a high-level programming language developed by Guido van Rossum at the National Research Institute of Mathematics and Computer Science in the Netherlands in the late 1980s. Python is used to develop software in companies and organizations such as Google, Yahoo, CERN, NASA. Python is free software and can be easily downloaded from <http://python.org/>.

Python has simple and elegance syntax. It has many standard libraries. And it has powerful scientific libraries such as NumPy, SciPy, SymPy, and Matplotlib too. It can run on many platforms such as Mac OS X, Linux, and Windows. Python is a general-purpose programming language that provides system programming, graphical user interface programming for desktop, database programming, numerical and scientific programming and much more. Since Python is an object-oriented language, it allows developing web projects. To develop a web project, each programming language has its own web application framework. So, what is the web framework?

The web framework is the project infrastructure that makes the skeleton of the software which is used to make the users more healthy and fast coding. There are web frameworks such as Flask, Bottle, and Django for web programming with Python. In this study, a web project was developed by using the Django web framework. Django is an open-source web framework using the MTV architecture pattern written in Python programming language to develop dynamic websites, web applications, and web services.

Django provides an environment for the rapid development of powerful and dynamic web applications. The URL system in Django is very flexible and powerful. It allows define patterns for the URLs and to define Python functions to handle each pattern. Django comes with an administration interface that ready to use. This interface provides the management of application's data. It is also highly flexible and customizable. Django also comes with a web server for development and testing and provides a very nice development environment. When debug mode is enabled, Django provides very detailed error messages along with debugging information. More information about Python and Django, see [13, 15, 23, 26, 28, 31].

5. Web Interface

The programs that can run from the command screen with the Python language has been written in order to implement the step size strategies given in section 3. Any user who will use the programs written should have some preliminaries such as knowing Python language, understanding the structure of the algorithm and knowing how to make data entry of the Cauchy problem to the related program. But then any user may not have these preliminaries. To overcome this problem, it has been designed a graphical user interface which is accessible online. Thus it has been provided that any user which has no programming skills can easily use the programs written for the step size strategies. Let us introduce the web interface designed.

In the top menu on the web interface designed, there are menus such as main page, strategies, calculations, simulations, user guide, contact, registration, log in. There are articles about the related step size strategies, contact information, and links of the programs used in designing the web interface in the lower right corner of the web interface.

If the user logs in to the website after registration, some download buttons become active in *Calculations* and *Simulations* on the website. The user can use the download buttons which becomes active to download the graphics of the numerical solution values obtained and the excel file of the numerical solution values obtained after any Cauchy problem is solved.

Let us introduce the menus of the web interface designed.

5.1. Home

The home page contains information about what the site provides to the user and the programming language which is used in the designing of the web interface. It has been given the view of the homepage in Figure 5.1.

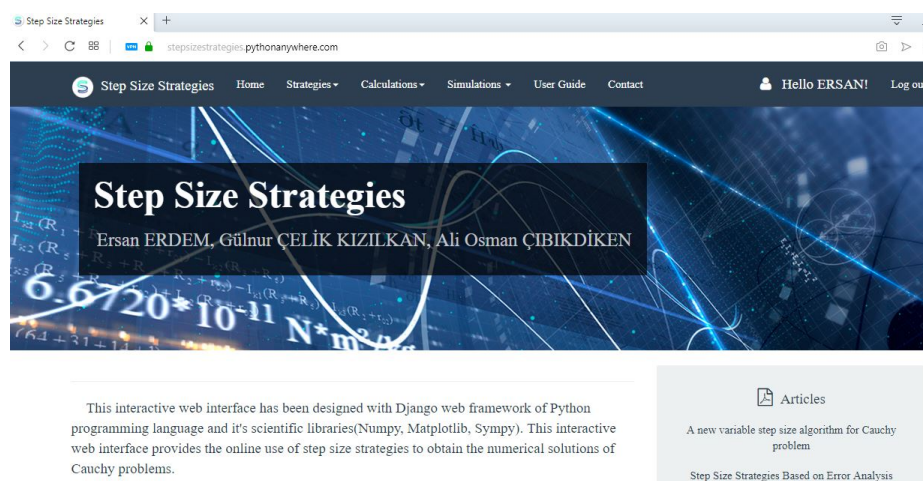


Figure 5.1: Home page.

5.2. Strategies

In this section of the web interface, it has been introduced the step size strategies given in Section 3. If any step size strategy is selected from the *Strategies* menu, comes the page which is introduced the related step size strategy. Note that there is h^* input area in the all input sections

on the web interface. Here h^* is the user-defined practical step size parameter. More information see [7].

5.3. Calculations

It is the section that provides to be applied the step size strategies given in Section 3 and to be obtained the numerical solution values of any Cauchy problem.

In order to obtain a numerical solution for a given Cauchy problem, it is first determined to be used which step size strategy. When the step size strategy which will be used from *the calculations menu* in the top menu bar on the web interface is selected, it is redirected to the page of the step size strategy to be used. It is made data entry of the related Cauchy problem to the input section in the page which came across and it is clicked on the *calculate* button. After it clicks on the *calculate* button, the calculation operation of the related Cauchy problem's numerical solution starts. When the all computing operations finished, the numerical solution values obtained is shown as a table on the same page.

5.4. Simulations

It is the section that provides to plotted graphics of the numerical solution values obtained with the step size strategies given in section 3 of any Cauchy problem. As it is made calculation section, it is chosen the related strategy from *the simulations menu* in the top menu bar of the web interface and made data entry of the Cauchy problem to the input section and it is clicked on the *simulate* button. After it clicks on the *simulate* button, the graphics plot operation of the related Cauchy problem's numerical solution values obtained starts. When the all graphic plot operations finished, the graphics obtained is shown in the same page.

5.5. User Guide

Arithmetic operation operators, mathematical constants and functions to be used when entering data to the data entry screen in *the Simulations menu* and *the Calculations menu* are given in Table 2, Table 3, and Table 4, respectively. Decimal expressions must be used with dot.

Table 2: Data entry of arithmetic operators

Addition	Subtraction	Multiplication	Division
+	-	*	/

Table 3: Data entry of functions

Functions	Data Entry
x^n	$x^{**}n$
\sqrt{x}	$sqr(x)$
e^x	$exp(x)$
$\ln(x)$	$log(x)$
$\log_n(x)$	$logn(x)$
$\sin(x), \cos(x), \tan(x), \cot(x)$	$\sin(x), \cos(x), \tan(x), \cot(x)$
$\arcsin(x), \arccos(x), \arctan(x)$	$\arcsin(x), \arccos(x), \arctan(x)$
$\sinh(x), \cosh(x), \tanh(x), \coth(x)$	$\sinh(x), \cosh(x), \tanh(x), \coth(x)$
$\operatorname{arcsinh}(x), \operatorname{arccosh}(x), \operatorname{arctanh}(x)$	$\operatorname{arcsinh}(x), \operatorname{arccosh}(x), \operatorname{arctanh}(x)$

Table 4: Data entry of mathematical constants

Mathematical constants	Data Entry
π	pi
e	$exp(1)$

6. Numerical Examples

Example 6.1. Consider the Cauchy problem (1.1) which its solution is undefined at $t = 4$ point which we discussed above. Since the solution exact of Cauchy problem (1.1) does not exist at $t = 4$ point, the using of step size strategy based on Picard theorem and error analysis will be appropriate for this problem. Let us calculate the step size and the numerical solution of Cauchy problem (1.1) using step size strategy based on Picard and error analysis on the web interface. Let us take $h^* = 10^{-4}$.

The step size strategy based on Picard theorem and error analysis from *the calculations menu* in the top menu bar on the web interface is selected and the page of the step size strategy comes. Data entry of the Cauchy problem to the input section is made as shown in Figure 6.1 and then it is clicked on the *calculate* button. In this case, data entered is sent to the program of the related strategy which runs in the background and the computing operation of the numerical solution is started. When all computing operations finished, the numerical solution values obtained is shown as a table on the same web page as shown in Figure 6.2. If it is clicked on the *output file* button, it can be downloaded the numerical solution values obtained as an excel file.

Step Size Strategies Home Strategies Calculations Simulations User Guide Contact

Step Size Strategy Based on Picard Theorem and Error Analysis

Input

$f(t, x) = x^{**2}$ $t_0 = 3$ $x_0 = 1$ $a = 2$ $b = 5$

$h^* = 0.0001$

Calculate Output file

Output

Show 10 entries

i	h_i	t_i	x_i
No data available in table			

Figure 6.1: Data entry of the Cauchy problem (1.1) to the area of calculation

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Step Size Strategy Based on Picard Theorem and Error Analysis

Input

$f(t, x) = x^{**2}$ $t_0 = 3$ $x_0 = 1$ $a = 2$ $b = 5$

$h^* = 0.0001$

Simulate Numeric Solution Step Size

Output

Show 10 entries

i	h_i	t_i	x_i
1	0.14802226	3.13888889	1.13888889
2	0.15443268	3.28691115	1.33088381
3	0.15056556	3.44134383	1.60442295
4	0.12131382	3.59190939	1.99200476
5	0.06103690	3.71322321	2.47338807
6	0.01197094	3.77426011	2.84679040
7	0.00042391	3.78623106	2.94380551

Showing 1 to 7 of 7 entries Previous 1 Next

Figure 6.2: The numerical solution values of the Cauchy problem (1.1)

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Step Size Strategy Based on Picard Theorem and Error Analysis

Input

$f(t, x) = x^{**2}$ $t_0 = 3$ $x_0 = 1$ $a = 2$ $b = 5$

$h^* = 0.0001$

Simulate Numeric Solution Step Size

Figure 6.3: Data entry of the Cauchy problem (1.1) to the area of simulation

If it is desired to plot the graph of the solution values obtained, it is chosen the related strategy from *the simulations menu* in top menu bar of the web interface and data entry of the Cauchy problem to the input section is made as shown in Figure 6.3 and then it is clicked on the *simulate* button. And the graphics plot operation is started. When the all graphic plot operations finished, the alteration graphic of the step size as shown in Figure 6.4 and the graphic of the numerical solution of Cauchy problem as shown in Figure 6.5 on the same web page. If it is clicked on the *numerical solution* button and the *step size* button, it can be downloaded the graphic of the numerical solution and the graphic of the step size respectively.

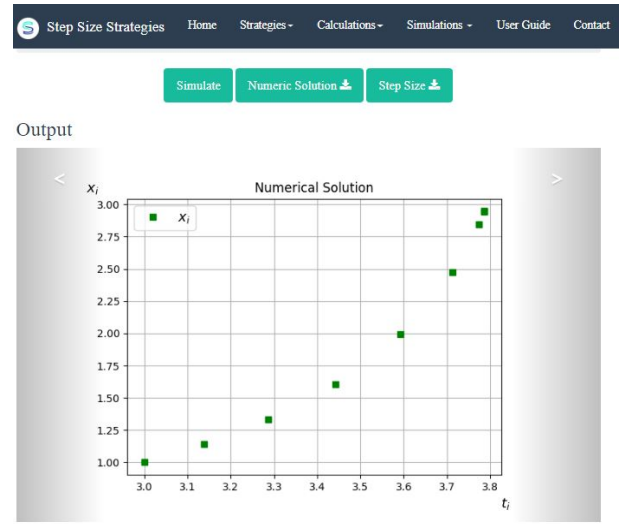
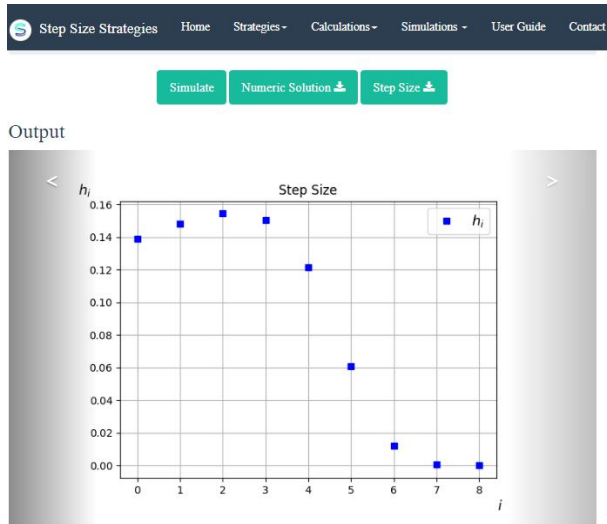


Figure 6.4: The graphic of the step size values obtained for Cauchy problem (1.1)

Figure 6.5: The graphic of the numerical solution values obtained for Cauchy problem (1.1)

Notice that the alteration graphic of the step size at given in Figure 6.4. While it has been used small step sizes in the region which the solution is changing rapidly, it has been used big step sizes in the region which the solution is changing slowly. Since the exact solution of Cauchy problem (1.1) does not exist at $t = 4$ point, it has not been computed the solution at $t = 4$ point thanks to the step size strategy used. For this reason, the computed solution can be used instead of the exact solution of the Cauchy problem (1.1).

Example 6.2. The well-known SIR epidemic model which is one of the real world scenario is given by the equation (6.1).

$$\begin{aligned} \frac{dS}{dt} &= -\frac{\beta SI}{N} \\ \frac{dI}{dt} &= \frac{\beta SI}{N} - \gamma I \\ \frac{dR}{dt} &= \gamma I, \end{aligned} \tag{6.1}$$

where β is contact rate, γ is mean recovery rate, N is number of the population [1, 20, 21].

The SIR model is an epidemiological model that mathematical description of the spread of a disease in a fixed population. This model divides the population into three groups as **Susceptible** (people who are susceptible of the disease but not yet infected with the disease), **Infected** (people who are infected with the disease), **Recovered** (people who have acquired a persistent immunity).

Let us take $\beta = 0.3$, $\gamma = 0.1$, $N = 1000$, $I(0) = 1$, $R(0) = 0$, $S(0) = N - I(0) - R(0)$. In this case the sytem of non-linear differential equations which given by (6.1) can rewritten as

$$\begin{pmatrix} S' \\ I' \\ R' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0.1 & 0 \end{pmatrix} \begin{pmatrix} S \\ I \\ R \end{pmatrix} + \begin{pmatrix} -0.0003SI \\ 0.0003SI \\ 0 \end{pmatrix}, \quad \begin{pmatrix} S(0) \\ I(0) \\ R(0) \end{pmatrix} = \begin{pmatrix} 999 \\ 1 \\ 0 \end{pmatrix} \tag{6.2}$$

where $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0.1 & 0 \end{pmatrix}$, $\varphi = \begin{pmatrix} -0.0003SI \\ 0.0003SI \\ 0 \end{pmatrix}$ for applying step size strategy for the numerical integration of systems of non-linear differential equations. Consider the Cauchy problem (6.2) on $D = \{t \in [0, 180], |y_i - y_{i0}| \leq 10\}$.

Let us apply step size strategy for the numerical integration of system of non-linear differential equations to Cauchy problem (6.2) using the values of $h^* = 10^{-4}$, $\delta_L = 0.08$.

It can be made the calculation for the systems of differential equations which its dimension is 2x2, 3x3 and 4x4 on the web interface. Firstly, it is determined the dimension of the Cauchy problem and is determined whether it's linear or not. According to the dimension of the Cauchy problem and whether it's linear or not, it is selected an appropriate the step size strategy from *the calculations menu* in the top menu bar on the web interface.

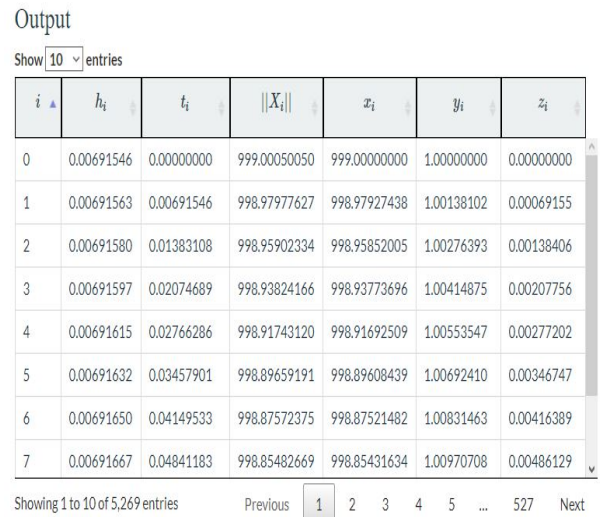
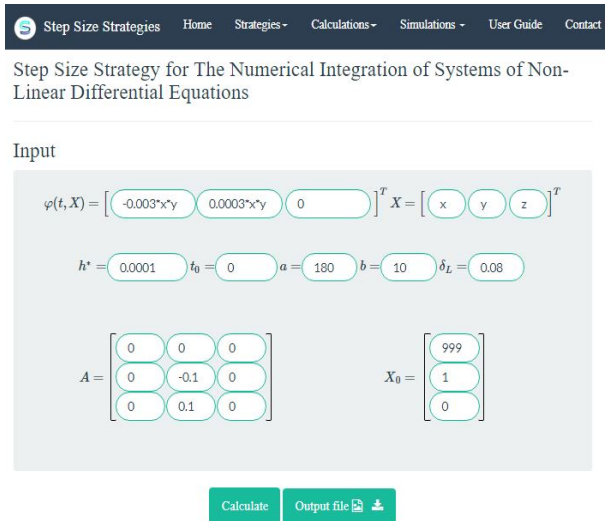


Figure 6.6: Data entry of the Cauchy problem (6.1) to the area of calculation Figure 6.7: The numerical solution values obtained for Cauchy problem (6.1)

It is chosen the step size strategy which is used for the numerical integration of systems of non-linear differential equations which its dimension is 3x3 from the calculations menu in the top menu bar on the web interface for the Cauchy problem (6.1). Data entry of the Cauchy problem (6.1) to the input section is made as shown in Figure 6.6. Here S, I, R is x, y, z respectively. And then it is clicked on the calculate button.

After the all computing operations finished, the numerical solution values obtained is shown as a table on the same web page as shown in Figure 6.7. If it is clicked on the output file button, it can be downloaded the numerical solution values obtained as a excel file. If it is desired to plot the graph of the solution values obtained, it is chosen the related strategy from the simulations menu in top menu bar of the web interface and data entry of the Cauchy problem (6.1) to the input section is made as shown in Figure 6.8 and then it is clicked on the simulate button. After the all graphic plot operations finished, the graphic of the norm of Cauchy problem's numerical solution, the alteration graphic of the step size, the graphic of the Cauchy problem's first component, the graphic of the Cauchy problem's second component, the graphic of the Cauchy problem's third component as shown in Figure 6.9, Figure 6.10, Figure 6.11, Figure 6.12, Figure 6.13 on the same web page respectively.

If it is clicked on the norm of numerical solution button, the step size button, the first component button, the second component button, the third component button, it can be downloaded the graphic of the numerical solution's norm, the graphic of the step size, the graphic of first component, the graphic of second component, the graphic of third component respectively.

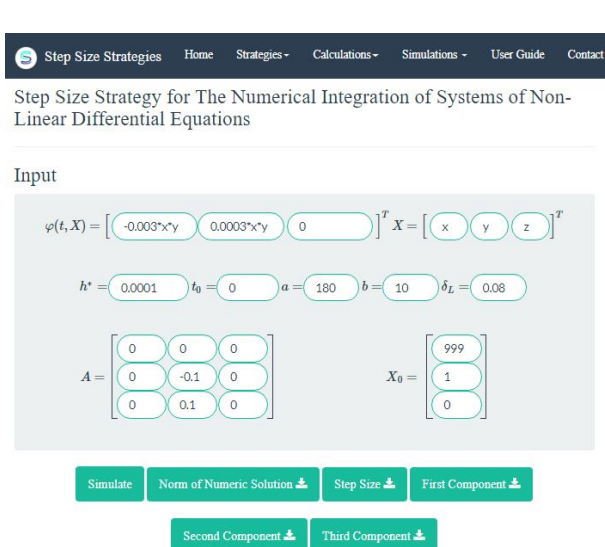


Figure 6.8: Data entry of the Cauchy problem (6.1) to the area of simulation Figure 6.9: The graphic of the numerical solution values obtained for Cauchy problem (6.1)

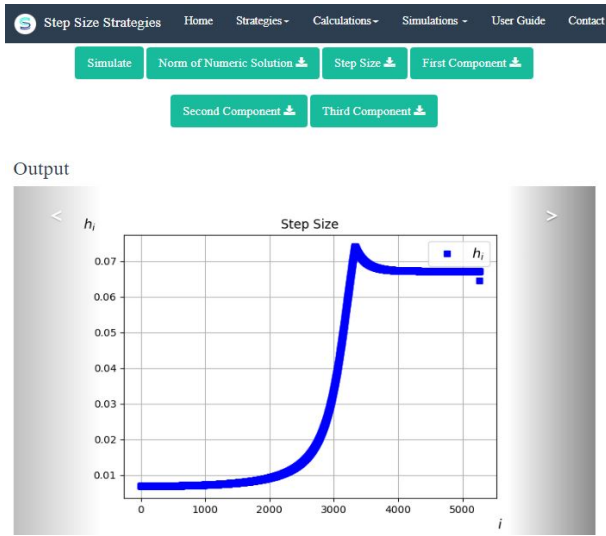


Figure 6.10: The graphic of the step size values obtained for Cauchy problem (6.1)

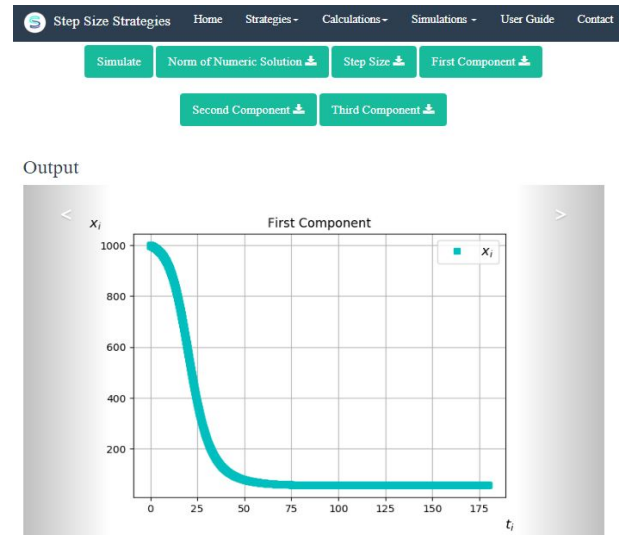


Figure 6.11: The graphic of the numerical solution values obtained for the first component of Cauchy problem (6.1)

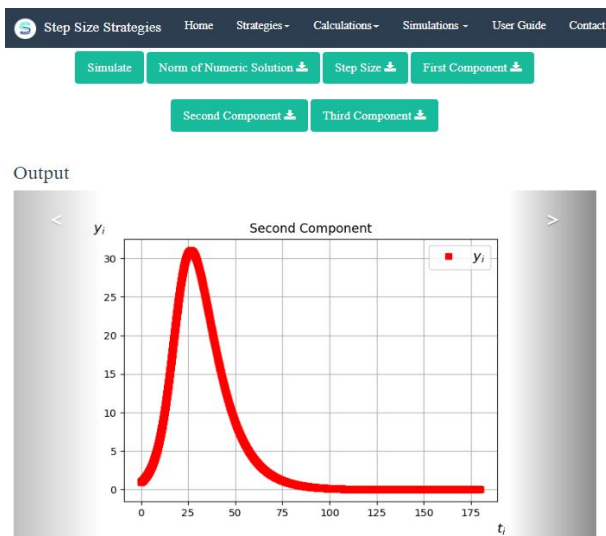


Figure 6.12: The graphic of the numerical solution values obtained for the second component Cauchy problem (6.1)

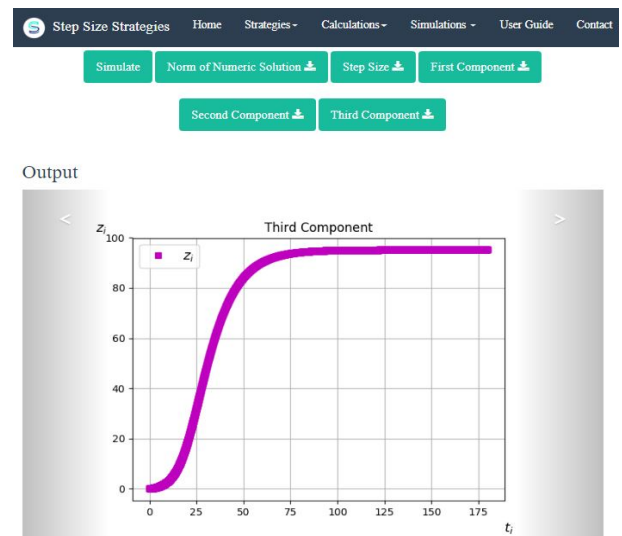


Figure 6.13: The graphic of the numerical solution values obtained for the third component of Cauchy problem (6.1)

7. Conclusions

In this study, an interactive web interface has been designed with the web framework called Django by using the power of Python language to create awareness about step size strategies and to expand the use of these strategies. This web interface takes the data of the Cauchy problem as input and solves the problem and then gives the solution values and graphs of these values as output. In addition, the obtained solution values as excel files and graphs of these values as pictures can be downloaded. This web interface is available online at <https://stepsizestrategies.pythonanywhere.com>.

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