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THE RELATIONSHIP BETWEEN UTILITY AND RISK: EXAMPLES FROM ECONOMICS

Fayda ve Risk Arasındaki İlişki: İktisattan Örnekler

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Abstract: When the issues in economics are analyzed deeply, the notion of utility is seen at the heart. As an important concept utility research has stayed popular so far. There are several research examining utility from different perspectives.

One of the important topics that utility is associated with is risk preferences. Utility perception deviates from what economic theory predicts when it comes to make a choice among alternatives involving risk. Besides utility perception and risk preferences variate if the alternative is a monetary gain or a monetary loss violating principal rules on utility maximization.

This study presents a detailed outlook of the concept of utility within the context of attitudes towards risk. It aims to explain that utility consideration differentiates according to the parameters of the situation rather than to the mathematical expected value calculation referring to the St. Petersburg Paradox.

Keywords: Utility, Risk, St. Petersburg Paradox.

Öz: İktisattaki konular analiz edildiğinde fayda kavramı bu konuların kalbinde yer almaktadır. Önemli bir konsept olarak fayda araştırması şimdiye kadar popülerliğini korumuştur. Faydayı farklı perspektiflerden inceleyen çeşitli araştırmalar bulunmaktadır.

Faydanın ilişkili olduğu önemli konulardan biri risk tercihleridir. Fayda algısı, risk içeren alternatifler arasında seçim yapmak sözkonusu olduğunda iktisat teorisinin öngördüklerinden sapmaktadır. Bunun yanında fayda algısı ve risk tercihleri sözkonusu alternatifin parasal bir kazanç ya da parasal bir kayıp olmasına göre fayda maksimizasyonuna ilişkin temel prensipleri ihlal ederek değişmektedir.

Bu araştırma fayda konseptinin detaylı bir bakış açısını riske karşı tavırlar çerçevesinde sunmaktadır. Çalışma fayda değerlendirmesinin matematiksel beklenen fayda hesaplamasından ziyade durumun parametrelerine göre farklılaştığını St. Ptersburg Paradoksuna atfen açıklamayı amaçlamaktadır.

Anahtar kelimeler: Fayda, Risk, St. Petersburg Paradox.

INTRODUCTION

Utility has been the most popular research topic in economics. Because all of the other structures built upon the concept of utility; it has been always attractive to the researchers from economics and from several other disciplines such as psychology, biology and neuroscience. Measuring utility has been the most controversial issue in economics. As a numeric measure and an indicator of a person's happiness and well-being utility has been conceptualized as "cardinal utility" enables quantitative measurement of utility and "ordinal utility" (enables relative ranking of preferences (Varian, 1987:54). How to measure utility had been discussed in classical economics before Pareto and the neoclassicals abandoned; Francis Y. Edgeworth (1845-1926) and Frank Ramsey (1903-1930) dreamed of a "hedonimeter" and "psychogalvanometer" respectively that could measure utility directly and Irving Fisher (1867-1947) wrote extensively due to frustration about direct utility measurement (Camerer, 2007:40). While economists such as Allen, Hicks and Samuelson were against to the concept of cardinal utility; von Neumann and Morgenstern, Arrow , Friedman and Savage and Markowitz were engaged in a groundwork for cardinal utility (Allen and Hicks, 1934; Samuelson, 1938; von Neumann and Morgenstern, 1944; Friedman and Savage, 1948; Markowitz, 1952; Arrow, 1971; Mas-Collel et al., 1995; Andreas, 2010; Friedman and Sunder 2011).

Although risk is usually defines as uncertainty in daily life; these two notions are different from each other in economic analysis and were first distinguished by Frank Knight in 1921(Knight, 1921). Basically if probability values of a state is known then this state is called to be risky if not uncertain. In economic theory attitudes towards risk are examined using utility functions. There are three types of attitudes towards risk; risk aversiveness, risk neutrality and risk seeking. These three behavior variations towards risk are decided comparing the utility values attached to the situations. For example a risk averse individiual attaches higher utility value to a certain option than the risky one, on the contrary a risk- lover attaches higher utility to the risky option and finally for a risk neutral individual the utility values both attached to the certain and risky alternatives are the same. Regarding to their attitudes towards risk these three types of individuals' utility functions have different shapes as their indifference curves. While the utility function of a risk averse individual is concave, it is convex for a risk lover and is linear for a risk-neutral.

Insurance and gambling are also connected to risk and utility perception together. Besides the higher levels of insurance industry's value added to GDP; there have been and there still are countries where large amounts of revenue yielded from the gambling industry. Insurance sector prevents individuals, investors and firms from risk corresponding to a premium payment. While the premium is paid to

make insurance to avoid from risk, it is also paid to gamble to take risk which is not consistent with a so called rational individual who prefers both at the same time (Friedman and Savage, 1948).

Risk and utility have been the most popular research topics in economic analysis. There is a considerable research examining these two concepts together or separately. The aim of the paper is to explain the relationship between utility and risk using St. Petersburg Paradox. In accordance with this purpose theoretical roots concerning to the relationship between risk and utility is revealed within the context of St. Petersburg Paradox. Final section concludes.

Theoretical Roots Concerning To The Relationship Between Risk And Utility: St. Petersburg Paradox

Theoretical roots of the relationship between utility and risk can be seen clearly from the rise of the St. Ptersburg Paradox. Up to the presentation of the paradox with a paper titled "Exposition Of A New Theory On The Measurement Of Risk" which is read to the Imperial Academy of Sciences in Petersburg by Daniel Bernoulli; so many scientists from so many different disciplines involving physicians, mathematicians, astrologers, physicists, astronomers and a priest were included in the process. Luca Pacioli, Gerolamo Cardano, Pierre de Fermat and Blaise Pascal, Chevalier de Mere, Boltzmann, Halley, Graunt, Christiaan Huygens, Nikolaus Bernoulli, Pierre Reymon De Montmort, Gabriel Cramer, Daniel Bernoulli, Georges Louis Leclerc Comte de Buffon, Isaac Todhunter, William Allen Whitworth, Kolmogorov made valuable contributions (Pacioli, 1494; Pascal and de Fermat, 1654; Huygens, 1656; Graunt, 1662; Halley, 1693; De Montmort, 1713; Cramer, 1728; Bernoulli, 1738; Buffon, 1777; Todhunter, 1865; Boltzmann, 1871; Whitworth, 1901; Ehrenfest and Ehrenfest 1912; Keynes, 1921; Kolmogorov, 1933, Menger, 1934; Debreu, 1951; Markowitz, 1952; Allais, 1953; Ore, 1953; Samuelson, 1960, 1977;Arrow, 1971; Bassett, 1987; Tversky, and Kahneman 1992Cohen, 1996; Dehlig, 1997; Stigler, 1999; Gell-Mann and Lloyd, 2004; Szekely and Richards, 2004; Sharpe, 2007; Devlin, 2008; Pickover, 2009; Peters, 2011,Mukhopadhyay, 2012; Shlesinger, 2012; Garcia, 2013; Salov, 2014, Britannica, 2019).

Mathematicians Pascal and de Fermat were interested in calculation the value of games of chance. After mathematician Girolamo Cardano published "The Book of Games of Chance" in 1565, they were involved in solving the problem proposed by Chevalier deMe´re´ in 1654(Akyıldırım and Soner, 2014). After Pascal and Fermat's ideas inspired Christian Huygens who presented the "value" of a game for the first time; they suggested that the attractiveness of a gamble is related to its expected value and in 1713, St. Petersburg Paradox and its solution was sent to De Montmort by Nikolaus Bernoulli and following the engagement of Gabriel Cramer to the paradox Daniel Bernoulli got involved (Dehling, 1997). The scenerio of the paradox simply based on flipping a fair coin until getting tails. If the first flip is a tails, it is won \$2; if tails comes on the second flip, it is won \$4; if tails is on the third flip, it is won \$8...So the expected value of the game is:

$$\sum_{n=1}^{\infty}(1/2)^n 2^n=\infty$$

The expected value of the paradox is infinite. Because the game offers an infinite expected gain it is expected that any rational individual is expected to take the gamble and to be willing to pay any price to take it. However it is seen that the number of the individuals who are willing to take the gamble is minuscule when it is compared to the infinite expected value of the game. That is what makes the situation as a paradox. Georges Louis Leclerc, Comte de Buffon who is also famous with the problem which is known as "Buffon's Needle" had a child played the game 2,048 times and Buffon concluded that the fair entrance fee is approximately \$10(Sz´ekelyand Richards, 2004).

To solve the paradox Bernoulli suggested log utility rather than a linear one, emphasizing the difference of the impact of a spesific amount of money to a rich and a poor referring to the diminishing marginal utility characteristics which also includes a moral approach. So risk decisions are expected to related to the wealth however gambling addiction provides evidence a counterexample to this rational expectation. The discovery of Daniel Bernoulli was the consideration of the utility not the price.

Although this scenerio is criticized as being unrealistic as there is not an infinite amount of money in the world to make such a payment and as it carries an infinite expected loss for the lottery-seller (Cramer, 1728; Samuelson, 1960; Peters, 2011); St. Paradox reveals the notable difference between expected value and expected-utility which was not clarified in those days yet.

John Von Neumann and Oskar Morgenstern were the ones to clarify that individuals consider "expected utility" when making a decision between risky alternatives. They suggested "The Theory Of Expected Utility" in 1944 with a strictly increasing, concave utility function conserves cardinal property where the form of expected utility is preserved only by increasing linear transformations with objective probabilities and represents a consistent ranking of lotteries having at least two continuous derivatives(Kreps, 1990:70, 84; Mas-Colell et. al., 1995:173; Muñoz- Dierks, 2005:17; Slantchev, 2005:9; Herfert, 2006:18; Levin, 2006:6, 8, 29; Mobius, 2008:5; Yanoff, 2012:501; Garcia, 2017:337). According to the theory of expected utility decision makers choose between alternatives by comparing the expected utility values that are the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities (Mongin, 1998:171). Thus the expected utility for the two-outcome lottery L=(P, A, B) where the outcomes and their probabilities are denoted by A, B and P, (1-P) respectively is E[U(L)]=PU(A)+(1-P)U(B) (Henderson and Quandt, 1980:54). The theory of expected utility poses a set of axioms to be satisfied to define rational behavior and decision making. The axioms of the theory of expected utility are determined as invariance, completeness (complete-ordering), transitivity , continuity, independence, unequal-probability, Archimedean, monotonicity and substitution (independence of irrelevant alternatives)(Tversky, 1969; Kahneman and Tversky, 1979; Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Tversky and Kahneman, 1986; Holt, 1986; Machina, 1987; Loomes et. al. , 1991; Carlin, 1992; Abdellaoui, 2002:3; Kahneman, 2003; Tversky, 2004; Slantchev, 2005:8; Levin, 2006:5, 9; Mobius, 2008:5; Birnbaum and Schmidt, 2008; Shon, 2008:2; Dean, 2009:6; Board, 2009:2, 9; Day and Loomes, 2010; Föllmer et. al. , 2016:58,) to explain behavior under risk considering utility. The theory of expected utility had appeared almost as a final state of the St. Petersburg Paradox. However it didn't take too much for the theory to be questioned. The first counterexample was presented by Maurice Allain in 1953 which is known as "Allais Paradox" in the literature. Allais showed that economic behavior under risk violates the independence axiom which is known as the heart of the theory of expected utility. In 1979 using a variation of Allais' example Kahneman and Tversky suggested "Prospect Theory" to contribute to the theory of expected utility to better understand economic behavior under risk. Introducing the "value function" which is concave in the gain, convex and also steeper in the loss domains and has a kink at the origin; Kahneman and Tversky proved that economic behavior under risk differentiates according to monetary gains and losses. Kahneman and Tversky demonstrated that individuals are more sensitive to monetary losses compared to the same amount of monetary gains. Kahneman and Tversky also presented the "certainty effect" which contributes to risk aversiveness in sure gains and risk-seeking in sure losses. They called attention to a reference point where value attached to the gains and losses. By prospect theory it is provided evidence that individuals don't consider final asset position as it is assumed in canonical economic theory but they consider gain and loss. That is why monetary losses are

overweighted. While Kahneman and Tversky interested in value rather than utility in the prospect theory; several different expected utility variants with several parameters have been suggested in time (Schoemaker, 1982).

These explanations show that risk and utility are both included together in theoretical journey. This finding sheds light on that if the future work aims to carry a step further existing models and theories it will need to embody utility and risk together rather than ignoring one or adopting pure ceteris-paribus.

Conclusion

From the two mathematicians Blaise Pascal and Pierre de Fermat's research on calculating the expected value of all kinds of games of chance; decision making under risk has been investigated. Decision making under risk has been the key concerns in economics as well. St. Petersburg Paradox is the bridge between Blaise Pascal and Pierre de Fermat's research in the 17th century and John von Neumann and Oskar Morgenstern's in 1944. In other words the paradox is the bridge between the notions "expecped value" and "expected utility".

Risk perception has considerable outcomes from both macroeconomic and microeconomic perspectives. Because financial industry structures frequently can encourage risk, exceeding risk or the optimal risk; risk has an important role at the macro level as taking risk triggers GDP growth whereas sometimes the reverse is valid. Attitudes towards risk has also important implications for market failures and market inequalities. At the micro level as affected by individual assessments; financial products of the market are expected to effect financial system as a whole. To interpret stock-market, banking system, exchange-rates and even the interest rates without risk analysis will be incompleted. To make proper risk analysis it is necesseary to put utility into the analysis accurately. Risk measurement without a utility assessment may lead not only to credit system collapse but also financial bankruptcies, unemployment and other serious economic problems. Economic behavior is a complicated system of rationality, mood, neurons and emotions. Thus it is needed to make more comprehensive analysis to decide how to get position towards to risky assests to achieve the best possible outcome.

Every decision includes some risk; from a simple decision of going out of the house on a sunny day to having a surgery or to investing in a spesific stock. At the same time every decision is a result of a comparison of the utilities of the alternatives. Risk has an important role in economic life because it is included in every step of economic decisions. Utility is such a concept that underlies economic theory. So utility and risk cannot be investigated separately. The relationship between risk and utility has always been worth examining and has given rise to the theories. The St. Petersburg Paradox is a milestone in the field in this manner enables researchers to discover that individuals consider more than just expected value. And now recent studies show that individuals consider more than utility. It is provided evidence that decision makers are interested in gains and losses, equality, trust and fairness rather than pure utility maximization goal. Thus evaluation of risk and utility needs to be updated in order to find appropriate recipes for real life economic problems. The standard theoretical view needs to be enriched by interdisciplinary perspectives to serve real life economic and financial problems at the national and international level.

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