New generalizations of modular spaces

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Abstract

In the present paper, we introduce the concept of $F$-modular, which is a generalization of the modular notion. Moreover, we introduce a $K_p$-modular and $K$-modular, and then compare these concepts together. Finally, we give a characterization of $F$-modulars.

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1. Introduction

A modular on a space $X$ is a mapping $\rho : X \to [0, \infty]$ satisfying the following properties:

(i) $\rho(x) = 0$ if and only if $x = 0$,

(ii) $\rho(\alpha x) = \rho(x)$ for every scaler $\alpha$ with $|\alpha| = 1$,

(iii) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y)$ for every $\alpha, \beta \geq 0$ such that $\alpha + \beta = 1$.

A modular $\rho$ defines a corresponding modular space, i.e., the vector space $X_\rho$ given by

$$X_\rho = \{x \in X : \rho(\lambda x) \to 0 \text{ as } \lambda \to 0\}.$$

The theory of modular spaces was founded by Nakano [15] and was intensively developed by Luxemburg [10], Koshi and Shimogaki [8] and Yamamuro [18] and their collaborators. In the present time the theory of modulars and modular spaces is extensively applied, in particular, in the study of various Orlicz spaces [16] and interpolation theory [9, 12], which in their turn have broad applications [13]. Shateri [17] introduced the notion of a $C^*$-valued modular, and investigated some fixed point theorems in such spaces.

Recently, many interesting extentsions of the metric space appeared. The notion of a $b$-metric space introduced by Czerwik [2]. Fagin, et al. [3] introduced $s$-relaxed metric spaces. Gähler [4] defined the notion of a 2-metric on the product set $X \times X \times X$. The reader can see more generalizations of the notion of a metric space in [1, 5, 7, 12, 14]. Jleli and Samet [6] introduced the $\mathcal{F}$-metric concept. They defined a natural topology in such spaces, and studied their topological properties.

In this paper, by using some ideas of [6] we introduce the $\mathcal{F}$-modular concept, which is a generalization of the modular space notion. We prove that any modular is an $\mathcal{F}$-modular, but the converse is not true in general, which shows that our concept is more general than the standard modular concept. Moreover, we introduce a $K_p$-modular and $K$-modular,

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and then compare these concepts together. Finally, we introduce the notion of Φ-modulars boundedness, which is used to provide a characterization of Φ-modular spaces.

2. Φ-modulars

We start by introducing the following set which plays an important role in our topic. Let Φ be the set of all functions \( f : (0, +\infty) \to \mathbb{R} \) which satisfy in the following conditions

(\( \Phi_1 \)) \( f \) is non-decreasing,
(\( \Phi_2 \)) For every sequence \( \{t_n\} \) in \((0, +\infty)\), \( \lim_{n \to +\infty} t_n = 0 \) if and only if \( \lim_{n \to +\infty} f(t_n) = -\infty \).

Now we define a new concept of a modular space.

**Definition 2.1.** Let \( X \) be a linear space, and let \( \delta : X \to [0, +\infty) \) be a given mapping. Suppose there exists \( (f, \gamma) \in \Phi \times [0, +\infty) \) such that

- \( (\delta_1) \delta(x) = 0 \) if and only if \( x = 0 \),
- \( (\delta_2) \delta(\alpha x) = \delta(x) \) for every scaler \( \alpha \) with \( |\alpha| = 1 \),
- \( (\delta_3) \) For each \( x, y \in X \), for each \( 2 \leq n \in \mathbb{N} \), and for every \( \{u_i\}_{i=1}^n \) in \( X \) with \( u_1 = x \) and \( u_n = y \), if \( \delta(\alpha x + \beta y) > 0 \) for \( \alpha, \beta > 0 \) in which \( \alpha + \beta = 1 \), then

\[
f(\delta(\alpha x + \beta y)) \leq f\left(\sum_{i=1}^n \delta(u_i)\right) + \gamma.
\]

Then \( \delta \) is called an \( \Phi \)-modular on \( X \), and the pair \( (X, \delta) \) is said to be an \( \Phi \)-modular space.

Note that if \( \rho \) is a modular on \( X \), then it is an \( \Phi \)-modular with \( f(t) = \ln t \) and \( \gamma = 0 \). Clearly \((\delta_1)\) and \((\delta_2)\) satisfied. On the other hand, for each \( x, y \in X \), for every \( 2 \leq n \in \mathbb{N} \), and for every \( \{u_i\}_{i=1}^n \) in \( X \) with \( u_1 = x \) and \( u_n = y \), we have

\[
\ln(\rho(\alpha x + \beta y)) \leq \ln(\rho(x) + \rho(y)) \leq \ln\left(\sum_{i=1}^n \rho(u_i)\right),
\]

for \( \alpha, \beta > 0 \) in which \( \alpha + \beta = 1 \).

In the following example we give an \( \Phi \)-modular space which is not a modular space.

**Example 2.2.** Let \( X = [1, \infty) \), define the mapping \( \delta : X \to [0, +\infty) \) as follows

\[
\delta(x) = \begin{cases} 
x^2 & x \in [1, 2), 
x & x \geq 2,
\end{cases}
\]

for all \( x \in X \). Then \( \delta \) is not a modular. Indeed, \( \delta \) does not satisfy (iii), because for \( x = 1, y = 2, \alpha = \frac{1}{5} \) and \( \beta = \frac{4}{5} \), we get

\[
\delta(\alpha x + \beta y) = \delta\left(\frac{1}{5} + \frac{8}{5}\right) = \delta\left(\frac{9}{5}\right) = \frac{81}{25} > \delta(x) + \delta(y) = 3.
\]

Now, we show that \( \delta \) is an \( \Phi \)-modular. Fix \( x, y \in X \), and let \( \{u_i\}_{i=1}^n \) in \( X \) with \( u_1 = x \) and \( u_n = y \). Put \( I = \{i = 1, \ldots, n; \ u_i \in [1, 2)\} \) and \( J = \{1, 2, \ldots\} - I \), then we have

\[
\sum_{i=1}^n \delta(u_i) = \sum_{i \in I} \delta(u_i) + \sum_{j \in J} \delta(u_j) = \sum_{i \in I} u_i^2 + \sum_{j \in J} u_j.
\]
Now we have two cases.

Case 1: If $\alpha x + \beta y \notin [1, 2)$, then

$$\delta(\alpha x + \beta y) = \alpha x + \beta y$$

$$\leq x + y \leq \sum_{i=1}^{n} u_i = \sum_{i \in I} u_i + \sum_{j \in J} u_j$$

$$\leq \sum_{i \in I} u_i^2 + \sum_{j \in J} u_j$$

$$= \sum_{i=1}^{n} \delta(u_i).$$

Case 2: If $\alpha x + \beta y \in [1, 2)$, then we have

$$\delta(\alpha x + \beta y) = (\alpha x + \beta y)^2$$

$$\leq 2(\alpha x + \beta y)$$

$$\leq 2(x + y)$$

$$\leq 2\left(\sum_{i \in I} u_i + \sum_{j \in J} u_j\right)$$

$$\leq 2\left(\sum_{i \in I} u_i^2 + \sum_{j \in J} u_j\right)$$

$$= 2 \sum_{i=1}^{n} \delta(u_i).$$

The above cases show that $\delta$ satisfies $(\delta_3)$ with $f(t) = \ln t$, $t > 0$ and $\gamma = \ln 2$. Therefore $\delta$ is an $\mathcal{F}$-modular.

Now we define a $K_p$-modular on a space $X$, also we provide an example of an $\mathcal{F}$-modular space that cannot be an $K_p$-modular space, which confirms that the class of $\mathcal{F}$-modular spaces is more large than the class of $K_p$-modular spaces.

**Definition 2.3.** Let $\Delta : X \to [0, +\infty)$ be a mapping satisfies $(\delta_1), (\delta_2)$, and $(\Delta_3)$ There exists $K \geq 1$ such that for every $x, y \in X$, for every $2 \leq n \in \mathbb{N}$, for every $\{u_i\}_{i=1}^{n}$ in $X$ with $u_1 = x$ and $u_n = y$, we have

$$\Delta(\alpha x + \beta y) \leq K \left(\sum_{i=1}^{n} \Delta(u_i)\right),$$

for $\alpha, \beta > 0$ in which $\alpha + \beta = 1$. Then $\Delta$ is called a $K_p$-modular, and $(X, \Delta)$ is said to be a $K_p$-modular space.

It is clear that $\Delta$ satisfies $(\delta_3)$ with $f(t) = \ln t$, $t > 0$ and $\gamma = \ln K$, and hence $\Delta$ is an $\mathcal{F}$-modular. Notice that the mapping $\delta$ in Example 2.2 satisfies in $(\Delta_3)$ with $K = 2$.

The following example shows that the class of $\mathcal{F}$-modulars is more large than the class of $K_p$-modulars.

**Example 2.4.** Let $X = \mathbb{Z}$. Define the mapping $\delta : X \to [0, +\infty)$ by

$$\delta(x) = \begin{cases} \frac{1}{e^{|x|}} & x \neq 0, \\ 0 & x = 0, \end{cases}$$

for all $x \in X$. Then $\delta$ is a $\mathcal{F}$-modular. It is clear that $\delta$ satisfies $(\delta_1)$ and $(\delta_2)$. In order to check $(\delta_3)$, let

$$f(t) = -\frac{1}{t}, \quad (t > 0).$$
It can be easily seen that \( f \in \mathcal{F} \). Fix \( x, y \in X \) and \( \alpha, \beta > 0 \) in which \( \alpha + \beta = 1 \) with \( \delta(\alpha x + \beta y) > 0 \). For every \( n \in \mathbb{N} \), and for every \( \{u_i\} \) in \( X \) with \( u_1 = x \) and \( u_2 = y \), we have

\[
1 + f\left(\sum_{i=1}^{n} \delta(u_i)\right) - f(\delta(x) + \delta(y))
= 1 - \frac{1}{\sum_{i=1}^{n} e^{u_i}} + \frac{1}{e^{\alpha x + \beta y}}
= 1 - \frac{1}{\sum_{i=1}^{n} e^{u_i}} + e^{\alpha x + \beta y}
> 1 + 1 + e^{\alpha x + \beta y}
\geq 0.
\]

Note that the first inequality holds because \( \sum_{i=1}^{n} \frac{1}{e^{u_i}} > 0 \) and so \( -\frac{1}{\sum_{i=1}^{n} e^{u_i}} > 1 \). Therefore we get

\[
f(\delta(x) + \delta(y)) \leq f\left(\sum_{i=1}^{n} \delta(u_i)\right) + 1.
\]

Consequently \( \delta \) is an \( \mathcal{F} \)-modular.

Next, we shall prove \( \delta \) is not a \( K_p \)-modular. Suppose that \( \delta \) satifies (\( \Delta_3 \)) with a certain \( K \geq 1 \). Consider \( u_1 = x = 4n, u_2 = y = 0 \) and \( \alpha = \beta = \frac{1}{2} \). Then we have

\[
\delta(\alpha x + \beta y) = \delta(2n) \leq K(\delta(x) + \delta(y)) = K \delta(4n),
\]

this implies that

\[
e^{2n} \leq K.
\]

Passing to the limit as \( n \to +\infty \), we obtain a contradiction. Therefore, \( \delta \) can not be a \( K_p \)-modular.

In following, we introduce another concept of a modular space which is more large than the class of \( \mathcal{F} \)-modular spaces and \( K_p \)-modular spaces.

**Definition 2.5.** Let \( X \) be a linear space, and let \( \rho : X \to [0, +\infty) \) be a mapping. Let there exists \( K \geq 1 \) such that

1. \( \rho(x) = 0 \) if and only if \( x = 0 \),
2. \( \rho(\alpha x) = \rho(x) \) for every scalar \( \alpha \) with \( |\alpha| = 1 \),
3. \( \rho(\alpha x + \beta y) \leq K(\rho(x) + \rho(y)) \), for \( \alpha, \beta > 0 \) in which \( \alpha + \beta = 1 \).

Then \( \rho \) is called a \( K \)-modular.

Notice that, each modular is a \( K \)-modular with \( K = 1 \). Also every \( K_p \)-modular is a \( K \)-modular. In following we give an example that shows the converse is not true in general.

**Example 2.6.** Let \( X = [0, 1] \), and let \( \delta : X \to [0, +\infty) \) be the mapping defined by

\[
\delta(x) = \begin{cases} x^2 & x \in [0, 1), \\ 0 & x = 1. \end{cases}
\]

It can be easily seen that \( \delta \) is a \( K \)-modular with \( K = 2 \). Next, we prove that \( \delta \) is not an \( \mathcal{F} \)-modular. Suppose that there exists \( (f, \gamma) \) such that \( \delta \) satisfies (\( \Delta_3 \)). Let \( n \in \mathbb{N} \), and put

\[
x = u_1 = 0, y = u_n = 1, u_i = \frac{1}{n}, \quad i = 2, \ldots, n - 1.
\]

Then for \( \alpha = \beta = \frac{1}{2} \), (\( \Delta_3 \)) implies that

\[
f(\delta(x) + \frac{y}{2}) \leq f(\delta(u_1) + \delta(u_2) + \cdots + \delta(u_n)) + \gamma.
\]
Hence \[
f\left(\frac{1}{2}\right) = \frac{1}{4} \leq f\left(\frac{n-2}{n^2}\right) + \gamma.
\]
By (\(\mathcal{F}_2\)), we have
\[
\lim_{n \to +\infty} f\left(\frac{n-2}{n^2}\right) + \gamma = -\infty,
\]
which is a contradiction. Consequently \(\delta\) is not an \(\mathcal{F}\)-modular.
Moreover \(\delta\) is not a \(K_p\)-modular. In fact if \(\delta\) satisfies \((\Delta_3)\), and let
\[
x = u_1 = 0, y = u_n = 1, u_i = \frac{1}{n}, \quad i = 2, \ldots, n - 1,
\]
then for \(\alpha = \beta = \frac{1}{2}\), by \((\Delta_3)\) we conclude that
\[
\delta\left(\frac{x}{2} + \frac{y}{2}\right) \leq \delta(u_1) + \delta(u_2) + \cdots + \delta(u_{n-1}) + \delta(u_n) + \gamma.
\]
Therefore
\[
\frac{1}{2} = \frac{1}{4} \leq \frac{n-2}{n^2}.
\]
By \((\mathcal{F}_2)\), we have
\[
\lim_{n \to +\infty} \frac{n-2}{n^2} = 0,
\]
which is a contradiction.

**Remark 2.7.** One can easily see that the mapping \(\delta\) defined by (2.1) in Example 2.4, is not also a \(K\)-modular on \(\mathcal{X}\).

3. \(\mathcal{F}\)-modular boundedness

In this section, we introduce the concept of \(\mathcal{F}\)-modular boundedness, which is used to provide a characterization of \(\mathcal{F}\)-modular spaces.

**Definition 3.1.** Let \(\mathcal{X}\) be a linear space, and let \(\delta : \mathcal{X} \to [0, +\infty)\) be a mapping satisfying \((\delta_1)\) and \((\delta_2)\). We call the pair \((\mathcal{X}, \delta)\) is \(\mathcal{F}\)-modular bounded with respect to \((f, \gamma)\) \(\in \mathcal{F} \times [0, +\infty)\), if there exists a modular \(\rho\) on \(\mathcal{X}\) such that for every \(x, y \in \mathcal{X}\), and for \(\alpha, \beta > 0\) in which \(\alpha + \beta = 1\), \(\delta(\alpha x + \beta y) > 0\) implies that
\[
f(\rho(\alpha x + \beta y)) \leq f(\delta(x)) + f(\delta(y)) \quad \text{and} \quad f(\rho(\alpha x + \beta y)) \leq f(\rho(\alpha x + \beta y)) + \gamma. \quad (3.1)
\]
We can prove the following result.

**Theorem 3.2.** Let \(\mathcal{X}\) be a space and let \(\delta : \mathcal{X} \to [0, +\infty)\) be a mapping satisfying \((\delta_1)\) and \((\delta_2)\). Let \((f, \gamma) \in \mathcal{F} \times [0, +\infty)\) such that \(f\) is continuous from the right. Then the followings are equivalent:

(i) \((\mathcal{X}, \delta)\) is an \(\mathcal{F}\)-modular on \(\mathcal{X}\) with \((f, \gamma)\) given above.

(ii) \((\mathcal{X}, \delta)\) is an \(\mathcal{F}\)-modular bounded with respect to \((f, \gamma)\).

**Proof.** Suppose that \((\mathcal{X}, \delta)\) is an \(\mathcal{F}\)-modular on \(\mathcal{X}\) with respect to \((f, \gamma)\). We define the mapping \(\rho : \mathcal{X} \to [0, +\infty)\) by
\[
\rho(\alpha x + \beta y) = \inf\left\{ \sum_{i=1}^{n} \delta(u_i) : n \in \mathbb{N}, n \geq 2, (u_i)_{i=1}^{n} \subset \mathcal{X}, u_1 = x, u_n = y \right\},
\]
for all \(x, y \in \mathcal{X}\) and for \(\alpha, \beta > 0\) in which \(\alpha + \beta = 1\). We show that \(\rho\) is a modular on \(\mathcal{X}\). Note that
\[
\rho(x) = \inf\left\{ \sum_{i=1}^{n} \delta(u_i) : n \in \mathbb{N}, n \geq 2, (u_i)_{i=1}^{n} \subset \mathcal{X}, u_1 = u_n = x \right\},
\]
so if \( x = 0 \), then \( \rho(x) = 0 \). Since \( \delta(\alpha x) = \delta(x) \), for each \( \alpha \) such that \( |\alpha| = 1 \), it follows from the definition of \( \rho \) that

\[
\rho(\alpha x) = \rho(x), \quad x \in \mathbb{X}.
\]

Now, let \( x \in \mathbb{X} \) be such that \( \rho(x) = 0 \). Suppose that \( x \neq 0 \). Given \( \varepsilon > 0 \), then there exists \( n \in \mathbb{N} \), \( n \geq 2 \), and \( \{u_i\}_{i=1}^n \subset \mathbb{X} \) with \( u_1 = u_n = x \) such that

\[
\sum_{i=1}^n \delta(u_i) < \varepsilon.
\]

By (\( \mathcal{F}_1 \)), we obtain

\[
f\left(\sum_{i=1}^n \delta(u_i)\right) \leq f(\varepsilon). \tag{3.2}
\]

Moreover, putting \( y = x \) in (\( \delta_2 \)) deduce that

\[
f(\delta(x)) \leq f\left(\sum_{i=1}^n \delta(u_i)\right) + \gamma. \tag{3.3}
\]

Using (3.2) and (3.3), we get

\[
f(\delta(x)) \leq f(\varepsilon) + \gamma.
\]

By (\( \mathcal{F}_2 \)), we obtain

\[
\lim_{\varepsilon \to 0^+} (f(\varepsilon) + \gamma) = -\infty,
\]

which is a contradiction. Now, let \( x, y \in \mathbb{X} \) and let \( \alpha, \beta > 0 \) be such that \( \alpha + \beta = 1 \). Suppose \( \varepsilon > 0 \) is arbitrary. Then by definition of \( \rho \), there exist \( \{u_i\}_{i=1}^n \) and \( \{v_j\}_{j=1}^m \) in \( \mathbb{X} \) such that \( u_1 = u_n = x \), \( v_1 = v_m = y \), and

\[
\sum_{i=1}^n \delta(u_i) < \rho(x) + \varepsilon, \quad \sum_{j=1}^m \delta(v_j) < \rho(y) + \varepsilon.
\]

Put \( w_1 = u_1 = x \), and \( w_i = u_i \) for every \( 2 \leq i \leq n \), \( w_i = v_{n+m-i} \) for every \( n+1 \leq i \leq n + m - 1 \), and \( w_{n+m} = u_m = y \). Then we obtain

\[
\rho(\alpha x + \beta y) \leq \sum_{i=1}^{n+m} \delta(w_i) = \sum_{i=1}^n \delta(u_i) + \sum_{j=1}^m \delta(v_j) < \rho(x) + \rho(y) + 2\varepsilon.
\]

Passing to the limit as \( \varepsilon \to 0^+ \), we get

\[
\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y).
\]

Now, we shall prove that \( \delta \) satisfies (3.1). For this, let \( x, y \in \mathbb{X} \), and for \( \alpha, \beta > 0 \) in which \( \alpha + \beta = 1 \), \( \delta(\alpha x + \beta y) > 0 \). It is clear that

\[
\rho(\alpha x + \beta y) \leq \delta(x) + \delta(y),
\]

and (\( \mathcal{F}_1 \)) implies that

\[
f(\rho(\alpha x + \beta y)) \leq f(\delta(x) + \delta(y)). \tag{3.4}
\]

Let \( \varepsilon > 0 \). Then, there exist \( n \in \mathbb{N} \), and \( \{u_i\}_{i=1}^n \subset \mathbb{X} \) with \( u_1 = x \) and \( u_n = y \) such that

\[
\sum_{i=1}^n \delta(u_i) < \rho(\alpha x + \beta y) + \varepsilon.
\]
Hence
\[ f\left(\sum_{i=1}^{n} \delta(u_i)\right) \leq f(\rho(\alpha x + \beta y) + \epsilon). \]

By \((\delta_3)\), we obtain
\[ f(\delta(\alpha x + \beta y)) \leq f(\rho(\alpha x + \beta y) + \epsilon) + \gamma. \]

Passing to the limit as \(\epsilon \to 0^+\), and using the right continuity of \(f\), we get
\[ f(\delta(\alpha x + \beta y)) \leq f(\rho(\alpha x + \beta y)) + \gamma. \] (3.5)

We deduce from (3.4) and (3.5) that
\[ f(\rho(\alpha x + \beta y)) \leq f(\delta(x) + \delta(y)) \leq f(\rho(\alpha x + \beta y)) + \gamma. \]

Therefore \((\mathcal{X}, \delta)\) is \(\mathcal{F}\)-modular bounded with respect to \((f, \gamma)\).

Now, let \((\mathcal{X}, \delta)\) is \(\mathcal{F}\)-modular bounded with respect to \((f, \gamma)\), that is, there exists a modular \(\rho\) on \(\mathcal{X}\) such that (3.1) satisfied. Let \(x, y \in \mathcal{X}\), and let \(\alpha, \beta > 0\) be such that \(\alpha + \beta = 1\), and \(\delta(\alpha x + \beta y) > 0\). Suppose \(n \in \mathbb{N}\), and \(\{u_i\}_{i=1}^{n}\) with \(u_1 = x, u_n = y\). Since \(\rho\) is a modular, we have
\[ \rho(\alpha x + \beta y) \leq \rho(x) + \rho(y) \leq \sum_{i=1}^{n} \rho(u_i). \] (3.6)

Using \((\mathcal{F}_1)\) and the fact that if \(\delta(x) + \delta(y) > 0\), and
\[ f(\rho(\alpha x + \beta y)) \leq f(\delta(x) + \delta(y)), \]
we deduce that
\[ \rho(\alpha x + \beta y) \leq \delta(x) + \delta(y). \] (3.7)

By (3.6) and (3.7), we get
\[ f(\rho(\alpha x + \beta y)) \leq f(\delta(x) + \delta(y)) \quad \text{and} \quad f(\delta(\alpha x + \beta y)) \leq f(\rho(\alpha x + \beta y)) + \gamma. \]

By \((\mathcal{F}_1)\) we deduce that
\[ f(\rho(\alpha x + \beta y)) + \gamma \leq f\left(\sum_{i=1}^{n} \delta(u_i)\right) + \gamma. \]

On the other hand
\[ f(\delta(\alpha x + \beta y)) \leq f(\rho(\alpha x + \beta y)) + \gamma, \]
we conclude that
\[ f(\delta(\alpha x + \beta y)) \leq f\left(\sum_{i=1}^{n} \delta(u_i)\right) + \gamma. \]

Therefore, \(\delta\) is an \(\mathcal{F}\)-modular on \(\mathcal{X}\).

Theorem 3.2 gives a characterization of \(\mathcal{F}\)-modulars as follows.

**Corollary 3.3.** An \(\mathcal{F}\)-modular on a space \(\mathcal{X}\) is an \(\mathcal{F}\)-modular bounded mapping.

**Remark 3.4.** Note that in the proof of Theorem 3.2, the right continuity assumption of \(f\) is used only to prove that \((i) \Rightarrow (ii)\). However, \((ii) \Rightarrow (i)\) holds for any \(f \in \mathcal{F}\).
References