Hacettepe Journal of
Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 48 (6) (2019), 1838 – 1844
DOI : 10.15672/hujms.461032

Research Article

ATA method

Guckan Yapar*, Hanife Taylan Selamlar, Sedat Capar, Idil Yavuz

Department of Statistics, School of Science, Dokuz Eylul University, Izmir, Turkey

Abstract

In this study, the forecasting accuracy of a new forecasting method that is alternative to two major forecasting approaches: exponential smoothing (ES) and ARIMA, will be evaluated. Using the results from the M3-competition, the forecasting performance of this method will be compared to not only these two major approaches but also to other successful methods derived from these two approaches with respect to simplicity and cost in addition to accuracy.

Mathematics Subject Classification (2010). 62M10

Keywords. exponential smoothing, forecasting, initial value, M3-competition, smoothing parameter

1. Introduction

“Better predictions remain the foundation of all science..." [1]. There are still two major univariate forecasting approaches: exponential smoothing (ES) and ARIMA [2]. Exponential smoothing is inarguably one of the most widely used forecasting methods available due to its simplicity, adaptiveness and accuracy [3]. The main idea behind ES is to assign recent observations more weight compared to the distant past when obtaining forecasts. The ETS state space models [4–6] brought exponential smoothing to a higher level by providing the method with a solid theoretical background. They extended the earlier classifications by [7] and [8] so that there are 30 potential ES models for various types of trend, seasonality and errors. The most popular of these are the simple ES, Holt’s linear trend model and Holt-Winter’s model. Later damped trend model was proposed [8] to help deal with over-trending. The popularity of exponential smoothing can also be attributed to its proven record against more sophisticated approaches [1, 9, 10]. For a model to be considered as an alternative model to ES, it should be simpler, more accurate, faster than ES and should not be a special case of it.

The decomposition based Theta method [11], later shown to be equivalent to a simple exponential model with drift by [12], stood out in the M3-competition. [11] removed the curvature of the original time series and called the resulting series that maintained only the mean and slope of the original series Theta-lines. They decomposed the original series into two or more Theta-lines and extrapolated these lines separately to obtain forecasts that

*Corresponding Author.
Email addresses: guckan.yapar@deu.edu.tr (G. Yapar), hanife.taylan@deu.edu.tr (H. T. Selamlar), sedat.capar@deu.edu.tr (S. Capar), idil.yavuz@deu.edu.tr (I. Yavuz)
Received: 18.09.2018; Accepted: 18.07.2019

in the end are combined to produce forecasts for the 3003 series in the M3-competition. As confirmed once again in [11], it is well known that combining forecasts [13, 14] under certain circumstances improves forecasting accuracy [15–18]. Because of this, the research since the beginning of this competition mainly focuses on certain transformations, decompositions, rules and combinations of ES and ARIMA (a few examples are [14, 19–21]) to improve the forecasting performance rather than proposing new forecasting methods.

Recently [22] proposed a simple modification of the exponential smoothing model, generalized here as the ATA method, which produces surprising results in terms of forecasting accuracy and simplicity. The ATA method eliminates the initialization problem and is easier to optimize compared to its counter ES models. This method is shown to have better forecasting accuracy for the M-competition data when equivalent parameterizations of simple exponential smoothing and ATA models are compared on post sample forecasting powers. Later [23] extended these ideas to Holt's linear trend method. They compared four versions of ATA (a modified simple exponential smoothing model, a modified Holt's trend model with trend parameter equal to 1, the best out of these two models and finally a fully optimized version) and showed that the proposed modification performs as well as and in most cases much better than the other pure major (exponential smoothing and ARIMA based) methods for the M3-competition data.

Even though ATA can be adapted to all ES models, in this paper we focus on just the additive ATA model with linear trend component and combine forecasts from this model for two pre-determined trend parameter values (0 and 1). The fact that the combination of forecasts from this one model alone can compete with all other approaches proves how much potential ATA has.

2. ATA method

The ATA method has similar form to ES but the smoothing parameters are modified so that when obtaining a smoothed value at a specific time point the weights among the observations are distributed taking into account how many observations can contribute to the value being smoothed. Therefore the smoothing parameter for this method is a function of t unlike exponential smoothing where no matter where the value you are smoothing resides on the time line, the observations receive weights only depending on their distances from the value being smoothed. In this paper, the proposed combination will be obtained from the forecasts from two parameterizations of the $ATA(p,q)$ model. For the series X_t, $t = 1, \dots, n$, the $ATA(p,q)$ model can be written as:

$$S_t = \left(\frac{p}{t}\right) X_t + \left(\frac{t-p}{t}\right)(S_{t-1} + T_{t-1}) \tag{2.1}$$

$$T_t = \left(\frac{q}{t}\right)(S_t - S_{t-1}) + \left(\frac{t-q}{t}\right) T_{t-1} \tag{2.2}$$

$$\hat{X}_t(h) = S_t + hT_t, \tag{2.3}$$

for $p \in \{1, \dots, n\}$, $q \in \{0, 1, \dots, p\}$ and $t > p \geq q$. For $t \leq p$ let $S_t = X_t$, for $t \leq q$ let $T_t = X_t - X_{t-1}$ and let $T_1 = 0$. It is worth pointing out that when $q = 0$ $ATA(p,q)$ reduces to a simple model that has similar form to simple ES, i.e. for $t > p$:

$$S_t = \left(\frac{p}{t}\right) X_t + \left(\frac{t-p}{t}\right) S_{t-1}, \tag{2.4}$$

and $S_t = X_t$ for $t \leq p$.

The $ATA(p,q)$ model defined in equations (2.1)- (2.3) has similar form to the Holt linear trend model. While the functional form of ATA models are generally very similar to those of exponential smoothing models, there are distinctive features of ATA that separate it from ES. The main difference lies in the weights assigned to observations by these two approaches. ATA can be parameterized so that all past observations receive equal weights while this is not possible for any ES model. Also when the ATA and ES models that assign equal weights to the most recent observation are compared, it can be seen that ATA tends to assign more weight to the other recent observations while assigning less weight to the distant past compared to ES. While all ES models require initialization and the initial values affect the quality of forecasts especially for small values of n and α, ATA does not require initialization and the optimization of the other parameters are simpler and faster since the parameter values are restricted to integers.

3. Results from the M3-competition

Since the M-3 competition data [1] is still one of the the most recent and comprehensive time-series data collections available, the performance of the proposed combination will be evaluated by applying the proposed method to this collection. The results from this competition are verified except for slight inconsistencies between the errors for the Theta method reported in [11] and the ones obtained from the M-3 data set with forecasts for all competing methods available in the International Journal of Forecasting's website. For some forecasting horizons and some subsets of data, the errors obtained from the data set are slightly bigger. To stay consistent with other papers in this area, we compare our findings to the results presented in [11].

Before ATA method is applied, the data sets were deseasonalized by the classical multiplicative decomposition method, when necessary. The parameters are optimized by minimizing the in-sample one-step-ahead sMAPE and to stay consistent with rest of the literature forecasts up to 18 steps ahead (the number of steps as specified in the M3-competition) are computed and again sMAPE for all forecast horizons are computed and averaged across all 3003 series.

Results from three different applications of the ATA method will be considered here.

(i) $ATA(p, 0)$ where p is optimized for $q = 0$
(ii) $ATA(p, 1)$ where p is optimized for $q = 1$
(iii) $ATA - comb$ where a simple average of the forecasts from the two models in (i) and (ii) is used as a forecast

Reseasonalized forecasts are produced when necessary for all versions for as many steps ahead as required. The results are given in Tables 1- 7.

Table 1. Average symmetric MAPE across different forecast horizons: all 3003 series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	10.5	11.3	13.6	15.1	15.1	15.9	14.5	16.0	19.3	20.7	12.62	13.57	13.76	14.24	14.81	15.47
Single	9.5	10.6	12.7	14.1	14.3	15.0	13.3	14.5	18.3	19.4	11.73	12.71	12.84	13.13	13.67	14.32
Holt	9.0	10.4	12.8	14.5	15.1	15.8	13.9	14.8	18.8	20.2	11.67	12.93	13.11	13.42	13.95	14.60
B-J automatic	9.2	10.4	12.2	13.9	14.0	14.6	13.0	14.1	17.8	19.3	11.42	12.39	12.52	12.78	13.33	13.99
ForecastPro	8.6	9.6	11.4	12.9	13.3	14.2	12.6	13.2	16.4	18.3	10.64	11.67	11.84	12.12	12.58	13.18
THETA	8.4	9.6	11.3	12.5	13.2	13.9	12.0	13.2	16.2	18.2	10.44	11.47	11.61	11.94	12.41	13.00
RBF	9.9	10.5	12.4	13.4	13.2	14.1	12.8	14.1	17.3	17.8	11.56	12.26	12.40	12.76	13.24	13.74
ForcX	8.7	9.8	11.6	13.1	13.2	13.8	12.6	13.9	17.8	18.7	10.82	11.72	11.88	12.21	12.80	13.48
ETS	8.8	9.8	12.0	13.5	13.9	14.7	13.0	14.1	17.6	18.9	11.04	12.13	12.32	12.66	13.14	13.77
$ATA(p, 0)$	8.9	10.0	12.1	13.7	13.9	14.7	12.8	13.9	17.3	18.9	11.16	12.21	12.34	12.64	13.13	13.77
$ATA(p, 1)$	8.4	9.7	11.5	12.9	13.6	14.2	12.9	15.4	18.9	20.9	10.64	11.72	11.94	12.66	13.32	14.09
$ATA - comb$	8.5	9.6	11.4	12.8	13.0	13.6	12.0	13.1	16.3	17.4	10.56	11.47	11.58	11.94	12.40	12.94

Table 2. Average symmetric MAPE across different forecast horizons: 862 seasonal series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	8.0	8.1	9.5	9.5	9.9	11.5	12.1	11.0	14.0	15.5	8.77	9.41	10.12	10.54	10.91	11.40
Single	7.1	7.4	8.8	8.7	9.3	10.9	11.3	10.7	13.1	14.6	8.02	8.71	9.42	9.78	10.13	10.62
Holt	6.5	6.9	8.2	8.4	9.4	10.6	11.2	11.5	13.2	15.3	7.50	8.33	9.15	9.66	10.09	10.67
B-J automatic	7.1	7.4	8.0	8.8	9.2	10.3	10.5	10.5	13.3	14.5	7.82	8.46	9.03	9.31	9.79	10.37
ForecastPro	6.2	6.6	7.5	8.1	8.4	9.7	10.0	9.6	11.5	13.1	7.12	7.76	8.38	8.64	8.98	9.45
THETA	6.5	6.9	7.8	8.0	8.9	10.2	9.9	10.2	12.0	13.6	7.30	8.05	8.64	9.03	9.37	9.84
RBF	8.0	8.0	8.7	8.6	8.7	10.1	10.5	10.6	12.4	13.3	8.30	8.68	9.23	9.59	9.92	10.29
ForcX	6.4	6.8	7.6	8.3	8.6	10.0	10.5	10.0	12.5	13.7	7.26	7.93	8.63	8.93	9.35	9.86
ETS	6.2	6.4	7.7	8.2	8.9	10.2	10.6	10.1	12.0	14.0	7.12	7.93	8.67	9.01	9.35	9.87
$ATA(p,0)$	6.5	6.9	8.0	8.3	9.2	10.9	11.1	10.7	12.3	14.2	7.45	8.30	9.09	9.44	9.75	10.22
$ATA(p,1)$	6.2	7.0	7.7	8.0	9.1	10.2	10.2	10.8	12.3	14.0	7.23	8.03	8.69	9.20	9.59	10.09
$ATA-comb$	6.3	6.8	7.6	7.9	8.9	10.2	10.3	10.4	11.8	13.4	7.18	7.97	8.66	9.05	9.36	9.81

Table 3. Average symmetric MAPE across different forecast horizons: 2141 non-seasonal series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	11.5	12.6	15.3	17.3	17.1	17.5	15.9	19.2	22.8	24.1	14.17	15.22	15.32	15.97	16.73	17.54
Single	10.4	11.9	14.3	16.3	16.3	16.5	14.5	17.0	21.8	22.5	13.22	14.28	14.30	14.70	15.40	16.21
Holt	10.0	11.9	14.8	17.4	18.1	18.5	16.2	17.8	23.5	24.8	13.52	15.10	15.23	15.69	16.43	17.26
B-J automatic	10.0	11.6	13.9	15.9	16.0	16.4	14.4	16.4	20.7	22.4	12.87	13.97	14.04	14.43	15.09	15.85
ForecastPro	9.6	10.8	13.0	14.9	15.3	15.9	14.1	15.6	19.5	21.7	12.05	13.25	13.34	13.78	14.37	15.09
THETA	9.2	10.6	12.7	14.3	14.9	15.4	13.2	15.1	19.0	21.2	11.71	12.85	12.90	13.32	13.91	14.62
RBF	10.6	11.6	13.9	15.3	15.0	15.6	14.1	16.3	20.4	20.7	12.87	13.69	13.78	14.27	14.88	15.51
ForcX	9.6	11.1	13.2	15.1	15.1	15.4	13.8	16.5	21.2	22.0	12.25	13.24	13.29	13.77	14.51	15.34
ETS	9.9	11.2	13.7	15.6	15.9	16.6	14.4	16.7	21.3	22.2	12.61	13.83	13.91	14.39	15.03	15.77
$ATA(p,0)$	9.8	11.3	13.7	15.8	15.8	16.3	13.8	16.0	20.6	21.9	12.66	13.79	13.76	14.16	14.81	15.58
$ATA(p,1)$	9.3	10.8	13.1	14.9	15.4	15.8	14.4	18.4	23.2	25.4	12.02	13.21	13.36	14.30	15.17	16.15
$ATA-comb$	9.3	10.7	12.9	14.7	14.7	15.0	12.9	14.8	19.3	19.9	11.92	12.88	12.86	13.31	13.90	14.55

Table 4. Average symmetric MAPE across different forecast horizons: 645 annual series

	Forecasting horizons						Averages	
Method	1	2	3	4	5	6	1-4	1-6
Naive2	8.5	13.2	17.8	19.9	23.0	24.9	14.85	17.88
Single	8.5	13.3	17.6	19.8	22.8	24.8	14.82	17.82
Holt	8.3	13.7	19.0	22.0	25.2	27.3	15.77	19.27
B-J automatic	8.6	13.0	17.5	20.0	22.8	24.5	14.78	17.73
ForecastPro	8.3	12.2	16.8	19.3	22.2	24.1	14.15	17.14
THETA	8.0	12.2	16.7	19.2	21.7	23.6	14.02	16.90
RBF	8.2	12.1	16.4	18.3	20.8	22.7	13.75	16.42
ForcX	8.6	12.4	16.1	18.2	21.0	22.7	13.80	16.48
ETS	9.3	13.6	18.3	20.8	23.4	25.8	15.48	18.53
$ATA(p,0)$	9.1	13.5	17.6	19.9	22.8	25.1	15.04	18.00
$ATA(p,1)$	8.3	12.2	16.8	18.6	21.5	23.3	13.95	16.78
$ATA-comb$	8.4	12.3	16.5	18.3	21.0	22.7	13.87	16.54

Table 5. Average symmetric MAPE across different forecast horizons: 756 quarterly series.

	Forecasting horizons							Averages		
Method	1	2	3	4	5	6	8	1-4	1-6	1-8
Naive2	5.4	7.4	8.1	9.2	10.4	12.4	13.7	7.55	8.82	9.95
Single	5.3	7.2	7.8	9.2	10.2	12.0	13.4	7.38	8.63	9.72
Holt	5.0	6.9	8.3	10.4	11.5	13.1	15.6	7.67	9.21	10.67
B-J automatic	5.5	7.4	8.4	9.9	10.9	12.5	14.2	7.79	9.10	10.26
ForecastPro	4.9	6.8	7.9	9.6	10.5	11.9	13.9	7.28	8.57	9.77
THETA	5.0	6.7	7.4	8.8	9.4	10.9	12.0	7.00	8.04	8.96
RBF	5.7	7.4	8.3	9.3	9.9	11.4	12.6	7.69	8.67	9.57
ForcX	4.8	6.7	7.7	9.2	10.0	11.6	13.6	7.12	8.35	9.54
ETS	5.0	6.6	7.9	9.7	10.9	12.1	14.2	7.32	8.71	9.94
$ATA(p,0)$	5.2	7.1	7.8	9.7	10.1	11.8	13.5	7.45	8.62	9.71
$ATA(p,1)$	5.3	6.8	7.6	9.1	9.9	11.0	12.4	7.19	8.28	9.24
$ATA-comb$	5.1	6.8	7.5	9.0	9.6	10.9	12.3	7.10	8.13	9.07

Table 6. Average symmetric MAPE across different forecast horizons: 1428 monthly series.

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	15.0	13.5	15.7	17.0	14.9	14.4	15.6	16.0	19.3	20.7	15.30	15.08	15.26	15.55	16.16	16.89
Single	13.0	12.1	14.0	15.1	13.5	13.1	13.8	14.5	18.3	19.4	13.53	13.44	13.60	13.83	14.51	15.32
Holt	12.2	11.6	13.4	14.6	13.6	13.3	13.7	14.8	18.8	20.2	12.95	13.11	13.33	13.77	14.51	15.36
B-J automatic	12.3	11.7	12.8	14.3	12.7	12.3	13.0	14.1	17.8	19.3	12.78	12.70	12.86	13.19	13.95	14.80
ForecastPro	11.5	10.7	11.7	12.9	11.8	12.0	12.6	13.2	16.4	18.3	11.72	11.78	12.02	12.43	13.07	13.85
THETA	11.2	10.7	11.8	12.4	12.2	12.2	12.7	13.2	16.2	18.2	11.54	11.75	12.09	12.48	13.09	13.83
RBF	13.7	12.3	13.7	14.3	12.3	12.5	13.5	14.1	17.3	17.8	13.49	13.14	13.36	13.64	14.19	14.76
ForcX	11.6	11.2	12.6	14.0	12.4	12.0	12.8	13.9	17.8	18.7	12.32	12.28	12.44	12.81	13.58	14.44
ETS	11.5	10.6	12.3	13.4	12.3	12.3	13.2	14.1	17.6	18.9	11.93	12.05	12.43	12.96	13.64	14.45
$ATA(p,0)$	11.5	10.8	12.6	13.8	12.6	12.5	12.9	13.9	17.3	18.9	12.20	12.33	12.78	12.98	13.67	14.49
$ATA(p,1)$	11.0	10.9	12.2	13.4	12.8	12.8	13.8	15.4	18.9	20.9	11.86	12.16	12.60	13.87	14.39	15.33
$ATA-comb$	11.1	10.7	12.1	13.1	12.0	11.9	12.4	13.1	16.3	17.4	11.74	11.81	12.04	12.48	13.07	13.75

Table 7. Average symmetric MAPE across different forecast horizons: 174 other series.

	Forecasting horizons							Averages		
Method	1	2	3	4	5	6	8	1-4	1-6	1-8
Naive2	2.2	3.6	5.4	6.3	7.8	7.6	9.2	4.38	5.49	6.30
Single	2.1	3.6	5.4	6.3	7.8	7.6	9.2	4.36	5.48	6.29
Holt	1.9	2.9	3.9	4.7	5.8	5.6	7.2	3.32	4.13	4.81
B-J automatic	1.8	3.0	4.5	4.9	6.1	6.1	7.5	3.52	4.38	5.06
ForecastPro	1.9	3.0	4.0	4.4	5.4	5.4	6.7	3.31	4.00	4.60
THETA	1.8	2.7	3.8	4.5	5.6	5.2	6.1	3.20	3.93	4.41
RBF	2.7	3.8	5.2	5.8	6.9	6.3	7.3	4.38	5.12	5.60
ForcX	2.1	3.1	4.1	4.4	5.6	5.4	6.5	3.42	4.10	4.64
ETS	2.0	3.0	4.0	4.4	5.4	5.1	6.3	3.37	3.99	4.51
$ATA(p,0)$	2.1	3.5	5.4	6.3	7.8	7.5	9.1	4.34	5.45	6.26
$ATA(p,1)$	1.9	2.9	4.1	4.8	6.0	5.7	7.1	3.46	4.26	4.87
$ATA-comb$	1.9	3.0	4.5	5.1	6.3	5.8	6.8	3.62	4.42	4.94

The results from all 3003 data sets can be summarized as in Table 1. Here when the methods are compared based on the average sMAPE for forecasting horizons 1-18, $ATA - comb$ stands out from the rest, ranking first. It performs better than not just the pure approaches like ETS and ARIMA, it outperforms all existing methods. It is worth noting that $ATA-comb$ is more accurate than ETS and ARIMA for all individual forecasting horizons, not just on average.

The success of ATA is evident even when just the results from the simplest version of the method ($ATA(p,0)$) are studied. This simple version performs better than SES for all forecasting horizons and on average and its average sMAPE for horizons $1-18$ is the same as ETS' sMAPE (13.77).

ATA method does not perform as well when the results are averaged just for the seasonal series. This can be attributed to the fact that the ATA models we considered here do not model seasonality like the other competitors. For non-seasonal data however, ATA models inarguably perform as well as Theta and much better than the other methods.

For annual data $ATA-comb$ performs close to RBF and ForecastX. For quarterly data, $ATA - comb$ ranks second right after Theta when sMAPE is averaged for horizons $1-4$, $1-6$ and $1-8$ and for monthly data it outperforms all other methods when errors are averaged for horizons $1-18$.

4. Discussion

In this study a combination of forecasts from the ATA method is proposed and the proposed approach's forecasting performance is investigated. Even though the models from the ATA method have similar form to their counter ES models, the proposed combination's predictive performance is much better for the M3 data sets. The optimum parameter values, forecasts and errors for the proposed method can be reached from the website https://atamethod.wordpress.com.

The results presented in this paper do not reflect the end performance of ATA and on the contrary these results are just the initial findings. In this paper we competed with just one ATA model (linearly trended with additive errors) and combined forecasts from only two parameterizations of it. Incorporating other types of trend, seasonality, error terms and using different accuracy measures will surely increase the method's performance. Like other approaches, the method can also benefit from certain transformations [21], other types of more involved combinations, outlier detection and other more complicated model selection strategies. The fact that this simple combination can perform better than existing methods is fascinating and this further strengthens the idea that simplicity is indeed a prerequisite for forecasting accuracy.

Funding

This work was funded by the TUBITAK-3001 support under grant 118F174.

References

[1] S. Makridakis and M. Hibon, *The M3-Competition: results, conclusions and implications*, Int. J. Forecast, **16** (4), 451–476, 2000.

[2] J. De Gooijer and R. Hyndman, *25 years of IIF time series forecasting: A selective review*, Int. J. Forecast, **22** (3), 443–473, 2006.

[3] P. Goodwin, *The holt-winters approach to exponential smoothing: 50 years old and going strong*, Foresight, **19**, 30–33, 2010.

[4] R. Hyndman, A. Koehler, R. Snyder and S. Grose, *A state space framework for automatic forecasting using exponential smoothing methods*, Int. J. Forecast, **13** (3), 439–454, 2002.

[5] R. Hyndman, A. Koehler, J. Ord and R. Snyder, *Forecasting with exponential smoothing: the state space approach*, Springer-Verlag, 2008.
[6] R. Hyndman and G. Athanasopoulos, *Forecasting: principles and practice*, OTexts, 2014.
[7] C. Pegels, *On startup or learning curves: An expanded view*, AIIE Transactions, **1** (3), 216–222, 1969.
[8] E. Gardner Jr and E. McKenzie, *Forecasting trends in time series*, Management Science, **31** (10), 1237–1246, 1985.
[9] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, J. Newton, E. Parzen and R. Winkler, *The forecasting accuracy of major time series methods*, Wiley, 1984.
[10] A. Koning, P. Franses, M. Hibon abd H. Stekler, *The M3 competition: Statistical tests of the results*, Int. J. Forecast, **21** (3), 397–409, 2005.
[11] V. Assimakopoulos and K. Nikolopoulos, *The theta model: a decomposition approach to forecasting*, Int. J. Forecast, **16** (4), 521–430, 2000.
[12] R. Hyndman and B. Billah, *Unmasking the Theta method*, Int. J. Forecast, **19** (2), 287–290, 2003.
[13] J. Bates and C. Granger, *The combination of forecasts*, Journal of the Operational Research Society, **20** (4), 451–468, 1969.
[14] R. Clemen, *Combining forecasts: A review and annotated bibliography*, Int. J. Forecast, **5** (4), 559–583, 1989.
[15] J. Armstrong, *Combining forecasts: The end of the beginning or the beginning of the end?*, Int. J. Forecast, **5** (4), 585–588, 1989.
[16] J. Armstrong, *Principles of forecasting: a handbook for researchers and practitioners*, Springer Science & Business Media, 2001.
[17] S. Makridakis and R. Winkler, *Averages of forecasts: Some empirical results*, Management Science, **29** (9), 987–996, 1983.
[18] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, J. Newton, E. Parzen and R. Winkler, *The accuracy of extrapolation (time series) methods: Results of a forecasting competition*, J. Forecast., **1** (2), 111–153, 1982.
[19] R. Cleveland, W. Cleveland, J. McRae and I. Terpenning, *STL: A seasonal-trend decomposition procedure based on loess*, Journal of Official Statistics, **6** (1), 3–73, 1990.
[20] M. Adya, J. Armstrong, F. Collopy and M. Kennedy, *An application of rule-based forecasting to a situation lacking domain knowledge*, Int. J. Forecast, **16** (4), 477–484, 2000.
[21] C. Bergmeir, R. Hyndman and J. Benitez, *Bagging exponential smoothing methods using STL decomposition and Box–Cox transformation*, Int. J. Forecast, **32** (2), 303–312, 2016.
[22] G. Yapar, *Modified simple exponential smoothing*, Hacet. J. Math. Stat., **47** (3), 741–754, 2018.
[23] G. Yapar, S. Capar, H. Selamlar and I. Yavuz, *Modified Holt's linear trend method*, Hacet. J. Math. Stat., **47** (5), 1394–1403, 2018.