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SIMPLE COMPUTATIONAL FORMULAS FOR INCLUSION PROBABILITIES IN RANKED SET SAMPLING

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Abstract

In this study, we derived new formulations for the first and second order inclusion probabilities of a ranked set sample in a finite population setting. Gökpınar and Özdemir (2010) developed a formula to calculate the first order inclusion probabilities. However, the formula given in this study is much easier than the one given by Gökpınar and Özdemir (2010). Second order inclusion probabilities are computed based on the formulas which are used for the calculation of first order inclusion probabilities. Also, we give a numerical example to show the calculation of the formulas and Matlab codes which give first and second inclusion probabilities for any set and population sizes.

Keywords: Ranked Set Sampling, First Order Inclusion Probability, Second Order Inclusion Probability, Finite Population Setting.

2000 AMS Classification: 62D05, 65C60

1. Introduction

Ranked Set Sampling (RSS) is an efficient sampling technique than the simple random sampling (SRS) for improving the accuracy of the estimation of means. RSS was first introduced by McIntyre (1952) for estimate the mean of pasture yields. In recent years, RSS is used in many fields such as the environment, ecology and agriculture. Some applications in these fields can be found in the studies of Johnson et.al. (1993) and Al-Saleh et al(2000). Also, some recent ideas about RSS can be found in Bouza(2005).

In RSS, the inclusion probabilities of the population units are different from each other, and it is difficult to determine the inclusion probabilities for all sample sizes. Al-Saleh and Samawi (2007) obtained the inclusion probabilities in RSS for the set size 2 and 3. Özdemir and Gökpınar (2007) obtained the inclusion probabilities in RSS for all set sizes when the cycle size is one, and Özdemir and Gökpınar (2008) have adapted

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this procedure to Median Ranked Set Sampling (MRSS) with any set and cycle sizes. Gökpınar and Özdemir (2010) generalized the formula of inclusion probabilities in RSS for all cycle and set sizes.

Jafari et. al. (2010) derived the first and second order inclusion probabilities for Level 0 RSS procedure (sampling with replacement) of Deshpande et. al. (2006) and developed several designs based estimators of the population mean. Recently, Gökpınar and Özdemir (2011) defined the Horvitz-Thompson (HT) estimator of the population mean using the inclusion probabilities of a ranked set sample in a finite population setting. Furthermore, they give a calculation formula of the second order inclusion probabilities which is required to calculate the variance of the HT estimator.

In this study, we give a simple formula to calculate the first and second order inclusion probabilities in RSS. In the second section of this study, we give the selection procedure, required definitions, and the formulas of these inclusion probabilities in RSS. In the third section, a numerical example is given to show the calculation of the formula. Concluding remarks are given in section 4. Also in the appendix, we give Matlab codes to calculate the first and second inclusion probabilities for any set and population sizes.

2. Inclusion Probabilities in RSS

Let the population units be $X_1 < X_2 < \ldots < X_N$ and let a ranked set sample from this population be $Y_1, Y_2, \ldots, Y_m$ based on the level 1 sampling procedure. Level 1 sampling procedure is given as follows (Deshphande et al. 2006, Al-Saleh and Samawi, 2007):

In the g^{th} selection,

1. A simple random sample of size m is selected without replacement from the population.

2. The sampled units are ranked with respect to the variable of interest and the g^{th} order statistic is selected for measurement.

3. All other m-1 units are returned to the population.

4. The steps 1-3 are repeated for $g=1,2,\ldots,m$ to obtain a ranked set sample of size m.

The entire cycle may be repeated, if necessary, r times to produce a ranked set sample of size mr=n. In this study, we only considered the case of r=1. A generalization for r>1 can be easily derived.

To calculate the first and second order inclusion probabilities, some basic definitions are required.

A_i is the event of selecting the ith population unit in the sample (i=1,2,...,N).

A_j is the event of selecting the jth population unit in the sample (j=1,2,...,N).

$$l_g(i,j) = \begin{cases} 1 & t < i \\ 2 & t > j \\ 3 & i < t < j \end{cases}$$

where i<j and t is the rank of the population unit which is selected in the gth selection. If i=j, then $l_g(i,i) = l_g(i)$ can be defined as;

$$l_g(i) = \begin{cases} 1 & t < i \\ 2 & t > i \end{cases}$$

$B_g^1(i,j)$is the event of selecting smaller population unit than the ith population unit in the gth selection ($l_g(i,j)$=1). If i=j, then $B_g^1(i,i) = B_g^1(i)$.

$B_g^2(i,j)$is the event of selecting greater population unit than the j^{th} population unit in the g^{th} selection ($l_g(i,j)$=2). If i=j, then $B_g^2(i,i) = B_g^2(i)$.

$B_g^3(i,j)$is the event of selecting greater population unit than i^{th} and smaller population unit than the jth population unit in the gth selection ($l_g(i,j)$=3).

$a_g(i)$ is the number of smaller population units than the i^{th} population unit selected before the g^{th} selection.

$a_g(j)$is the number of smaller population units than the j^{th} population unit selected before the g^{th} selection.

So there is a relationship between $a_g(i)$ and $\{l_1(i), l_2(i), ..., l_{g-1}(i)\}$as given below

$$a_g(i) = 2(g-1) - \sum_{u=1}^{g-1} l_u(i)$$

By using these definitions, the probability of selecting the i^{th} population unit in the sample can be obtained as

$$\pi_N(A_i) = 1 - \pi_N(A_i^c) \qquad i = 1, 2, \ldots, N \tag{2.1}$$

where

$$\pi_N(A_i^c) = \sum_{l_1(i), l_2(i), \ldots, l_m(i)=1}^{2} P\left(B_1^{l_1(i)}(i) \cap B_2^{l_2(i)}(i) \cap \ldots \cap B_m^{l_m(i)}(i)\right)$$

$$= \sum_{l_1(i), l_2(i), \ldots, l_m(i)=1}^{2} \prod_{g=1}^{m} P_{a_g(i)}\left(B_g^{l_g(i)}(i) | B_1^{l_1(i)}(i) \cap B_2^{l_2(i)}(i) \cap \ldots \cap B_{g-1}^{l_{g-1}(i)}(i)\right) \tag{2.2}$$

We derive $P_{a_g(i)}\left(B_g^{l_g(i)}(i) | B_1^{l_1(i)}(i) \cap B_2^{l_2(i)}(i) \cap \ldots \cap B_{g-1}^{l_{g-1}(i)}(i)\right)$ in the following theorems.

2.1. Theorem. *The probability,* $P_{a_g(i)}\left(B_g^{1}(i) | B_1^{l_1(i)}(i) \cap B_2^{l_2(i)}(i) \cap \ldots \cap B_{g-1}^{l_{g-1}(i)}(i)\right)$ *in Eq. (2.2), can be written as follows when* $a_g(i)=0$*;*

$$P_0\left(B_g^1(i) | B_1^2(i) \cap B_2^2(i) \ldots \cap B_{g-1}^2(i)\right) = \begin{cases} 0 & i = 1, 2, ..., g \\ \sum_{u=g}^{m} \dfrac{\binom{i-1}{u}\binom{N-i-g+2}{m-u}}{\binom{N-g+1}{m}} & i = g+1, ... N-m+1 \\ 1 & i = N-m+2, ..., N. \end{cases} \tag{2.3}$$

Proof. $P_0\left(B_g^1(i) | B_1^2(i) \cap B_2^2(i) \cap \ldots \cap B_{g-1}^2(i)\right)$ means that the probability of selection of a smaller unit than the i-th population unit in the g-th selection under the condition that there is no a smaller population unit selected before the g-th selection. So, there are i-1 smaller population units and N-i+1-(g-1) =N-i-g+2 greater population units from the i-th population unit in the g-th selection. Also, we should choose at least g population units smaller than i-th population unit to choose a population unit smaller than the i-th population unit. So, smaller population units than any of the first g population units $(i = 1, 2, ..., g)$ have no chance to be selected in the g-th selection. On the other hand, greater population units than any of the last m-1 population units$(i = N-m+2, ..., N)$ have no chance to be selected in the g-th selection. Therefore, smaller population units than any of the last m-1 population units$(i = N-m+2, ..., N)$ have a %100 probability to be selected in the g-th selection. So,

$$
\begin{aligned}
&P_0\left(B_g^1(i)|B_1^2(i)\cap B_2^2(i)...\cap B_{g-1}^2(i)\right)\\
&=\frac{\begin{pmatrix} i-1\\ g\end{pmatrix}\begin{pmatrix} N-i-g+2\\ m-g\end{pmatrix}}{\begin{pmatrix} N-g+1\\ m\end{pmatrix}}+...+\frac{\begin{pmatrix} i-1\\ m\end{pmatrix}\begin{pmatrix} N-i-g+2\\ 0\end{pmatrix}}{\begin{pmatrix} N-g+1\\ m\end{pmatrix}}\\
&=\sum_{u=g}^{m}\frac{\begin{pmatrix} i-1\\ u\end{pmatrix}\begin{pmatrix} N-i-g+2\\ m-u\end{pmatrix}}{\begin{pmatrix} N-g+1\\ m\end{pmatrix}},\quad i=g+1,....,N-m+1.
\end{aligned}
$$

This completes the proof. □

The other probabilities required to calculate the inclusion probabilities can be obtained by using Theorem 2.1. The selection probability of the population unit smaller than $i'=i+a_g(i')$ $(a_g(i')=1,2,...,g-1)$ in the g-th selection when $a_g(i')>0$, is equal to the selection probability of the population unit smaller than the i-th population unit in the g-th selection when $a_g(i)=0$. This probability is stated at Theorem 2.2.

2.2. Theorem. $P_{a_g(i')}\left(B_g^1(i')|B_1^{l_1(i')}(i')\cap B_2^{l_2(i')}(i')...\cap B_{g-1}^{l_{g-1}(i')}(i')\right)$ *can be written as follows when* $i'=i+a_g(i')$ $(a_g(i')=1,2,...,g-1)$*.*

$$
\begin{aligned}
&P_{a_g(i')}\left(B_g^1(i')|B_1^{l_1(i')}(i')\cap...\cap B_{g-1}^{l_{g-1}(i')}(i')\right)=\\
&P_{a_g(i)=0}\left(B_g^1(i)|B_1^2(i)\cap...\cap B_{g-1}^2(i)\right).
\end{aligned}\tag{2.4}
$$

Proof. In the g-th selection, the number of population units smaller than i' are

$$i'-a_g(i')-1=i+a_g(i')-a_g(i')-1=i-1.$$

By the same way, the number of population units equal or greater than i' are

$$N-i'+1-(g-1-a_g(i'))=N-(i+a_g(i'))+1-(g-1-a_g(i'))=N-i-g+2.$$

So,

$$
\begin{aligned}
&P_{a_g(i')}\left(B_g^1(i')|B_1^{l_1(i')}(i')\cap B_2^{l_2(i')}(i')\cap...\cap B_{g-1}^{l_{g-1}(i')}(i')\right)\\
&=\sum_{u=g}^{m}\frac{\begin{pmatrix} i-1\\ u\end{pmatrix}\begin{pmatrix} N-i-g+2\\ m-u\end{pmatrix}}{\begin{pmatrix} N-g+1\\ m\end{pmatrix}}.
\end{aligned}
$$

This probability is equal to $P_{a_g(i)=0}\left(B_g^1(i)|B_1^2(i)\cap B_2^2(i)...\cap B_{g-1}^2(i)\right)$.

This completes the proof. □

We also required the probability of selecting of a greater unit from the i-th population unit. This probability is stated at Theorem 2.3.

2.3. Theorem. $P_{a_g(i)}\left(B_g^2(i)|B_1^{l_1(i)}(i)\cap B_2^{l_2(i)}(i)\cap...\cap B_{g-1}^{l_{g-1}(i)}(i)\right)$ *can be written as follows:*

$$
\begin{aligned}
&P_{a_g(i)}\left(B_g^2(i)|B_1^{l_1(i)}(i)\cap B_2^{l_2(i)}(i)\cap...\cap B_{g-1}^{l_{g-1}(i)}(i)\right)\\
&=1-P_{a_g(i+1)=a_g(i)}\left(B_g^1(i+1)|B_1^{l_1(i+1)}(i+1)\cap...\cap B_{g-1}^{l_{g-1}(i+1)}(i+1)\right).
\end{aligned}\tag{2.5}
$$

Proof. From the basic complement rule of probability, $P(A^c) = 1 - P(A)$, we know that $P_{a_g(i)}\left(\{B_g^2(i) \ |B_1^{l_1(i)}(i) \cap B_2^{l_2(i)}(i) ... \cap B_{g-1}^{l_{g-1}(i)}(i)\}\right)$ is the selection probability of a greater unit from i-th population unit (i+1,i+2,...,N) when $a_g(i)$ is known and $P_{a_g(i+1)=a_g(i)}\left(B_g^1(i+1)|B_1^{l_1(i+1)}(i+1) \cap ... \cap B_{g-1}^{l_{g-1}(i+1)}(i+1)\right)$ is the selection probability of a smaller unit from (i+1)-*th* population unit (1,2,... i) when $a_g(i+1) = a_g(i)$. So, these probabilities are complement to each other. This completes the proof. $\square$

By using these definitions, the probability of selecting both the ith and jth population units in the sample can be obtained as

$$\begin{aligned} \pi_N(A_i \cap A_j) &= 1 - \pi_N\left((A_i \cap A_j)^c\right) \qquad i,\ j = 1, 2, \ldots, N\ (i < j) \\ &= 1 - \left[\pi_N\left(A_i^c\right) + \pi_N\left(A_j^c\right) - \pi_N\left(A_i^c \cap A_j^c\right)\right] \end{aligned} \tag{2.6}$$

where $\pi_N(A_i^c)$ and $\pi_N\left(A_j^c\right)$ probabilities can be calculated from the Theorems 2.1, 2.2, 2.3. The probability $\pi_N(A_i^c \cap A_j^c)$ can be defined as follows;

$$\begin{aligned} \pi_N(A_i^c \cap A_j^c) &= \sum_{l_1(i,j), l_2(i,j), \ldots, l_m(i,j)=1}^{3} P\left(B_1^{l_1(i,j)}(i,j) \cap ... \cap B_m^{l_m(i,j)}(i,j)\right) \\ &= \sum_{l_1(i,j), l_2(i,j), \ldots, l_m(i,j)=1}^{3} \prod_{g=1}^{m} P_{a_g(i), a_g(j)} \\ &\qquad \left(B_g^{l_g(i,j)}(i,j)|B_1^{l_1(i,j)}(i,j) \cap ... \cap B_{g-1}^{l_{g-1}(i,j)}(i,j)\right) \end{aligned} \tag{2.7}$$

The conditional probability of $B_g^{l_g(i,j)}(i,j)$ can be calculated from Theorems 2.1, 2.2, 2.3. when $l_g(i,j)$=1 and $l_g(i,j)$=2. When $l_g(i,j)$=3, the conditional probability of $B_g^{l_g(i,j)}(i,j)$ is given as following Theorem 2.4.

2.4. Theorem. $P_{a_g(i),a_g(j)}\left(B_g^3(i,j)|B_1^{l_1(i,j)}(i,j) \cap B_2^{l_2(i,j)}(i,j) ... \cap B_{g-1}^{l_{g-1}(i,j)}(i,j)\right)$ *can be written as follows:*

$$\begin{aligned} &P_{a_g(i),a_g(j)}\left(B_g^3(i,j)|B_1^{l_1(i,j)}(i,j) \cap ... \cap B_{g-1}^{l_{g-1}(i,j)}(i,j)\right) \\ &= P_{a_g(j)}\left(B_g^1(j)|B_1^{l_1(j)}(j) \cap ... \cap B_{g-1}^{l_{g-1}(j)}(j)\right) \\ &\qquad - P_{a_g(i+1)=a_g(i)}\left(B_g^1(i+1)|B_1^{l_1(i+1)}(i+1) \cap ... \cap B_{g-1}^{l_{g-1}(i+1)}(i+1)\right) \end{aligned} \tag{2.8}$$

Proof. $P_{a_g(j)}\left(B_g^1(j)|B_1^{l_1(j)}(j) \cap ... \cap B_{g-1}^{l_{g-1}(j)}(j)\right)$ is the probability of selecting smaller population unit than the j^{th} population unit in the g^{th} selection when there are a$_g(j)$ smaller unit then jth population unit. Also,

$$P_{a_g(i+1)=a_g(i)}\left(B_g^1(i+1)|B_1^{l_1(i+1)}(i+1) \cap ... \cap B_{g-1}^{l_{g-1}(i+1)}(i+1)\right)$$

is the probability of selecting a smaller population unit than the $(i+1)^{th}$ population unit in the g^{th} selection when there are $a_g(i+1)= a_g(i)$ smaller units then $(i+1)^{th}$ population unit. So, from the basic rules of probability, the probability of a population unit between i^{th} and j^{th} unit including in a ranked set sample can be obtained by using the difference of these two probabilities. This completes the proof. $\square$

By using Theorem 2.1, 2.2, 2.3 and 2.4 we can obtain the inclusion probabilities given in Eq. (2.1) and (2.6). A simple example for calculation is given in the following section.

3. Computation of the Formula

By using the formulas in previous section, the inclusion probabilities for the all units in the population can be derived easily. For example, when N=5 and m=3, the population consists of $X_1<X_2<X_3<X_4<X_5$ elements. The inclusion probability of X_i (i=1, 2, 3, 4, 5) can be written using Eq. (2.1) as follows:

$\pi_N(A_i) = 1 - \pi_N(A_i^c)$ i=1,2,3,4,5

where

$$\pi_N(A_i^c) = \sum_{l_1,l_2,l_3=1}^{2} P\left(B_1^{l_1}(i) \cap B_2^{l_2}(i) \cap B_3^{l_3}(i)\right)$$
$$= \sum_{l_1,l_2,l_3=1}^{2} P_{a_3(i)}\left(B_3^{l_3}(i) | B_1^{l_1}(i) \cap B_2^{l_2}(i)\right) P_{a_2(i)}\left(B_2^{l_2}(i) | B_1^{l_1}(i)\right) P_{a_1(i)}\left(B_1^{l_1}(i)\right).$$

here $a_1(i) = 0$, $a_2(i) = 0, 1$ and $a_3(i) = 0, 1, 2$.

By using Theorem 2.1, the probability of selecting a smaller unit than the i-th population unit when g=1, can be written as follows;

$$P_0\left(B_1^1(i)\right) = \begin{cases} 0 & i = 1 \\ \sum_{u=1}^{3} \dfrac{\binom{i-1}{u}\binom{6-i}{3-u}}{\binom{5}{3}} & i = 2, 3 \\ 1 & i = 4, 5. \end{cases}$$

$$P_0\left(B_1^1(1)\right) = 0;\ P_0\left(B_1^1(2)\right) = 6/10;$$
$$P_0\left(B_1^1(3)\right) = 9/10;\ P_0\left(B_1^1(4)\right) = 1;\ P_0\left(B_1^1(5)\right) = 1.$$

From Theorem 2.3, it can be written as follows;

$$P_0\left(B_1^2(1)\right) = 4/10;\ P_0\left(B_1^2(2)\right) = 1/10;$$
$$P_0\left(B_1^2(3)\right) = 0;\ P_0\left(B_1^2(4)\right) = 0;\ P_0\left(B_1^2(5)\right) = 0.$$

We can write the other inclusion probabilities by the same way. In Table 1, the inclusion probabilities of the all population units are given in the g^{th} selection for all possible combinations of (l_1, l_2, l_3). In Table 1, P_{l_1,l_2,l_3} is defined as follows;

$$P_{l_1,l_2,l_3} = P\left(B_1^{l_1}(i) \cap B_2^{l_2}(i) \cap B_3^{l_3}(i)\right)$$
$$= P_{a_3(i)}\left(B_3^{l_3}(i) | B_1^{l_1}(i) \cap B_2^{l_2}(i)\right) P_{a_2(i)}\left(B_2^{l_2}(i) | B_1^{l_1}(i)\right) P_{a_1(i)}\left(B_1^{l_1}(i)\right).$$

The obtained inclusion probabilities in Table 1 are the same as the inclusion probabilities which are given in the study of Gökpınar and Özdemir (2010). But this formula is much easier and simpler than the formula of the inclusion probabilities given in Gökpınar and Özdemir (2010).

By the same way, the second order inclusion probabilities can be obtained as given in Table 2.

As seen from Table 1, the extreme units have greater inclusion probabilities than the others. The following figures are constructed for different population and set sizes.

As seen from Figures 1-6, units from both extremes (e.g. X_1, X_N) have greater second order inclusion probabilities than the others for all set and population sizes. Also units in the mid section of the population have smaller second order inclusion probabilities. The effects of first and second order inclusion probabilities on HT estimator under populations with different coefficient of variation and skewness values are investigated at Gökpınar

Table 1. The first order inclusion probabilities of the population units with N=5, m=3

X_i	(l_1,l_2,l_3)	(1,1,1)	(1,1,2)	(1,2,1)	(1,2,2)	(2,1,1)	(2,1,2)	(2,2,1)	(2,2,2)	$\pi_N(A_i^c)$	$\pi_N(A_i)$
X_1	g=1	0	0	0	0	0.40	0.40	0.40	0.40	0.40	0.60
	g=2	0	0	1	1	0	0	1	1		
	g=3	0	1	0	1	0	1	0	1		
P_{l_1,l_2,l_3}		0	0	0	0	0	0	0	0.40		
X_2	g=1	0.60	0.60	0.60	0.60	0.10	0.10	0.10	0.10	0.65	0.35
	g=2	0	0	1	1	0	0	0.50	0.50		
	g=3	0	1	0	1	0	1	0	1		
P_{l_1,l_2,l_3}		0	0	0	0.60	0	0	0	0.05		
X_3	g=1	0.90	0.90	0.90	0.90	0	0	0	0	0.45	0.55
	g=2	0	0	0.50	0.50	0.50	0.50	0	0		
	g=3	0	1	0	1	0	1	0	0		
P_{l_1,l_2,l_3}		0	0	0	0.45	0	0	0	0		
X_4	g=1	1	1	1	1	0	0	0	0	0.50	0.50
	g=2	0.50	0.50	0	0	1	1	0	0		
	g=3	0	1	0	0	0	0	1	1		
P_{l_1,l_2,l_3}		0	0.50	0	0	0	0	0	0		
X_5	g=1	1	1	1	1	0	0	0	0	0	1
	g=2	1	1	0	0	1	1	0	0		
	g=3	0	0	1	0	1	0	0	0		
P_{l_1,l_2,l_3}		0	0	0	0	0	0	0	0		

Table 2. The second order inclusion probabilities of the population units with N=5, m=3

$\pi_N(A_i \cap A_j)$	X_1	X_2	X_3	X_4	X_5
X_1	-	0	0.30	0.30	0.60
X_2	0	-	0.20	0.15	0.35
X_3	0.30	0.20	-	0.05	0.55
X_4	0.30	0.15	0.05	-	0.50
X_5	0.60	0.35	0.55	0.50	-

and Özdemir(2012). The results of assigning larger probabilities to the extremes are also discussed at Gökpınar and Özdemir(2012).

4. Concluding Remarks

In this study, we give a new formula for the first and the second order inclusion probabilities in RSS which is simpler and easier than the previous ones. This formula can be adapted to other modifications of RSS and can be generalized for any cycle sizes. Furthermore, a MATLAB code is given for calculate the inclusion probabilities in the Appendix.

Figure 1. The second order inclusion probabilities of the population units with N=20, m=3

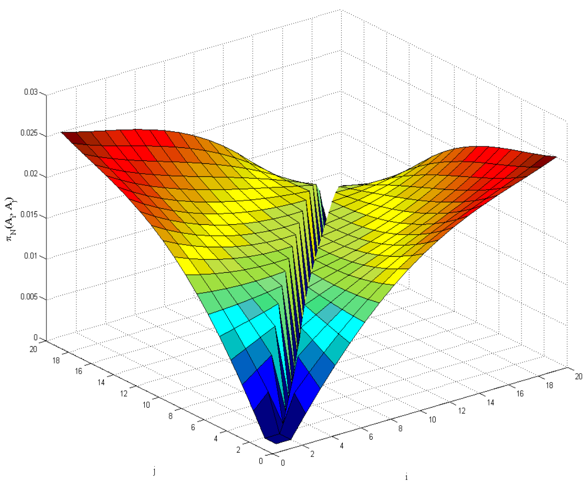


Figure 2. The second order inclusion probabilities of the population units with N=20, m=5

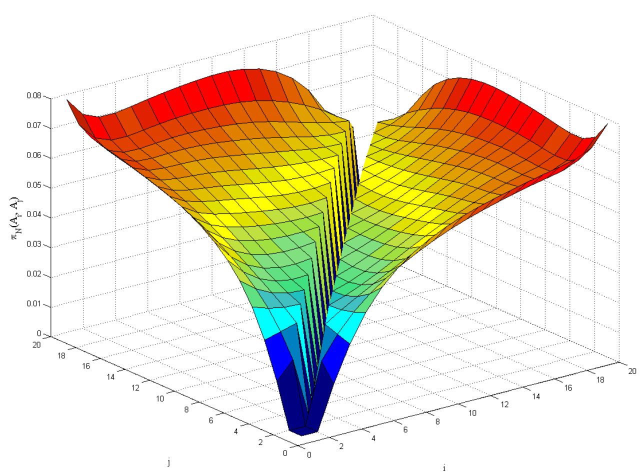


FIGURE 3. The second order inclusion probabilities of the population units with N=20, m=7

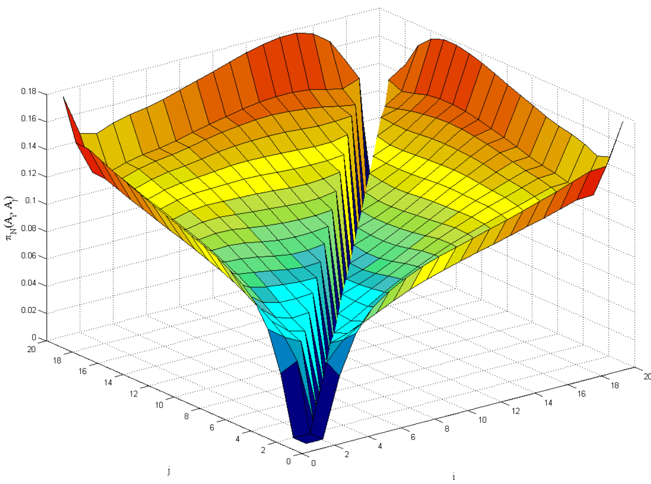


FIGURE 4. The second order inclusion probabilities of the population units with N=50, m=3

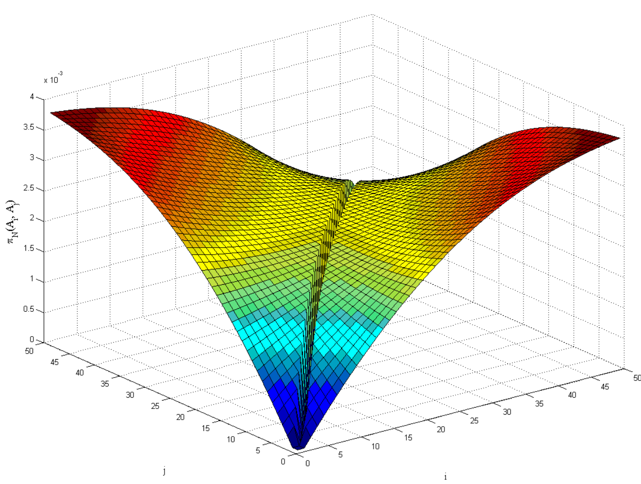


Figure 5. The second order inclusion probabilities of the population units with N=50, m=5

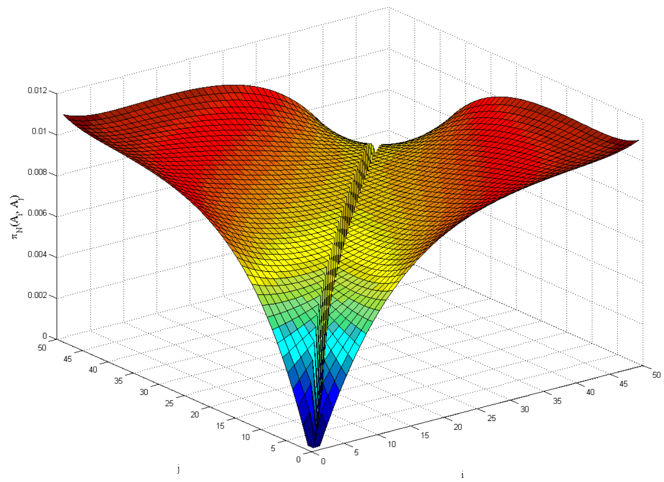
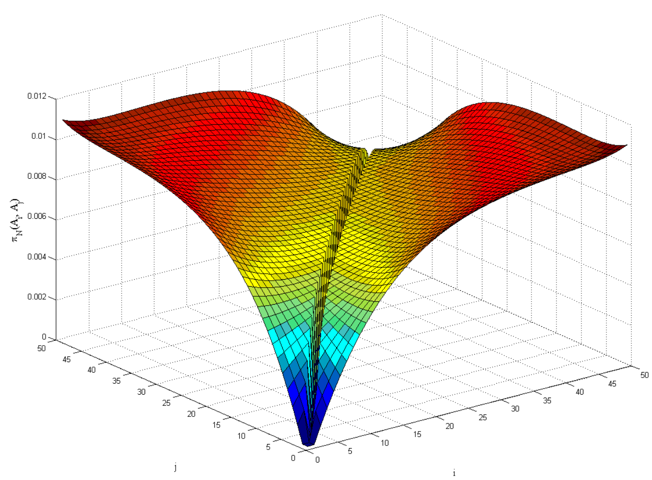


Figure 6. The second order inclusion probabilities of the population units with N=50, m=7



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Appendix A. Matlab Code for First Order Inclusion Probabilities

```
function P =firstinc(N,m)
B(1:m,1:m,1:N,1:2)=0;
for i=1:N
for g=1:m
for u=g:m
B(1,g,i,1)=B(1,g,i,1)+nck(i-1,u)*nck(N-i-g+2,m-u)/nck(N-g+1,m);
if i>1
B(1,g,i-1,2)=1-B(1,g,i,1);
end
end
end
end
for ag=2:m
for i=1:N
for g=ag:m
for u=g:m
B(ag,g,i+ag-1,1)=B(1,g,i,1);
if i>1
B(ag,g,i-1,2)=1-B(ag,g,i,1);
end
end
end
```

```
end
end
A=allperm([1 2],m);
for i=1:N
AT(:,1,i)=B(1,1,i,A(:,1));
end
for i=1:N
for j=2:m
for t=1:2^m
AT(t,j,i)=B(2*j-1-sum(A(t,1:j-1)),j,i,A(t,j));
end
end
end
for i=1:N
for t=1:2^m
c(i,t)=1;
for j=1:m
c(i,t)=c(i,t)*AT(t,j,i);
end
end
end
P=1-sum(c');
```

B. Matlab Code for Second Order Inclusion Probabilities

```
function P2=secondinc(N,m)
B(1:m,1:m,1:N,1:2)=0;
B3(1:m,1:m,1:m,1:N,1:N)=0;
for i=1:N
for g=1:m
for u=g:m
B(1,g,i,1)=B(1,g,i,1)+nck(i-1,u)*nck(N-i-g+2,m-u)/nck(N-g+1,m);
if i>1
B(1,g,i-1,2)=1-B(1,g,i,1);
end
end
end
end
for ag=2:m
for i=1:N
for g=ag:m
for u=g:m
B(ag,g,i+ag-1,1)=B(1,g,i,1);
if i>1
B(ag,g,i-1,2)=1-B(ag,g,i,1);
end
end
end
end
end
for aig=1:m
for ajg=aig:m
for i=1:N
```

```
for j=i+1:N
for g=1:m
B3(aig,ajg,g,i,j)=B(ajg,g,j,1)-B(aig,g,i+1,1);
if B3(aig,ajg,g,i,j)<0
B3(aig,ajg,g,i,j)=0;
end
end
end
end
end
end
A=allperm([1 2 3],m);
for k=1:size(A,1)
for l=1:size(A,2)
if A(k,l)==1;
AA{k,l}={1 1};
elseif A(k,l)==3;
AA{k,l}={2 1};
elseif A(k,l)==2;
AA{k,l}={2 2};
end
end
end
for i=1:N-1
for j=i+1:N
for k=1:3^m
if A(k,1)==1;
AT(i,j,k,1)=B(1,1,i,1);
elseif A(k,1)==2;
AT(i,j,k,1)=B(1,1,j,2);
elseif A(k,1)==3;
AT(i,j,k,1)=B3(1,1,1,i,j);
end
end
end
end
for i=1:N-1
for j=i+1:N
for l=2:size(A,2)
for k=1:3^m
if A(k,l)==1;
aa=2*l-1;
for t=1:l-1
aa=aa-AA{k,t}{1};
end
AT(i,j,k,l)=B(aa,l,i,AA{k,l}{1});
elseif A(k,l)==2
aa=2*l-1;
for t=1:l-1
aa=aa-AA{k,t}{2};
end
```

```
AT(i,j,k,l)=B(aa,l,j,AA{k,l}{2});
elseif A(k,l)==3
aai=2*l-1;
for t=1:l-1
aai=aai-AA{k,t}{1};
end
aaj=2*l-1;
for t=1:l-1
aaj=aaj-AA{k,t}{2};
end
AT(i,j,k,l)=B3(aai,aaj,l,i,j);
end
end
end
end
end
for i=1:N-1
for j=i+1:N
for k=1:3^m
c(i,j,k)=1;
for l=1:m
c(i,j,k)=c(i,j,k)*AT(i,j,k,l);
end
end
P(i,j)=sum(c(i,j,:));
end
end
P(N,1:N)=0;
P1=firstinc(N,m);
for i=1:N-1
for j=i+1:N
P2(i,j)=1-((1-P1(i))+(1-P1(j))-P(i,j));
end
end
```