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BAYESIAN UNIT-ROOT TESTING IN STOCHASTIC VOLATILITY MODELS WITH CORRELATED ERRORS

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Abstract

A series of returns are often modeled using stochastic volatility models. Many observed financial series exhibit unit-root non-stationary behavior in the latent AR(1) volatility process and tests for a unit-root become necessary, especially when the error process of the returns is correlated with the error terms of the AR(1) process. In this paper, we develop a class of priors that assigns positive prior probability on the non-stationary region, employ credible interval for the test, and show that Markov Chain Monte Carlo methods can be implemented using standard software. Several practical scenarios and real examples are explored to investigate the performance of our method.

Keywords: Contemporaneous financial correlation, Markov chain Monte Carlo, Gibbs sampling, unit-root test, WinBUGS, financial data

2000 AMS Classification: 62M10, 62P20

1. Introduction

The time evolving feature of volatility is often modeled using the so-called Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family and Stochastic Volatility (SV) models. Bollerslev et al. [3] and Bera and Higgins [1] present an extensive survey on GARCH/ARCH literature. In a basic SV model, the mean corrected return is modeled as a product of two independent stochastic terms one of which is the volatility (the conditional variance of the logarithmic return) and the other one is a zero mean Gaussian white noise with unit variance. The stochastic nature of the volatility is modeled as a

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log normal autoregressive (AR) process with errors independent of the white noise of the returns. In many real life applications SV models offer greater flexibilities over GARCH family of models Fridman and Harris [7]. However a basic SV model is too restrictive to allow the financial leverage effect in the model. In order to incorporate the leverage effect which is seen in many financial series into the model, Harvey and Shephard [8] extended the basic SV model to include a correlation between the two error terms namely the white noise in the return and the error in the volatility process. In the rest of the article bSV (basic SV) and eSV (extended SV) refer to SV model with uncorrelated and correlated return error and conditional variance error, respectively. We should also note that there are different characterizations for the correlations and they provide alternative specifications to the leverage effect e.g. see Jacquier et al. [10] and Yu [24]. However by eSV throughout the article we refer to a SV model that incorporates nonzero correlation without making any distinction among the alternative specifications. More detailed specification of the eSV is presented in Sect. 2.

Degree of the persistency of past volatility on the current one is represented by the AR coefficient in the SV models. Therefore investigating efficient estimation techniques in SV models has been a considerable research interest in finance, econometrics, and statistics. Literature on frequentist and Bayesian estimation methodologies for bSV can be found among the references given in for example So and Li [23]. In terms of the estimation in eSV, Sandman and Koopman [22] discussed the Monte Carlo maximum likelihood method of estimating the parameters of bSV and showed that the same approach would be applicable to eSV without modification. A Bayesian MCMC algorithm for eSV was developed by Jacquier et al. [10] where the volatilities were sampled univariately. Yu [24] refined the eSV model characterized by Jacquier et al. [10] and adjusted the MCMC method of Meyer and Yu [14] to accommodate the correlation. Omori et al. [17] extended the method of Kim et al. [13] for estimation in eSV. This algorithm later was further improved by Nakajima and Omori [15].

There are several financial data analyses in the literature where bSV or eSV models are used. In most cases the estimates of the AR coefficients are found to be extremely close to unity. For instance Harvey and Shephard [8] and Kim et al. [13] analyzed the daily observations of weekday close exchange rates for the UK pound, Deutsche Mark, Japanese Yen, and Swiss Frank versus US dollar and estimated the coefficient of the AR process close to 1 for each of these four series, namely 0.9912, 0.9646, 0.9948, and 0.9575 respectively. Chib et al. [5] employed their bSV estimation algorithm on stock market data (S&P500 daily returns from 1980 to 1987) and interest rate data (bank discount rates on three month treasury bills from 1962 to 1995) and they too estimated the AR coefficients for these series to be close to unity (greater than 0.98). The eSV analysis of Jacquier et al. [10] on daily series including two stock indices (S&P500 daily returns from 1980 to 1987 and returns on value weighted CRSP indices from 1962 to 1987) and two exchange rates (UK pound and Deutsche Mark versus US dollar) delivered persistency estimates all close to unity (greater than 0.97). Nakajima and Omori [15] analyzed the daily stock returns data, the S&P500 index from 1970 to 2004, using their MCMC approach for eSV and the estimate for AR coefficient came out to be close to unity (greater than 0.98). Such data analyses including a wide variety of financial series, such as stock market indices and interest rates suggest strong empirical evidence that volatility is highly persistent. Two major questions are raised by these findings at this point. One of them is whether the volatility follows a random walk. The other one is how different financial series of interest compare with each other with respect to the volatility persistence within. Persistence of volatility and its implications in econometrics and finance are investigated in the literature (see e.g. Poterba and Summers [21], Pindyck

[19], Pindyck [20], Chou [6], Bolerslev and Engle [2]). Statistical testing for a hypothesis of unit-root in volatility process aids in providing answers to these questions.

So and Li [23] proposed a Bayes factor based unit-root testing procedure for bSV model that uses the MCMC output obtained by using the algorithm of Kim et al. [13]. Kalaylıoğlu and Ghosh [12] proposed a Bayesian unit-root test in bSV model that was based on the posterior interval for the persistency parameter. This article extends their Bayesian testing scheme for unit root testing to apply in eSV and investigates the behavior of the proposed unit-root test versus the magnitude of the underlying correlation between the errors in the return and the errors in the volatility process.

The rest of the article is organized as follows: Section 2 describes the eSV model of interest. It also presents our Bayesian approach including the motivation for our choice of prior densities for the persistency parameter, Gibbs sampling for our eSV via WinBUGS, and Bayesian unit-root testing based on the posterior interval. A Monte Carlo simulation experiment studying the power and the errors of the unit-root test under the correlations of various different strength is presented in Sect. 3 while the proposed test is used in Sect. 4 on couple of financial series to detect the presence of on unit-root. Finally Sect. 5 gives a summary of the results and future directions of research.

2. Bayesian inference for eSV model

The SV model with correlated errors is given below. The hypothesis of interest is whether the volatility process has a significant unit root. This hypothesis is tested through the use of a posterior credible interval constructed via Gibbs sampling algorithm.

2.1. Model. We consider Euler time discretization of the continuous time stochastic volatility model (see Johannes and Polson [11]) given by

$$\begin{aligned} &r_t = e^{h_{t-1}/2}\epsilon_t \\ (2.1)\qquad &h_t - \mu = \phi(h_{t-1} - \mu) + \nu_t, \text{t=2,...,n} \end{aligned}$$

where ϕ represents the persistency of past volatility on the current volatility, r_t is the observed mean corrected return, h_t is the unobserved volatility, $r_1 \sim N(0, e^{\mu})$, $h_1 \sim N(\mu, \frac{\sigma_\nu^2}{1-\phi^2})$ when $|\phi| < 1$, $h_1 \sim N(\mu, \sigma_\nu^2)$ when $\phi = 1$, and

$$\begin{pmatrix} \epsilon_t \\ \nu_t \end{pmatrix} \sim \text{i.i.d. } N(0, \Sigma), \Sigma = \begin{pmatrix} 1 & \rho\sigma_\nu \\ \rho\sigma_\nu & \sigma_\nu^2 \end{pmatrix}$$

Here ρ is the correlation between ϵ_t and ν_t and we call model (2.1) as our eSV. In order to understand the relevance of ρ in a financial time series, denote first $w_t = \frac{\nu_t/\sigma_\nu - \rho\epsilon_t}{\sqrt{1-\rho^2}}$. Then using w_t we can rewrite (2.1) as

$$\begin{aligned} &r_t = e^{h_{t-1}/2}\epsilon_t \\ (2.2)\qquad &h_t - \mu = \phi(h_{t-1} - \mu) + \sigma_\nu\rho\epsilon_t + \sigma_\nu w_t\sqrt{1-\rho^2} \end{aligned}$$

where

$$\begin{pmatrix} \epsilon_t \\ w_t \end{pmatrix} \sim N(0, I_2)$$

and I_2 is the 2×2 identity matrix. Notice that, from (2.2) we obtain

$$E(h_t|r_t, h_{t-1}) = \mu + \phi(h_{t-1} - \mu) + \sigma_\nu\rho\, e^{-h_{t-1}/2}r_t \qquad (2.3)$$

In (2.3), ρ represents the linear relationship between r_t and $E(h_t|r_t, h_{t-1})$ holding the other parameters constant. That is, a financial contemporaneous correlation between the conditional mean of volatility and return is induced by including a correlation between ϵ_t and ν_t in the SV model. Model (2.1) is different from both Jacquier et al. [10] and Yu [24] in the way the correlation is specified. The statistical interest lies in testing $H_0 : \phi$=1 versus $H_0 : |\phi| < 1$.

2.2. Prior Distributions. The parameter of interest for unit-root hypothesis testing in eSV is the persistency parameter ϕ. Thus we start with our choice of prior densities for ϕ. In the literature for Bayesian estimation in SV models, continuous distributions with the support between 0 and 1 such as Uniform (a completely flat prior), Beta, and truncated Normal are used as prior distributions for ϕ. Phillips [18] had shown that flat priors defined on (0,1) are indeed informative for ϕ in AR time series models, contrary to the intention, and downweights the unit root alternative in the posterior distribution. Motivated by this finding, Kalaylıoğlu and Ghosh [12] proposed for Bayesian unit-root test in bSV the use of a prior distribution that has non-null mass on the unity. This allows a non-null mass on unity in the posterior distribution also and prevents the posterior inference on the volatility process from being biased towards stationarity. Also this approach enables the use of $(1 - \alpha)\%$ posterior credible interval as a unit-root testing mechanism. They showed that unit-root test in bSV based on posterior interval provides a more straightforward and statistically less erroneous (owing to smaller combined Type I and Type II errors) alternative to the test based on the Bayes Factor of So and Li [23]. The priors we used for ϕ in the rest of the article are in line with Kalaylıoğlu and Ghosh [12] and are uniformly distributed between 0 and 1+c. We considered c = 0, 0.001, and 0.1 for c in order to perform a sensitivity analysis. The parameters μ, σ^2_ν, and ρ are assumed independent apriori. We used a uniform prior for μ, Inverse-Gamma distribution with shape and scale parameters equal to $2 + 10^{-10}$ and 0.1 which gives the mean about 0.1 for σ^2_ν, and uniform prior with support between -1 and 1 for ρ.

2.3. Gibbs sampling. MCMC sampling algorithms for bSV models can be considered in two separate groups. One group of MCMC methods are based on the single move Gibbs sampler developed in Jacquier et al. [9] in which unobserved volatilities are sampled univariately. The other group follow the multi-move algorithm developed in Kim et al. [13] in which volatilities are sampled as a group; this procedure is based on taking the log-squared transformations of returns and approximating the exact distribution of log-squared transformed errors (log $\chi^2_{(1)}$) by a mixture of normal densities. These approaches were later adopted for eSV; see Yu [24] for extension of single-move sampling and Omori et al. [17] for multi-move sampling. Since single move sampling is based on univariate sampling of the unobserved volatilities, it is less simulation-efficient than multi-move MCMC algorithm. However Yu [24] showed that this is less of a problem for eSV. We prefer using the single-move algorithm and employ WinBUGS to carry it out. We present measures of empirical efficiency of our implementation in Sect. 4.

Joint posterior distribution representation for model (2.1) is

$$f(\vec{h}, \mu, \phi, \sigma^2_\nu, \rho|\vec{r}) \propto \Pi^n_{t=1} f(r_t|h_t, h_{t-1}, \mu, \phi, \sigma^2_\nu, \rho)$$
$$\Pi^n_{t=2} f(h_t|h_{t-1}, \mu, \phi, \sigma^2_\nu) f(h_1|\mu, \phi, \sigma^2_\nu) f(\mu) f(\phi) f(\sigma^2_\nu) f(\rho)$$

where $\vec{h}$ and $\vec{r}$ represent the vectors of unobserved volatilities and the observed return data respectively from t=1 to n,

$$f(r_t|h_t, h_{t-1}, \mu, \phi, \sigma^2_\nu, \rho) \sim N(\sigma_\nu^{-1} \rho^{-1} e^{h_{t-1}/2}((h_t - \mu) - \phi(h_{t-1} - \mu)), \rho^{-2}(1 - \rho^2) e^{h_{t-1}})$$
$$f(h_t|h_{t-1}, \mu, \phi, \sigma^2_\nu) \sim N(\mu(1 - \phi) + \phi h_{t-1}, \sigma^2_\nu)$$

and $f(\mu)$, $f(\phi)$, $f(\sigma_\nu^2)$, and $f(\rho)$ are prior densities for the corresponding parameters. Full conditional density of ρ is

$$f(\rho|\underline{h}, \mu, \phi, \sigma_\nu^2, \underline{r}) \sim \Pi_{t=2}^{n} \rho(1-\rho^2)^{-1/2} exp(-\frac{1}{2}\rho^2(1-\rho^2)^{-1}$$
$$exp(-h_{t-1})(r_t - c_t\rho^{-1})^2)$$

where $c_t = \sigma_\nu^{-1}\rho^{-1} exp(h_{t-1}/2)((h_t-\mu)-\phi(h_{t-1}-\mu))$. We used WinBUGS - version 1.4. to do the posterior computations through Gibbs sampling. The full conditional density of ρ is not a kernel of a well known distribution and not log-concave. In such cases WinBUGS adopts a Metropolis Hastings algorithm. The full conditional densities of the other parameters and the latent volatilities are given in Kalaylıoğlu and Ghosh [12]. The application and details of the Gibbs sampling algorithm for SV models in WinBUGS are given in Meyer and Yu [14].

2.4. Testing the unit-root hypothesis. In order to test for the unit-root hypothesis $H_0 : \phi = 1$ versus $H_0 : |\phi| < 1$ in (2.1), we extend the use of posterior credible interval approach in Kalaylıoğlu and Ghosh [12]. The test rejects the null hypothesis if the marginal posterior interval of 95% confidence level for ϕ does not include the unity. Rejection of the unit-root hypothesis implies that the volatility process is stationary and hence one can proceed with the application of inferential techniques in SV models that are developed under the stationarity assumption. On the other hand failure to reject the null hypothesis implies that the volatility process is nonstationary and shocks to volatility have long-term effects.

3. Simulation study

TABLE 1. Number of correct decisions (NCD) on unit-root hypothesis and total error rates (based on 100 MC replications), true $\rho = -1$

ϕ	Prior	NCD	Total error rate
1	U(0,1)	0	-
	U(0,1.001)	71	-
	U(0,1.1)	62	-
0.98	U(0,1)	100	1
	U(0,1.001)	91	0.38
	U(0,1.1)	90	0.48
0.95	U(0,1)	100	1
	U(0,1.001)	99	0.30
	U(0,1.1)	99	0.39
0.80	U(0,1)	100	1
	U(0,1.001)	100	0.29
	U(0,1.1)	100	0.38

We conducted a Monte Carlo (MC) simulation experiment in order to assess the performance of the proposed unit-root test based on the posterior interval of ϕ in several scenarios of our eSV. We set the values of the true parameters at $\mu = -9$ and $\sigma_\nu = 0.32$ and these are the values used in So and Li [23] and Kalaylıoğlu and Ghosh [12]. In order to investigate the magnitudes of the errors of Types I and II this unit-root test commits under different scenarios on contemporaneous correlation, we choose various

Table 2. Number of correct decisions (NCD) on unit-root hypothesis and total error rates (based on 100 MC replications), true $\rho = -0.8$

ϕ	Prior	NCD	Total error rate
1	U(0,1)	0	-
	U(0,1.001)	77	-
	U(0,1.1)	82	-
0.98	U(0,1)	100	1
	U(0,1.001)	94	0.29
	U(0,1.1)	93	0.25
0.95	U(0,1)	100	1
	U(0,1.001)	99	0.24
	U(0,1.1)	99	0.19
0.80	U(0,1)	100	1
	U(0,1.001)	100	0.23
	U(0,1.1)	100	0.18

Table 3. Number of correct decisions (NCD) on unit-root hypothesis and total error rates (based on 100 MC replications), true $\rho = -0.5$

ϕ	Prior	NCD	Total error rate
1	U(0,1)	0	-
	U(0,1.001)	87	-
	U(0,1.1)	92	-
0.98	U(0,1)	100	1
	U(0,1.001)	94	0.19
	U(0,1.1)	89	0.19
0.95	U(0,1)	100	1
	U(0,1.001)	100	0.13
	U(0,1.1)	100	0.08
0.80	U(0,1)	100	1
	U(0,1.001)	100	0.13
	U(0,1.1)	100	0.08

values for ϕ and ρ. We consider $\phi = 1, 0.98, 0.95, 0.8$; $\phi = 1$ implies unit-root in the volatility process. The literature documents that stock returns are negatively correlated with the changes in volatility e.g., Nelson [16]. Thus we consider $\rho = -1, -0.8, -0.5, 0$; $\rho = -1$ implies perfect negative correlation between the disturbance to the return and the disturbance to volatility. For each scenario which is based on a combination of ϕ and ρ in the data generation process, we generated a set of observed mean corrected returns over n=1000 days using the model (2.1) and conducted the MCMC sampling to obtain the posterior distributions of (μ, ϕ, ρ, σ_ν^2). We also ran the experiment by simulating data from a bSV (i.e. SV model with ρ=0) and fit model (2.1) for the Bayesian analysis to further assess the robustness of the proposed unit-root test to the underlying SV model. The experiment for each scenario was repeated for 100 times. The data were generated using a code written in Matlab (version 7.7). For each data set generated, matBUGS

TABLE 4. Number of correct decisions (NCD) on unit-root hypothesis and total error rates (based on 100 MC replications), true $\rho = 0$

ϕ	Prior	NCD	Total error rate
1	U(0,1)	0	-
	U(0,1.001)	86	-
	U(0,1.1)	96	-
0.98	U(0,1)	100	1
	U(0,1.001)	89	0.25
	U(0,1.1)	78	0.26
0.95	U(0,1)	100	1
	U(0,1.001)	99	0.15
	U(0,1.1)	99	0.05
0.80	U(0,1)	100	1
	U(0,1.001)	100	0.14
	U(0,1.1)	100	0.04

TABLE 5. Number of correct decisions (NCD) on unit-root hypothesis and total error rates (based on 100 MC replications), true $\rho = -0.5$

ϕ	Prior	NCD	Total error rate
1	U(0,1)	0	-
	U(0,1.001)	92	-
	U(0,1.1)	96	-
0.98	U(0,1)	100	1
	U(0,1.001)	89	0.25
	U(0,1.1)	86	0.21
0.95	U(0,1)	100	1
	U(0,1.001)	99	0.15
	U(0,1.1)	99	0.15
0.80	U(0,1)	100	
	U(0,1.001)	100	0.14
	U(0,1.1)	100	0.07

(the Matlab interface to WinBUGS) was used to call WinBUGS from within Matlab for posterior calculations.

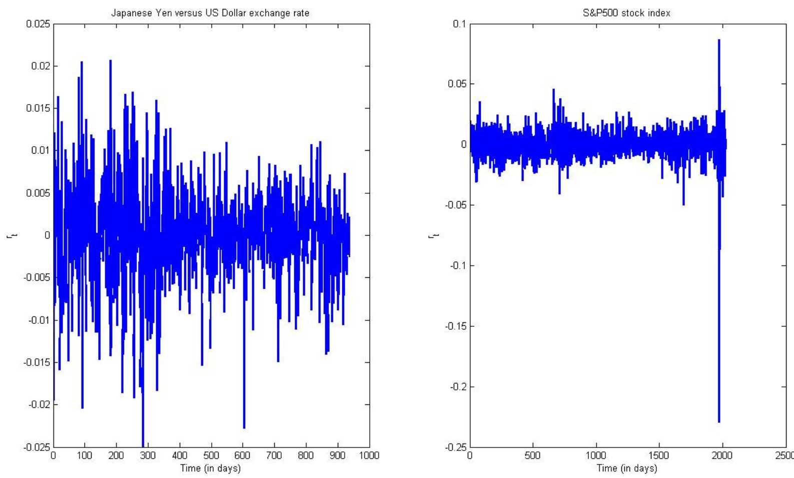
For the scenario with $(\phi, \rho) = (1, -1)$, we discarded the first 4000 iterations for the burn-in period in our Gibbs samples. For the other (ϕ, ρ) scenarios, we discarded first 2500 iterations as the burn-in period. We made these selections based on the trace plots of three different Markov chains starting from three different sets of initial parameter values. MCMC sample size we used for inference after the burn-in point was 5000. We should note that MCMC chain size of 5000 is somehow smaller than what is generally used in the MCMC literature for eSV. The reason for adopting a relatively smaller MCMC size for the present simulation study is that the total number of different scenarios we considered is many, namely 48 corresponding to 4 different values for ρ, 4 for ϕ, and 3 types of prior densities for ϕ. Thus the overall computational cost would increase considerably for a larger MCMC sample size. Nevertheless Gelman-Rubin statistic computed (see Brooks and Gelman [4]) in WinBUGS converged to 1 indicating the employed size ensures sufficient precision in our MCMC estimates. The MC error estimates for the posterior means of all the parameters and unobserved volatilities being so small (which is varing between 0.00001 and 0.03) also indicates that the employed chains size for posterior inference was adequate.

Through the Tables 1 to 5 we list the results of our simulation study on the number of correct decisions made by our unit-root test among 100 MC replications and the total error rate. *The number of correct decisions is the number of MC replications for which the value 1 is included in the* 95% *posterior credible interval of* ϕ *when the true value of* ϕ *is 1. When the true value of* ϕ *is less than 1, the number of correct decisions is calculated as the number of* 95% *posterior intervals that do not include 1. The total error rate is computed as Type I error rate + Type II error rate.* For the Tables 1 to 3, model (2.1) was used for both the data generation and the Bayesian analysis. For the Table 4, the data were generated from bSV and eSV was used for the Bayesian inference. For the Table 5, the data were generated from (2.1) and bSV was used for the Bayesian inference. The results show that U(0,1+c) type of prior density for ϕ enables the test based on posterior interval to detect the underlying unit root in eSV as opposed to U(0,1) prior. Tables 1 to 4 imply that in the presence of strong underlying correlation between the innovations in the volatility and the innovations in the returns (e.g for ρ= -1), the test is more prone to comitting Type I error compared to a milder true correlation (e.g. for ρ=-0.5). Comparing Tables 3 versus 4, we see that overfitting the model (i.e. including a correlation parameter in the model whereas the two error terms are actually uncorrelated) makes the test more vulnerable to committing Type II error. Also the results show that strength of the correlation affects the test's ability to correctly determine the unit-root; all the total error rates committed under the perfect true correlation is higher. For all the ρ values investigated except for ρ=-1, using U(0,1.1) as opposed to U(0,1.001) prior for the persistency parameter decreases the total error rates. In Table 5, we investigated the total error committed by the proposed unit root test when the model is underfit. That is, we generated the data from model (2.1) with ρ=-0.5 and used bSV for the analysis. The error rates increased as seen in Table 5 compared to Table 3; the probability of making an erroneous conclusion about the unit root is sensitive to the assumption about the existence of correlation in the modeling stage. We repeated the same experiment with true ρ=-1. Ignoring this severe correlation in the analysis resulted in an increase in the total error rates.

4. Empirical application

We studied two sets of financial data one of which is an exchange rate series and the other one is a series of stock indices. The exchange rate series is the daily exchange rate of Japanese Yen versus US Dollar from Oct/1/1981 to June/28/1985. The stock index is the S&P500 stock returns from January/2/1980 to December/31/1987. The Japanese Yen versus US dollar exchange rate series was investigated previously by Kalaylıoğlu and Ghosh [12] using their SV model; they fitted the bSV and conducted the Bayesian unit root testing based on the posterior interval. The S&P500 index considered here was analyzed in Jacquier et al. [10] using their SV model for estimation of volatility persistence. We used equation (2.1) to model these data sets and the proposed Bayesian unit root test is used to determine whether the log volatility process possesses a unit-root for these series.

FIGURE 1. Time series plots of the returns



We used WinBUGS to conduct the Gibbs sampling for estimation of model (2.1) for these series. We used two distinct sets of initial values to start the Markov chain for the model parameters. We let WinBUGS to generate the initial values for the latent volatilities $\{h_t\}_{t=1}^{t=n}$. We discarded the first 5000 MCMC iterations and used the remaining 15000. In order to reduce the autocorrelation within the MCMC series for the correlation parameter ρ we used every 5th MCMC iteration for posterior computations. The posterior inference for ϕ and ρ are given in Table 6. Let p_t denote the exchange rate or stock return at time t. Mean corrected log return is $r_t = (\log p_t - \log p_{t-1}) - \frac{1}{n}\sum(\log p_t - \log p_{t-1})$. These mean corrected returns are plotted against time in Fig. 1.

The results in Table 6 show that there is a significant evidence for unit-root in log-volatility model for the exchange rate series (the corresponding 95% posterior intervals include the point 1). On the other hand there is no such evidence for the S&P500 index. This result implies that the effect of volatility in the Japanese Yen vs. US dollar exchange rate stays persistent for a long time. For the exchange rate data, where the underlying volatility is nonstationary, the posterior inference about ρ seems sensitive to the prior density chosen for ϕ. Same behaviour is not observed for the stock index series. The MCMC updates took about 3500 minutes for the exchange rate data and about 8000 minutes for the S&P500 data. The posterior credible intervals for ρ for the exchange rate series imply that the correlation between the error in the return and the error in the volatility may be ignorable. In this case, model (2.1) is an overfit due to the inclusion of the correlation term. However this does not constitute a disadvantage for discovering the unit-root in latent volatility; the proportion of correct decisions made when the true $\phi = 1$ in Table 4 are 86% and 96% respectively for priors U(0,1.001) and U(0,1.1).

5. Summary

In this paper, we considered the SV models, where the volatility terms (i.e., the conditional variances) are modeled by a latent first order autoregressive (AR(1)) process and the error process of the log-returns is correlated with the error terms of the latent AR(1) process. We proposed a Bayesian testing procedure for unit-root in SVM; we

Table 6. Posterior means and 95% posterior intervals for ϕ and ρ.

Data	Prior for ϕ	ϕ	ρ
Yen vs. US dollar	U(0,1.001)	0.9865(0.9501,1.000)	0.1186(-0.1763,0.3897)
	U(0,1.1)	0.9748(0.8908,1.003)	0.1087(-0.1789,0.3604)
	U(0,1)	0.9792(0.9014,0.9993)	0.1449(-0.1494,0.3953)
S&P500	U(0,1.001)	0.9627(0.9325,0.9823)	-0.3069(-0.4362,-0.1402)
	U(0,1.1)	0.9652(0.9373,0.9844)	-0.3044(-0.4710,-0.1502)
	U(0,1)	0.9615(0.9295,0.9826)	-0.3062(-0.4495,-0.1691)

developed a suitable class of priors that assigns positive prior probability on the non-stationary region and employed posterior credible interval for the hypothesis testing decision criterion. We conducted an extensive simulation study which demonstrated the superior performance of the proposed test over some of the methods that use continuous prior for the persistence parameter that is defined only on the stationary region. The simulation results suggest that the proposed test has much smaller total error (type I plus type II) than that obtained by default priors.

We also found out that the ability of the proposed test to correctly concluding in a unit root is affected by the underlying correlation structure. Therefore in practice prior to applying the unit root test in SVM, a preliminary analysis should be conducted to obtain an insight on the strength of the correlation.

Comparing Table 1 to Table 5 we see that in the presence of perfect correlation and the unit root in the data, fitting eSV makes the proposed unit root test commit larger type I error. i.e. the unit root test in eSV tends to reject the null hypothesis more often compared to the unit root test in bSV. A further investigation is needed to understand this behaviour. One may consider more specialized priors for ρ that would allow positive prior probability for $\rho = 0$ and continuous on remaining values of ρ. Although we used the default 95% posterior intervals to make all of our conclusions, further investigations are required to develop more rigorous criteria to select the cut off value for the posterior probability of the null hypothesis.

The major limitation of our study is the number of replications used in the Monte Carlo simulation experiment. Although a small number of Monte Carlo replications is common in the literature for SV estimation, these replications may have to be increased in order to get more insight on the unit root testing qualities for certain scenarios.

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