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On bi-ideals of ordered Γ- semigroups – A Corrigendum

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This is a Corrigendum of the paper in [1]. We keep the numbering given in [1] and we add our remarks and corrections.

We give, among others, the correct definition of a strongly regular *po*-Γ-semigroup, and we correct the proof of 2.19 Lemma given in [1]. As one can see below, each of the papers in the References of the present paper contains its corresponding result in [1] and only such papers are cited in the References of this corrigendum and nothing more.

1.1 Definition Adding the uniqueness condition in the definition given by Sen and Saha in [12], we get the 1.1 Definition in [1] first introduced in [5] (cf. also [6,7]) which, for an ordered Γ-semigroup M, allows us in an expression of the form, say $A_1\Gamma A_2\Gamma, ..., A_n\Gamma$ to put the parentheses anywhere beginning with some A_i and ending in some A_j $(i, j \in N = \{1, 2, ..., n\})$ or in an expression of the form $a_1\Gamma a_2\Gamma, ..., a_n\Gamma$ or $a_1\gamma a_2\gamma, ..., a_n\gamma$ to put the parentheses anywhere beginning with some a_i and ending in some a_j ($A_1, A_2, ..., A_n$ being subsets and $a_1, a_2, ..., a_n$ elements of M). Unless the uniqueness condition (widely used by some authors in the past), in an expression of the form, say $a\gamma b\mu c\xi d\rho e$ or $a\Gamma b\Gamma c\Gamma d\Gamma e$, it was not known where to put the parentheses. As the sets M and Γ are different, in the property (1) of that Definition in [1] we have to express what the $a\gamma b \in M$ $(a, b \in M,\ \gamma \in \Gamma)$ means. So in the 1.1 Definition in [1] we add the following: For two nonempty sets M and Γ, define $M\Gamma M$ as the set of all elements of the form $m_1\gamma m_2$, where $m_1, m_2 \in M$, $\gamma \in \Gamma$. That is,

$M\Gamma M := \{m_1\gamma m_2 \mid m_1, m_2 \in M, \gamma \in \Gamma\}$.

Let now M and Γ be two nonempty sets. The set M is called a Γ-semigroup if the following assertions are satisfied:

(1) $M\Gamma M \subseteq M$.

(2) If $m_1, m_2, m_3, m_4 \in M$, $\gamma_1, \gamma_2 \in \Gamma$ such that $m_1 = m_3$, $\gamma_1 = \gamma_2$ and $m_2 = m_4$, then $m_1\gamma_1 m_2 = m_3\gamma_2 m_4$.

(3) $(m_1\gamma_1 m_2)\gamma_2 m_3 = m_1\gamma_1(m_2\gamma_2 m_3)$ for all $m_1, m_2, m_3 \in M$ and all $\gamma_1, \gamma_2 \in \Gamma$.

After the definition of ideals, on p. 794, lines –12 till –15, the authors wrote: "It is clear that the intersection of all ideals of a *po*-Γ-semigroup M is still an ideal of M" which is not true in general. The intersection of ideals is an ideal only if their intersection is nonempty. Then they say "We shall call this particular ideal, if exists, the kernel of M". But it is clear that such an intersection always exists. If there is no any proper ideal in M, then M itself is an ideal of M.

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1.6 Definition This is the definition of ordered Γ-semigroups (shortly *po*-Γ-semigroups) due to Sen and Seth [13].

The definition of the bi-ideals of a *po*-Γ-semigroup has been given by the authors on p. 794 as follows: A nonempty subset B of a *po*-Γ-semigroup M is called a *bi-ideal* of M if the following assertions are satisfied:

(1) $B\Gamma M\Gamma B \subseteq B$ and

(2) if $a \in M$ and $b \in M$ such that $b \leq a$, then $b \in B$.

On p. 795 the authors wrote "One can easily prove that $(a \cup a\Gamma a\Gamma a]$ is the bi-ideal generated by a" –which is correct. On the other hand, for the same subject, on p. 797 they wrote "if M is a *po*-Γ-semigroup and $\emptyset \neq A \subseteq M$, then one can easily prove that the set $(A \cup A\Gamma A \cup A\Gamma M\Gamma A]$ is the bi-ideal of M generated by A. In particular, for $A = \{a\}$, we write $B(a) = (a \cup a\Gamma a \cup a\Gamma M\Gamma a]$" –which is wrong, and used that second one in several parts of the paper.

It might be noted that some authors use the terms bi-ideal, subidempotent bi-ideal, some others the terms generalized bi-ideal, bi-ideal while the first one given in [1] is the best as in very exceptional cases we use the subidempotent bi-ideals.

On p. 794, l. –8, after the definition of bi-ideals the authors say "A bi-ideal A of M is called *subidempotent* if $A\Gamma A \subseteq A$". Besides, according to 2.3 Proposition the bi-ideals and the subidempotent bi-ideals are the same. These emphasize the fact that the authors use the concepts bi-ideals, subidempotent bi-ideals (and not the concepts generalized bi-ideals, bi-ideals) and that the definition of bi-ideals given in 1.1 Definition in [1] is correct.

1.7 Lemma In this lemma, property (4) should be corrected as $((A]] = (A]$ for every $A \subseteq M$. In property (7) the intersection of two ideals of M is an ideal of M if their intersection is nonempty.

On p. 795, for an element a of M the authors defined $a^2 = a\gamma a$, $a^3 = (a\gamma)^2 a$, $a^n = (a\gamma)^{n-1} a$. As in a^2 the a is an element of M, in $(a\gamma)^2$ the "$a\gamma$" has no meaning. If we write $(a\gamma)^2 a$ this means $(a\gamma)(a\gamma)a$ and, according to the definition of Γ-semigroups this is not true. The a^3 is the $(a\gamma a)\gamma a$ in other words the $a\gamma a\gamma a$ or the $a\gamma(a\gamma a)$.

In property (2) of 2.8 Definition, we have to write $A \subseteq (A\Gamma A\Gamma M]$ instead of $A \in (A\Gamma A\Gamma M]$. That is, a *po*-$\Gamma$-semigroup M is right regular if $A \subseteq (A\Gamma A\Gamma M]$ for all $A \subseteq M$. In the proof of the "$\Leftarrow$" part of 2.9 Lemma in [1] we have to write $(a\Gamma a)\Gamma(a\Gamma a) \subseteq M\Gamma(a\Gamma a)$ instead of $(a\Gamma a)\Gamma(a\Gamma a) \in M\Gamma(a\Gamma a)$, and $a\Gamma a \subseteq (M\Gamma(a\Gamma a)]$ instead of $a\Gamma a \in (M\Gamma(a\Gamma a)]$.

For 2.1 Lemma we refer to [2; Corollary 2], for 2.2 Theorem to [11; Theorem 2], for 2.3 Proposition to [8; p. 199], for 2.4 Theorem to [8; Proposition 1], for 2.9 Lemma to [10; Proposition 2].

2.11 Lemma (cf. [11; Lemma 9]) *Let M be a po-Γ-semigroup and $B(x)$, $B(y)$ the bi-ideals of M generated by the elements x and y of M, respectively, then we have $B(x)\Gamma M\Gamma B(y) \subseteq (x\Gamma M\Gamma y]$.* □

The proof given in [1] is wrong, we correct it as follows:

Proof. We have

$$B(x)\Gamma M\Gamma B(y) = (x \cup x\Gamma M\Gamma x]\Gamma(M]\Gamma(y \cup y\Gamma M\Gamma y] \subseteq ((x \cup x\Gamma M\Gamma x)\Gamma M\Gamma(y \cup y\Gamma M\Gamma y)] = (x\Gamma M\Gamma y].$$

□

The 2.12 Lemma in [1] (and its proof) should be corrected as follows:

2.12 Lemma (cf. [3; Remark 5] *A po-Γ-semigroup M is completely regular if and only if for every $a \in M$ there exist $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$ such that $a \leq (a\gamma a)\mu x\rho(a\xi a)$.*

Proof. $\Longrightarrow$. By hypothesis, we have

$$a \leq a\zeta t\omega a,\ a \leq a\gamma a\mu y \text{ and } a \leq z\rho a\xi a$$

for some $t, y, z \in M$ and $\zeta, \omega, \gamma, \mu, \rho, \xi \in \Gamma$. Then we have

$$a \leq a\zeta t\omega a \leq (a\gamma a\mu y)\zeta t\omega(z\rho a\xi a) = (a\gamma a)\mu(y\zeta t\omega z)\rho(a\xi a).$$

We have $(y\zeta t)\omega z \in M\Gamma M \subseteq M$. We put $x := y\zeta t\omega z$, and we have $a \leq (a\gamma a)\mu x\rho(a\xi a)$, where $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$.

$\Longleftarrow$. Let $a \in M$. By hypothesis, there exist $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$ such that

$$a \leq (a\gamma a)\mu x\rho(a\xi a) \leq a\gamma(a\mu x\rho a)\xi a,\ (a\gamma a)\mu(x\rho a\xi a),\ (a\gamma a\mu x)\rho(a\xi a)$$

where $a\mu x\rho a$, $x\rho a\xi a$, $a\gamma a\mu x \in M$, so M is regular, right regular and left regular. □

The authors give the following lemma:

2.13 Lemma (cf. [4; the Remark]) *A po-Γ-semigroup M is completely regular if and only if for every $A \subseteq M$, $A \subseteq ((A\Gamma A)\Gamma M\Gamma(A\Gamma A)]$.*

Equivalently, if for every $a \in M$, $a \in ((a\Gamma a)\Gamma M\Gamma(a\Gamma a)]$. □

This is from [1; p. 798, l. 10]: "From Lemma 2.12, it is obvious that the following Lemma 2.13 holds". But it is also obvious that from Lemma 2.13 Lemma 2.12 also holds. (According to 2.13 Lemma a *po*-Γ-semigroup M is completely regular if and only if, for every $a \in M$, $a \in ((a\Gamma a)\Gamma M\Gamma(a\Gamma a)]$ which means that, for every $a \in M$ there exist $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$ such that $a \leq (a\gamma a)\mu x\rho(a\xi a)$ and this is exactly the 2.12 Lemma). So 2.12 Lemma implies 2.13 Lemma and 2.13 Lemma implies 2.12 Lemma. So one of these lemmas should be deleted or combine them in one lemma as follows:

Lemma. *Let M be a po-Γ-semigroup. The following are equivalent:*

(1) *M is completely regular.*

(2) *For every $a \in M$ there exist $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$ such that $a \leq (a\gamma a)\mu x\rho(a\xi a)$.*

(3) *For every $A \subseteq M$, $A \subseteq ((A\Gamma A)\Gamma M\Gamma(A\Gamma A)]$.*

(3) *For every $a \in M$, $a \in ((a\Gamma a)\Gamma M\Gamma(a\Gamma a)]$.*

2.14 Theorem (cf. also the Theorem 2 in [3]) *A po-Γ-semigroup M is completely regular if and only if every bi-ideal B of M is semiprime.* □

In the " $\Rightarrow$ " part of 2.14 Theorem in [1] the authors instead of using 2.12 Lemma, they repeated its proof but they did the same mistake as in 2.12 Lemma by writing

$$a \leq a\alpha x\beta a \leq (a\gamma a\rho y)\alpha x\beta(z\mu a\gamma a)$$

for some $\alpha, \beta, \gamma, \rho, \mu \in \Gamma$ instead of

$$a \leq a\alpha x\beta a \leq (a\gamma a\rho y)\alpha x\beta(z\mu a\delta a)$$

for some $\alpha, \beta, \gamma, \rho, \mu, \delta \in \Gamma$ (that is, "γ" instead of "δ") so their proof of " $\Rightarrow$ " part of the theorem is wrong. As far as the " $\Leftarrow$ " part of the same theorem their proof is also wrong. To prove that the set $(a\Gamma a\Gamma M\Gamma a\Gamma a]$ is a bi-ideal of M, they should consider two elements x,y of $(a\Gamma a\Gamma M\Gamma a\Gamma a]$, an element $z \in M$, and two elements α, β in Γ and prove that $x\alpha z\beta y \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$. We have

$$x\alpha z\beta y \leq (a\gamma a\rho u\mu a\xi a)\alpha z\beta(a\omega a\delta v\sigma a\lambda a)$$

for some $u, v \in M$, $\gamma, \rho, \mu, \xi, \omega, \delta, \sigma, \lambda \in \Gamma$ and not

$$x\alpha z\beta y \leq (a\gamma a\rho u\mu a\gamma a)\alpha z\beta(a\gamma a\delta v\sigma a\gamma a)$$

for some $u, v \in M$, $\alpha, \beta, \gamma, \rho, \mu, \delta, \sigma \in \Gamma$. There is no "some" α, β there, the α, β should be fixed elements of Γ.

The proof of 2.14 Lemma given by the authors in [1] should be corrected according to the corrected form of 2.12 Lemma. Here is the correct proof of 2.14 Theorem:

$\Longrightarrow$. Let B be a bi-ideal of M, $a \in M$ and $a\Gamma a \subseteq B$. Since M is completely regular, by (the corrected form of) 2.12 Lemma, there exist $x \in M$ and $\gamma, \mu, \rho, \xi \in \Gamma$ such that

$$a \leq (a\gamma a)\mu x\rho(a\xi a) \in (a\Gamma a)\Gamma M\Gamma(a\Gamma a) \subseteq B\Gamma M\Gamma B \subseteq B,$$

so $a \in B$.
$\Longleftarrow$. Let $a \in M$. The nonempty set $(a\Gamma a\Gamma M\Gamma a\Gamma a]$ is a bi-ideal of M. In fact: Let $x, y \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$, $\alpha, \beta \in \Gamma$ and $z \in M$. We have $x \leq a\gamma a\rho u\mu a\xi a$ and $y \leq a\zeta a\delta v\sigma a\lambda a$ for some $u, v \in M$ and $\gamma, \rho, \mu, \xi, \zeta, \delta, \sigma, \lambda \in \Gamma$. Then we have

$$\begin{aligned} x\alpha z\beta y &\leq (a\gamma a\rho u\mu a\xi a)\alpha z\beta(a\zeta a\delta v\sigma a\lambda a) \\ &= a\gamma a\rho(u\mu a\xi a\alpha z\beta a\zeta a\delta v)\sigma a\lambda a \in a\Gamma a\Gamma M\Gamma a\Gamma a, \end{aligned}$$

so $x\alpha z\beta y \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$. Let now $x \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$ and $M \ni y \leq x$. Then, by (the corrected form of) the property (4) in 1.7 Lemma, we have $y \in ((a\Gamma a\Gamma M\Gamma a\Gamma a]] = (a\Gamma a\Gamma M\Gamma a\Gamma a]$. The rest of the proof is as in [1].

2.15 Lemma *Let M be a po-Γ-semigroup. The following are equivalent:*
(1) *M is completely regular.*
(2) *$B(a) = B(a\Gamma a) = B(a\Gamma a\Gamma M\Gamma a\Gamma a)$ $\forall$ $a \in M$.*
(3) *$a\mathcal{B}a\Gamma a$.* □

For 2.15 Lemma we refer to Lemma 1.4 in [14]. This lemma is without proof in [14] and it is not cited in the References of [1]. According to the definition of $\mathcal{B}$ given on p. 797, l. –9, we cannot write the property (3) of the lemma, as a is an element of M and $a\Gamma a$ a subset of M. So property (3) should be deleted and write $B(a) = B(a\Gamma a)$ instead. In the second line of the proof the word "Then" must be replaced by "and". This is because if M is a regular *po*-Γ-semigroup then, for any subset A of M, we have $B(A) = (A \cup A\Gamma M\Gamma A]$, so for an element a of M we have $B(a) = (a\Gamma M\Gamma a]$ and for the subset $a\Gamma a$ of M we have $B(a\Gamma a) = (a\Gamma a\Gamma M\Gamma a\Gamma a]$. On line 4 of the proof the authors wrote

$$a \in (a\Gamma M\Gamma a] \subseteq ((a\Gamma a\Gamma M]\Gamma M\Gamma(M\Gamma a\Gamma a]] \subseteq (a\Gamma a\Gamma M\Gamma a\Gamma a] \subseteq (a\Gamma M\Gamma a]$$

in an attempt to show that $(a\Gamma M\Gamma a] = (a\Gamma a\Gamma M\Gamma a\Gamma a]$. That "$a \in$" on line 4 of the proof should be deleted otherwise confusion is possible: As $(a\Gamma M\Gamma a]$ is a bi-ideal of M, if $a \in (a\Gamma M\Gamma a]$, then $B(a)$ is a subset of $(a\Gamma M\Gamma a]$, on the other hand, as M is regular, $B(a)$ is equal to $(a\Gamma M\Gamma a]$. They wrote $a^4 = (a\gamma)^3 a$ which is wrong. We remark that if M is a regular *po*-Γ-semigroup then, for any $A \subseteq M$, we have $B(A) = (A\Gamma M\Gamma A]$. Here is the correct proof of 2.15 Lemma.

Proof. (1) $\Longrightarrow$ (2). Let $a \in M$. Since M is regular, for the element a of M, we have $B(a) = (a\Gamma M\Gamma a]$ and, for the subset $a\Gamma a$ of M, we have $B(a\Gamma a) = (a\Gamma a\Gamma M\Gamma a\Gamma a]$. Since M is right regular and left regular, we have

$$\begin{aligned} (a\Gamma M\Gamma a] &\subseteq ((a\Gamma a\Gamma M]\Gamma M\Gamma(M\Gamma a\Gamma a]] = ((a\Gamma a\Gamma M]\Gamma(M]\Gamma(M\Gamma a\Gamma a]] \\ &\subseteq ((a\Gamma a\Gamma M)\Gamma M\Gamma(M\Gamma a\Gamma a)] \subseteq (a\Gamma a\Gamma M\Gamma a\Gamma a] \subseteq (a\Gamma M\Gamma a], \end{aligned}$$

so $(a\Gamma M\Gamma a] = (a\Gamma a\Gamma M\Gamma a\Gamma a]$. Thus we have

$$B(a) = (a\Gamma M\Gamma a] = (a\Gamma a\Gamma M\Gamma a\Gamma a] = B(a\Gamma a).$$

In addition,

$$\begin{aligned} B(a\Gamma a\Gamma M\Gamma a\Gamma a) &= ((a\Gamma a\Gamma M\Gamma a\Gamma a) \cup (a\Gamma a\Gamma M\Gamma a\Gamma a)\Gamma M\Gamma(a\Gamma a\Gamma M\Gamma a\Gamma a)] \\ &= (a\Gamma a\Gamma M\Gamma a\Gamma a] = B(a\Gamma a). \end{aligned}$$

Thus we obtain $B(a) = B(a\Gamma a) = B(a\Gamma a\Gamma M\Gamma a\Gamma a)$.
(3) $\Longrightarrow$ (1). Let $a \in M$. By hypothesis (the corrected form of (3)),

$$a \in B(a) = B(a\Gamma a) = (a\Gamma a \cup a\Gamma a\Gamma M\Gamma a\Gamma a] = (a\Gamma a] \cup (a\Gamma a\Gamma M\Gamma a\Gamma a].$$

If $a \le a\gamma a$ for some $\gamma \in \Gamma$, then

$$a \le (a\gamma a)\gamma(a\gamma a) \le a\gamma a\gamma a\gamma(a\gamma a) \in a\Gamma a\gamma M\gamma a\gamma a,$$

thus $a \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$ and, by Lemma 2.13 or (the corrected form of) Lemma 12 in [1], M is completely regular. If $a \in (a\Gamma a\Gamma M\Gamma a\Gamma a]$ then again by Lemma 2.13, M is completely regular. □

2.16 Theorem *A po-Γ-semigroup M is completely regular if and only if for each bi-ideal B of M, we have*

$$B = (B\Gamma B].$$

□

As we see below, the "⇒" part holds but the "⇐" does not seem to be true. One has to prove it for an ordered semigroup and put a Γ when necessary to get its analogous for a *po*-Γ-semigroup or to find a counterexample. For the ⇒-part the authors begin their proof as "Since B is a sub-Γ-semigroup of M", "Since B is an ideal of M", they had to clarify what they meant otherwise confusion is possible. The ⇐-part of the proof is wrong. $B(x)$ is not equal to $(x \cup x^2 \cup x\Gamma M\Gamma x]$. If they considered the bi-ideals subsemigroups, then this is in contrast with 2.3 Proposition according to which in some *po*-Γ-semigroups (regular in that proposition) the bi-ideals and the "subidempotent" by ideals are the same. They use the notation x^3, x^4, while they defined it (though not correctly) only for $x\gamma x\gamma x$, $x\gamma x\gamma x\gamma x$ and not for $x\Gamma x\Gamma x$, $x\Gamma x\Gamma x\Gamma x$, etc. They use 2.11 Lemma in [1], while its proof is not correct. They write $x^2 \in L$, $x^3 \in M\Gamma L$, $x^4 \in M\Gamma L$, while it is $x^2 \subseteq L$, $x^3 \subseteq M\Gamma L$, $x^4 \subseteq M\Gamma L$. On line –3 they write $(L \cup L\Gamma M] = (L]$ while L is a left (not right) ideal of M.
The following theorem holds:
Theorem. *Let M be an po-Γ- semigroup. If M is completely regular, then for each bi-ideal B of M, we have $B = (B\Gamma B]$. "Conversely", if M has the property $B = (B\Gamma B]$ for each bi-ideal B of M, then M is regular.*

Proof. $\Longrightarrow$. Let B be a bi-ideal of M. Then $B\Gamma M\Gamma B \subseteq B$. Since M is regular, we have $B \subseteq (B\Gamma M\Gamma B]$. Thus we have $B \subseteq (B\Gamma M\Gamma B] \subseteq (B] = B$, and $B = (B\Gamma M\Gamma B]$. Then we have

$$\begin{aligned} B\Gamma B = (B\Gamma M\Gamma B]\Gamma(B] &\subseteq ((B\Gamma M\Gamma B)\Gamma B] = (B\Gamma(M\Gamma B)\Gamma B] \\ &\subseteq (B\Gamma M\Gamma B] = B. \end{aligned}$$

and $(B\Gamma B] \subseteq (B] = B$. On the other hand, since M is completely regular, we have

$$\begin{aligned} B &\subseteq (B\Gamma B\Gamma M\Gamma B\Gamma B] \subseteq ((B\Gamma M\Gamma B)\Gamma B] \\ &= ((B\Gamma M\Gamma B]\Gamma B] = (B\Gamma B]. \end{aligned}$$

Therefore we have $B = (B\Gamma B]$.
$\Longleftarrow$. Let $a \in M$. By hypothesis, we have

$$\begin{aligned} a \in B(a) = (B(a)\Gamma B(a)] &= ((B(a)\Gamma B(a)]\Gamma B(a)] \\ &= ((B(a)\Gamma B(a))\Gamma B(a) \subseteq (B(a)\Gamma M\Gamma B(a)] \\ &= ((a \cup a\Gamma M\Gamma a]\Gamma M\Gamma(a \cup a\Gamma M\Gamma a]] \\ &= ((a \cup a\Gamma M\Gamma a]\Gamma(M]\Gamma(a \cup a\Gamma M\Gamma a]] \\ &= ((a \cup a\Gamma M\Gamma a)\Gamma M\Gamma(a \cup a\Gamma M\Gamma a)] \\ &= (a\Gamma M\Gamma a \cup a\Gamma M\Gamma a\Gamma M\Gamma a \cup a\Gamma M\Gamma a\Gamma M\Gamma a\Gamma M\Gamma a] \\ &= (a\Gamma M\Gamma a], \end{aligned}$$

so S is regular. □

2.17 Theorem (cf. [14; Theorem 2.1]) *Let M be a po-Γ-semigroup. The following are equivalent:*

(1) *M is completely regular.*
(2) *For every $a \in M$, $a \in (a\Gamma M\Gamma a] = (a\Gamma a\Gamma M\Gamma a\Gamma a]$.*
(3) *Every $\mathcal{B}$-class if M is a B-simple subsemigroup of M.*
(4) *Every $\mathcal{B}$-class if M is a subsemigroup of M.*
(5) *M is union of disjoint B-simple subsemigroups of M.*
(6) *M is union of B-simple subsemigroups of M.*
(7) *Every bi-ideal of M is semiprime.*
(6) *The set $\{(x_{\mathcal{B}}\} \mid x \in M\}$ coincides with the set of all maximal B-simple subsemigroups of M.* □

Concerning the 2.17 Theorem we remark the following: The proof of the implication (2) ⇒ (3) given in [1] is wrong. In an attempt to show that $(x)_{\mathcal{B}}$ is a subsemigroup of M ($x \in M$), the authors proved that for $a, b \in (x)_{\mathcal{B}}$, we have $B(x) = B(a\Gamma b)$ (cf. p. 801, l. 6), then they wrote "Moreover, $(x)_{\mathcal{B}}$ is a subsemigroup of M" obviously meaning "Thus $(x)_{\mathcal{B}}$ is a subsemigroup of M". It might be noted that while for ordered semigroups $B(x) = B(ab)$ implies $(x, ab) \in \mathcal{B}$ and so $ab \in (x)_{\mathcal{B}}$, this is not the case for ordered Γ-semigroups. So the proof that $(x)_{\mathcal{B}}$ is a subsemigroup of M given by the authors is not correct. Besides, to show that $(x)_{\mathcal{B}}$ is a subsemigroup of M one has to prove that for $a, b \in (x)_{\mathcal{B}}$ and $\gamma \in \Gamma$, we have $a\gamma b \in (x)_{\mathcal{B}}$. The authors begin the proof as follows: "Let $a, b \in (x)_{\mathcal{B}}$" (they do not consider any $\gamma \in \Gamma$ as they should). "By hypothesis, we have $a\Gamma b \subseteq ((a\Gamma b\Gamma a\Gamma b)\Gamma M\Gamma(a\Gamma b\Gamma a\Gamma b)]$". Here $a\Gamma b$ is a subset of M, while the hypothesis is: $a \in (a\Gamma M\Gamma a] = (a\Gamma a\Gamma M\Gamma a\Gamma a]$ for every $a \in M$. On p. 800, lines –6, –5, $((a\Gamma a\Gamma M\Gamma a\Gamma a]\Gamma M\Gamma y] \subseteq (a\Gamma a\Gamma M\Gamma a\Gamma a\Gamma M\Gamma y]$ should be replaced by $((a\Gamma a\Gamma M\Gamma a\Gamma a]\Gamma M\Gamma y] = (a\Gamma a\Gamma M\Gamma a\Gamma a\Gamma M\Gamma y]$, on line –2 on the same page the $((y\Gamma M\Gamma(b\Gamma b\Gamma M\Gamma b\Gamma b]] \subseteq (y\Gamma M\Gamma b\Gamma b\Gamma M\Gamma b\Gamma b]$ should be replaced by $(y\Gamma M\Gamma(b\Gamma b\Gamma M\Gamma b\Gamma b]] = (y\Gamma M\Gamma(b\Gamma b\Gamma M\Gamma b\Gamma b]$. In the proof that $(x)_{\mathcal{B}}$ is a B-simple subsemigroup of M, (p. 801, l. 12), the correct is: By (2), we have

$$\begin{aligned} y \in B(z) = (z \cup z\Gamma M\Gamma z] &\subseteq ((z\Gamma M\Gamma z] \cup z\Gamma M\Gamma z] \\ &= ((z\Gamma M\Gamma z]] = (z\Gamma M\Gamma z] \subseteq (B\Gamma M\Gamma B] \subseteq (B] = B. \end{aligned}$$

The implication (2) ⇒ (3) is used in the implication (1) ⇒ (8). On page 801, line –11, they write $\emptyset \neq B \cap S_\alpha$ (since $a\Gamma a \subseteq B$, $a\Gamma a \subseteq S_\alpha$), but the important point here is that $a\Gamma a$ is nonempty which should be added. The same on p. 802, l. 3, $\emptyset \neq (x\Gamma M\Gamma x] \cap T$ (since $x\Gamma x\Gamma x \subseteq x\Gamma M\Gamma x$, $x\Gamma x\Gamma x \subseteq T$) is written. On p. 801, l. –15 "$\forall\ a \in M$" should be replaced by "and $a \in M$". The last line of p. 801 should be deleted. On p. 802, l. 15 they should write $(B(x)\Gamma M\Gamma B(x)]$ instead of $B(x)\Gamma M\Gamma B(x)$, on l. 18 they should write $(B(y)\Gamma M\Gamma B(y)]$ instead of $B(y)\Gamma M\Gamma B(x)$. On p. 802, l. 18 $x \in B(y)$ implies $B(x) \subseteq B(y)$ and not $B(y) \subseteq B(x)$. On p. 802, l. –18, $T \subseteq \{(x)_{\mathcal{B}} \mid x \in M\}$ should be replaced by $T \in \{(x)_{\mathcal{B}} \mid x \in M\}$. We correct the proof given in lines –17 till –19 as follows: Let T be a maximal B-simple subsemigroup of M and $x \in T$. Then $T = (x)_{\mathcal{B}}$. In fact: In a similar way as above we prove that $T \subseteq (x)_{\mathcal{B}}$. By (1) ⇒ (3), $(x)_{\mathcal{B}}$ is a B-simple subsemigroup of M. Hence we have $T = (x)_{\mathcal{B}}$ and $T \in \{(x)_{\mathcal{B}} \mid x \in M\}$.

The proof of the implication (4) ⇒ (1) is not true. This is because although in ordered semigroups $x^5 \in (x)_{\mathcal{B}}$ (as in the paper by Zhu), in *po*-Γ-semigroups the set $x\Gamma x\Gamma x\Gamma x\Gamma x$ is a subset and not an element of $(x)_{\mathcal{B}}$ and so, according to the definition of $\mathcal{B}$ given by the authors $x\Gamma x\Gamma x\Gamma x \subseteq (x)_{\mathcal{B}}$ does not mean that $B(x) = B(x\Gamma x\Gamma x\Gamma x)$ as it is in ordered semigroups because x is an element and $x\Gamma x\Gamma x\Gamma x$ a subset of M.

Here is the correct proof of (4) ⇒ (1): Let $a \in M$. By hypothesis, $(a)_{\mathcal{B}}$ is a subsemigroup of M, so $(a)_{\mathcal{B}}\Gamma(a)_{\mathcal{B}} \subseteq (a)_{\mathcal{B}}$. Then $a^2 := a\Gamma a \subseteq (a)_{\mathcal{B}}\Gamma(a)_{\mathcal{B}} \subseteq (a)_{\mathcal{B}}$,

$a^3 := (a\Gamma a)\Gamma a \subseteq (a)_{\mathcal{B}}\Gamma(a)_{\mathcal{B}} \subseteq (a)_{\mathcal{B}}$,....., $a^5 := a\Gamma a\Gamma a\Gamma a\Gamma a \subseteq (a)_{\mathcal{B}}$. Take an element $\gamma \in \Gamma$ ($\Gamma \neq \emptyset$). Since $a\gamma a\gamma a\gamma a\gamma a \in a\Gamma a\Gamma a\Gamma a\Gamma a \subseteq (a)_{\mathcal{B}}$, we have $(a\gamma a\gamma a\gamma a\gamma a, a) \in \mathcal{B}$, and

$$a \in B(a) = B(a\gamma a\gamma a\gamma a\gamma a) \subseteq B(a\Gamma a\Gamma a\Gamma a\Gamma a) \subseteq B(a\Gamma a\Gamma a\Gamma a\Gamma a].$$

Since $(a\Gamma a\Gamma a\Gamma a\Gamma a]$ is a bi-ideal of M, we have $B(a\Gamma a\Gamma a\Gamma a\Gamma a] = (a\Gamma a\Gamma a\Gamma a\Gamma a]$. Then $a \in (a\Gamma a\Gamma a\Gamma a\Gamma a] \subseteq (a\Gamma a\Gamma M\Gamma a\Gamma a]$. By Lemma 2.13 or (the corrected form of) Lemma 2.12, M is completely regular.

The 2.18 Definition below is the definition given by the authors in [1] in an attempt to prove part of the Theorem in [9] in case of *po*-Γ-semigroups. But with the definition of strongly regular *po*-Γ-semigroups given by Hila-Pisha, the proof of the implication (1) ⇒ (2) in [1] is not correct. So, we introduce below the concept of strongly regular *po*-Γ-semigroup and we present a corrected proof of 2.19 Lemma.

2.18 Definition (cf. [9]) A *po*-Γ-semigroup M is called *strongly regular* if for every $a \in M$ there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that $a \leq a\gamma x\mu a$ and $a\gamma x = x\gamma a$ for all $\gamma \in \Gamma$.

This is the correct definition:

Definition. A *po*-Γ-semigroup M is called *strongly regular* if for every $a \in M$ there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that

$$a \leq a\gamma x\mu a \text{ and } a\gamma x = x\gamma a = x\mu a = a\mu x.$$

We remark that if M is a strongly regular *po*-Γ-semigroup, then it is left regular, right regular and regular. In fact: Let $a \in M$. Since M is strongly regular, there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that $a \leq a\gamma x\mu a$ and $a\gamma x = x\gamma a = x\mu a = a\mu x$. Since $a \leq (a\gamma x)\mu a = (x\gamma a)\mu a = x\gamma a\mu a$, M is left regular. Since $a \leq a\gamma(x\mu a) = a\gamma(a\mu x) = a\gamma a\mu x$, M is right regular. M is clearly regular as well, so M is completely regular.

2.19 Lemma (cf. the Theorem in [9])

Let M be a po-Γ-semigroup. The following are equivalent:

(1) *M is strongly regular.*
(2) *M is left regular, right regular, and $(M\Gamma a\Gamma M]$ is a strongly regular subsemigroup of M for every $a \in M$.*
(3) *For every $a \in M$, we have $a \in (M\Gamma a] \cap (a\Gamma M]$, and $(M\Gamma a\Gamma M]$ is a strongly regular subsemigroup of M.*

To prove this lemma, we need the following lemma:

Lemma. *Let M be a strongly regular po-Γ-semigroup. Then, for every $a \in M$, there exist $y \in M$ and $\gamma, \mu \in \Gamma$ such that*

$$a \leq a\gamma y\mu a,\ y \leq y\mu a\gamma y \text{ and } a\gamma y = y\gamma a = y\mu a = a\mu y.$$

Proof. Let $a \in M$. Since M is strongly regular, there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that $a \leq a\gamma x\mu a$ and $a\gamma x = x\gamma a = x\mu a = a\mu x$. Then we have

$$a \leq a\gamma x\mu a \leq (a\gamma x\mu a)\gamma x\mu a = a\gamma(x\mu a\gamma x)\mu a.$$

We put $y := x\mu a\gamma x$, and we have

$$a \leq a\gamma y\mu a$$

$$y = x\mu a\gamma x \leq x\mu(a\gamma x\mu a)\gamma x = (x\mu a\gamma x)\mu a\gamma x = y\mu a\gamma x \leq y\mu(a\gamma x\mu a)\gamma x$$
$$= y\mu a\gamma(x\mu a\gamma x) = y\mu a\gamma y,$$

so $y \leq y\mu a\gamma y$,

$$a\gamma y = a\gamma(x\mu a\gamma x) = (a\gamma x)\mu(a\gamma x) = (x\mu a)\mu(x\gamma a) = x\mu(a\mu x)\gamma a$$
$$= x\mu(a\gamma x)\gamma a = (x\mu a\gamma x)\gamma a = y\gamma a,$$

$$y\gamma a=(x\mu a\gamma x)\gamma a=x\mu a\gamma (x\gamma a)=x\mu a\gamma (x\mu a)=(x\mu a\gamma x)\mu a=y\mu a,$$

$$\begin{aligned}y\mu a&=(x\mu a\gamma x)\mu a=x\mu (a\gamma x)\mu a=x\mu (a\mu x)\mu a=(x\mu a)\mu (x\mu a)=(a\mu x)\mu (a\gamma x)\\&=a\mu (x\mu a\gamma x)=a\mu y.\end{aligned}$$

□

Proof of 2.19 Lemma
(1) $\Longrightarrow$ (2). Let $a\in M$. The set $(M\Gamma a\Gamma M]$ is a strongly regular subsemigroup of M. In fact: Let $b\in (M\Gamma a\Gamma M]$. Since $b\in M$ and M is strongly regular, by the Lemma, there exist $x\in M$ and $\gamma,\mu\in\Gamma$ such that $b\le b\gamma x\mu b$, $x\le x\mu b\gamma x$, and $b\gamma x=x\gamma b=x\mu b=b\mu x$. It is enough to prove that $x\in (M\Gamma a\Gamma M]$.

Since $b\in (M\Gamma a\Gamma M]$, there exist $z,t\in M$ and $\xi,\rho\in\Gamma$ such that $b\le z\xi a\rho t$. Then we have

$$x\le x\mu b\gamma x\le x\mu (z\xi a\rho t)\gamma x\in (M\Gamma M)\Gamma a\Gamma (M\Gamma M)\subseteq M\Gamma a\Gamma M,$$

so $x\in (M\Gamma a\Gamma M]$. Moreover, M is left regular and right regular, and (2) holds.
(2) $\Longrightarrow$ (3). Let $a\in M$. Since M is left regular, we have

$$a\in (M\Gamma a\Gamma a]\subseteq ((M\Gamma M)\Gamma a]\subseteq (M\Gamma a].$$

Since M is right regular, we have $a\in (a\Gamma a\Gamma M]\subseteq (a\Gamma (M\Gamma M)]\subseteq (a\Gamma M]$, thus we get $a\in (M\Gamma a]\cap (a\Gamma M]$, and condition (3) is satisfied.
(3) $\Longrightarrow$ (1). Let $a\in M$. Since $a\in (M\Gamma a]\cap (a\Gamma M]$, there exist $z,t\in M$ and $\xi,\rho\in\Gamma$ such that $a\le z\xi a$ and $a\le a\rho t$. Then we have

$$a\le a\rho t\le (z\xi a)\rho t=z\xi (a\rho t)\in M\Gamma a\Gamma M,$$

then $a\in (M\Gamma a\Gamma M]$. Since $(M\Gamma a\Gamma M]$ is strongly regular, there exists $x\in (M\Gamma a\Gamma M](\subseteq M)$ and $\gamma,\mu\in\Gamma$ such that $a\le a\gamma x\mu a$ and $a\gamma x=x\gamma a=x\mu a=a\mu x$, thus M is strongly regular.

According to the paper in [1], by 2.16 Theorem and 2.19 Lemma, we have the following:
2.20 Theorem *A po-Γ-semigroup M is strongly regular if and only if the following conditions hold true:*

(1) *For every bi-ideal B of M, we have $B=(B\Gamma B]$.*

(2) *$(M\Gamma a\Gamma M]$ is a strongly regular subsemigroup of M for every $a\in M$.*

By the "$\Rightarrow$" part of 2.16 Theorem and 2.19 Lemma, if a *po*-Γ-semigroup M is strongly regular, then for every bi-ideal B of M, we have $B=(B\Gamma B]$ and $(M\Gamma a\Gamma M]$ is a strongly regular subsemigroup of M for every $a\in M$, so conditions (1) and (2) are satisfied. But, as the proof of the "$\Leftarrow$" part of 2.16 Theorem is wrong, by 2.16 Theorem and 2.19 Lemma we cannot conclude that conditions (1) and (2) imply that M is strongly regular.
Note. It is better not to write, for example $a\alpha a$, $a\alpha b$, $a\alpha x\beta a$, $a\alpha a\alpha a$, $a\alpha a\alpha a\alpha a\beta b$ and use another letter of the Greek alphabet for the element α of Γ. We can write, for example, $a\lambda a$, $a\delta b$, $a\gamma x\beta a$, $a\xi a\zeta a$, $a\omega a\sigma a\mu a\beta b$ to be able for someone to read the paper. Also to avoid symbols like x^2, x^3, and write $x\gamma x$ or $x\Gamma x$, $x\gamma x\gamma x$ or $x\Gamma x\Gamma x$ instead. When we have expressions like $y\Gamma y\Gamma y\Gamma M\Gamma y\Gamma y\Gamma M\Gamma y\Gamma y$ (see p. 801), it is not harm to write $x\Gamma x\Gamma x$ instead of x^3.
Conclusion. If we want to get a result on a *po*-Γ-semigroup, we first solve it for a *po*-semigroup and then we have to be careful to define the analogous concepts in case of the *po*-Γ-semigroup (if they do not defined directly) and put the "Γ" in the appropriate place. We never solve the problem directly in *po*-Γ-semigroups.

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