Hacettepe Journal of Mathematics and Statistics
Volume 42 (4) (2013), 447 – 453

ON WEAK SYMMETRIES OF ALMOST KENMOTSU (κ, μ, ν)-SPACES

Nesip Aktan * Satılmış Balkan † Mustafa Yıldırım ‡

Received 04 : 07 : 2012 : Accepted 15 : 04 : 2013

Abstract

In this paper, we study on weak symmetries of almost Kenmotsu (κ, μ, ν)-spaces. For each α, γ, δ 1-forms and any vector field X we get $\alpha + \gamma + \delta = \dfrac{X(\kappa)}{\kappa}$, if a $(2n+1)$-dimensional almost Kenmotsu (κ, μ, ν)-space is weakly symmetric and for each ε, σ, ρ 1-forms we get $\varepsilon + \sigma + \rho = \dfrac{X(\kappa)}{\kappa}$, if a $(2n+1)$-dimensional almost Kenmotsu (κ, μ, ν)-space is weakly Ricci symmetric.

Keywords: Almost Kenmotsu Manifolds, Weak Symmetric Manifolds, Almost Kenmotsu (κ, μ, ν)-Spaces.

2000 AMS Classification: 53D10, 53C15, 53C25, 53C35.

1. INTRODUCTION

Weakly symmetric Riemannian manifolds are generalizations of locally symmetric manifolds and pseudo-symmetric manifolds. These are manifolds in which the covariant derivative DR of the curvature tensor R is a linear expression in R. The appearing coefficients of this expression are called associated 1-forms. They satisfy in the specified types of manifolds gradually weaker conditions.

Firstly, the notions of weakly symmetric and weakly Ricci-symmetric manifolds were introduced by L. Tamassy and T. Q. Binh in 1992 ([10] and [11]). In [10], the authors considered weakly symmetric and weakly projective-symmetric Riemannian manifolds. In 1993, the authors considered weakly symmetric and weakly Ricci-symmetric Einstein and Sasakian manifolds [11]. In 2000, U. C. De, et. all gave necessary conditions for

*Duzce University, Faculty of Art and Sciences, Department of Mathematics, Duzce/TURKEY, Email: nesipaktan@duzce.edu.tr
†Duzce University, Faculty of Art and Sciences, Department of Mathematics, Duzce/TURKEY, Email:satilmisbalkan@duzce.edu.tr
‡Duzce University, Faculty of Art and Sciences, Department of Mathematics, Duzce/TURKEY, Email: mustafay@duzce.edu.tr

the compatibility of several K-contact structures with weak symmetry and weakly Ricci-symmetry [13]. In 2002, C. Özgür, considered weakly symmetric and weakly Ricci-symmetric Lorentzian para-Sasakian manifolds [14]. Recently in [8], C. Özgür studied weakly symmetric Kenmotsu manifolds and in [15] Aktan and Görgülü studied on weak symmetries of almost r-paracontact Riemannian manifold of P-Sasakian type.

Manifolds known as Kenmotsu manifolds have been introduced and studied by K. Kenmotsu in 1972 [16]. Kenmotsu manifolds were studied by many authors such as [1]-[9] and many others. T. Koufogiorgos, et. all, introduced in [17] the notion of (κ,μ,ν)-contact metric manifold, where now the equation to be satisfied is

$$R(X,Y)\xi=\eta(Y)(\kappa I+\mu h+\nu\varphi h)X-\eta(X)(\kappa I+\mu h+\nu\varphi h)Y, \tag{1.1}$$

for some smooth functions κ,μ and ν on M. Lastly, H. Öztürk, N. Aktan and C. Murathan studied in [18] the almost α-cosymplectic (κ,μ,ν)-spaces under different conditions (like η-parallelism) and gave an interesting example in dimension 3. Some of their results are used in this paper.

These almost Kenmotsu manifolds whose almost Kenmotsu structures (φ,ξ,η,g) satisfy the condition

$$\begin{aligned} R(\xi,X)Y= {} & \kappa(g(Y,X)\xi-\eta(X)Y)+\mu(g(hY,X)\xi-\eta(Y)hX) \\ & +\nu(g(\varphi hY,X)\xi-\eta(Y)\varphi hX), \end{aligned} \tag{1.2}$$

for $\kappa,\mu,\nu\in\Re_n\left(M^{2n+1}\right)$, where $\Re_n\left(M^{2n+1}\right)$ be the subring of the ring of smooth functions f on M^{2n+1} for which $df\wedge\eta=0$.

A non-flat differentiable manifold M^{2n+1} is called weakly symmetric if there exist a vector field P and 1-forms $\alpha,\beta,\gamma,\delta$, on M such that

$$\begin{aligned} (\nabla_X R)(Y,Z,W)= {} & \alpha(X)R(Y,Z)W+\beta(Y)R(X,Z)W \\ & +\gamma(Z)R(Y,X)W+\delta(W)R(Y,Z)X+g(R(Y,Z)W,X)P, \end{aligned} \tag{1.3}$$

holds for all vector fields $X,Y,Z,W\in\chi\left(M^{2n+1}\right)$ ([10] and [11]). A weakly symmetric manifold $\left(M^{2n+1},g\right)$ is pseudo-symmetric if $\beta=\gamma=\delta=\frac{1}{2}\alpha$ and $P=A$, locally symmetric if $\alpha=\beta=\gamma=\delta=0$ and $P=0$. A weakly symmetric is said to be proper if at least one of the 1-forms $\alpha,\beta,\gamma,\delta$ is not zero or $P\neq 0$.

A differentiable manifold M^{2n+1} is called weakly Ricci-symmetric if there exists 1-forms ε,σ,ρ such that the condition

$$(\nabla_X S)(Y,Z)=\varepsilon(X)S(Y,Z)+\sigma(Y)S(X,Z)+\rho(Z)S(X,Y), \tag{1.4}$$

holds for all vector fields $X,Y,Z,W\in\chi\left(M^{2n+1}\right)$ ([10] and [11]). If $\varepsilon=\sigma=\rho$ then M^{2n+1} is called pseudo Ricci-symmetric [12].

From (1.4), an easy calculation shows that if M^{2n+1} is weakly symmetric then we have

$$\begin{aligned} (\nabla_X S)(Z,W)= {} & \alpha(X)S(Z,W)+\beta(R(X,Z)W) \\ & +\gamma(Z)S(X,W)+\delta(W)S(X,Z)+\rho(R(X,W)Z), \end{aligned} \tag{1.5}$$

where P is defined by $\rho(X)=g(X,P)$ for all $X\in\chi\left(M^{2n+1}\right)$ [11].

In this paper, we consider weakly symmetries and weakly Ricci symmetries for almost Kenmotsu (κ,μ,ν)-spaces.

2. ALMOST KENMOTSU (κ,μ,ν)-SPACES

Let $\left(M^{2n+1},\varphi,\xi,\eta,g\right)$ be a $(2n+1)$-dimensional almost contact Riemannian manifold, where φ is a $(1,1)$-tensor field, ξ is the structure vector field, η is a 1-form and g is Riemannian metric. It is well-known that φ,ξ,η,g satisfy

$$\eta(\xi)=1,\quad \varphi\xi=0,\quad \eta\circ\varphi=0, \tag{2.1}$$

$$\varphi^2X=-X+\eta(X)\,\xi,\quad \eta(X)=g(X,\xi), \tag{2.2}$$

$$g(\varphi X,\varphi Y)=g(X,Y)-\eta(X)\,\eta(Y), \tag{2.3}$$

for any vector fields X,Y on M^{2n+1}.

The 2-form Φ of M^{2n+1} defined by $\Phi(X,Y)=g(\phi X,Y)$, is called the fundamental 2-form of the almost contact metric manifold M^{2n+1}. Almost contact metric manifolds such that $d\eta=0$ and $d\Phi=2\eta\wedge\Phi$ are almost Kenmotsu manifolds. Finally, a normal almost Kenmotsu manifold is called Kenmotsu manifold. An almost Kenmotsu manifold is a nice example of an almost contact manifold which is not K-contact (and hence not a Sasakian) manifold.

Now let us recall some important curvature-properties of almost Kenmotsu manifolds satisfy (1.1) and (1.2) and the following properties:

$$(\nabla_{X\varphi})\,Y=g(\varphi X+hX,Y)\,\xi-\eta(Y)\,(\varphi X+hX), \tag{2.4}$$

$$\nabla_X\xi=-\varphi^2X-\varphi hX, \tag{2.5}$$

$$S(X,\xi)=2n\kappa\eta(X), \tag{2.6}$$

$$Q\xi=2n\kappa\xi, \tag{2.7}$$

$$l=-\kappa\varphi^2+\mu h+\nu\varphi h, \tag{2.8}$$

$$l\varphi-\varphi l=2\mu h\varphi+2\nu h, \tag{2.9}$$

$$h^2=\left(\kappa^2+1\right)\varphi^2,\text{for }\kappa\leq -1, \tag{2.10}$$

$$(\nabla_\xi h)=-\mu\varphi h+(v-2)\,h. \tag{2.11}$$

Here, l and Jacobi operator are defined by $l(X)=R(X,\xi)\,\xi$ and $h=\dfrac{1}{2}L_\xi\varphi$, where L is Lie derivative operator.

2.1. Theorem. *On almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, the functions κ, μ, ν only vary in the direction of ξ, i.e.*

$$X(\kappa)=X(\mu)=X(\nu)=0$$

for every vector field X orthogonal to ξ [18].

3. MAIN RESULTS

3.1. Theorem. *Let M be an almost Kenmotsu (κ,μ,ν)-space. If M is weakly symmetric then* $\alpha+\gamma+\delta=\dfrac{X(\kappa)}{\kappa}$.

Proof. Assume that M^{2n+1} is a weakly symmetric almost Kenmotsu (κ,μ,ν)-space. Putting $W=\xi$ in (1.5) we get

$$(3.1)\qquad \begin{aligned}(\nabla_X S)(Z,\xi)= {}& \alpha(X)\,S(Z,\xi)+\beta(R(X,Z)\xi)\\ &+\gamma(Z)\,S(X,\xi)+\delta(\xi)\,S(X,Z)+\rho(R(X,\xi)Z),\end{aligned}$$

So, using (1.1) (1.2) and (2.6) we have

$$(3.2)\qquad \begin{aligned}&(\nabla_X S)(Z,\xi)=\\ &2n\kappa\alpha(X)+\beta(\kappa)\eta(Z)X+\kappa\eta(Z)\beta(X)\\ &+\beta(\mu)\eta(Z)hX+\mu\eta(Z)\beta(hX)+\beta(\nu)\eta(Z)\varphi hX\\ &+\nu\eta(Z)\beta(\varphi hX)-\beta(\kappa)\eta(X)Z-\kappa\eta(X)\beta(Z)\\ &-\beta(\mu)\eta(X)hZ-\mu\eta(X)\beta(hZ)-\beta(\nu)\eta(X)\varphi hZ\\ &-\nu\eta(X)\beta(\varphi hZ)+2n\kappa\gamma(Z)\eta(X)+\delta(\xi)S(X,Z)\\ &-\rho(\kappa)(g(X,Z)\xi-\eta(Z)X)-\kappa(g(X,Z)\rho(\xi)-\eta(Z)\rho(X))\\ &-\rho(\mu)(g(hZ,X)\xi-\eta(Z)hX)-\mu(g(hZ,X)\rho(\xi)-\eta(Z)\rho(hX))\\ &-\rho(\nu)(g(\varphi hZ,X)\xi-\eta(Z)\varphi hX)\\ &-\nu(g(\varphi hZ,X)\rho(\xi)-\eta(Z)\rho(\varphi hX)).\end{aligned}$$

By the covariant differentiation of the Ricci tensor S, the left side can be written as

$$(\nabla_X S)(Z,\xi)=\nabla_X S(Z,\xi)-S(\nabla_X Z,\xi)-S(Z,\nabla_X\xi).$$

By the use of (2.5), (2.6)and the parallelity of the metric tensor g we have

$$(3.3)\qquad (\nabla_X S)(Z,\xi)=2nX(\kappa)\eta(Z)+2n\kappa g(Z,\nabla_X\xi)-S(Z,\nabla_X\xi).$$

Comparing the right hand sides of (3.2) and (3.3), we obtain

$$(3.4)\qquad \begin{aligned}&2nX(\kappa)\eta(Z)+2n\kappa g(Z,\nabla_X\xi)-S(Z,\nabla_X\xi)=\\ &2n\kappa\alpha(X)+\beta(\kappa)\eta(Z)X+\kappa\eta(Z)\beta(X)\\ &+\beta(\mu)\eta(Z)hX+\mu\eta(Z)\beta(hX)\\ &+\beta(\nu)\eta(Z)\varphi hX+\nu\eta(Z)\beta(\varphi hX)\\ &-\beta(\kappa)\eta(X)Z-\kappa\eta(X)\beta(Z)\\ &-\beta(\mu)\eta(X)hZ-\mu\eta(X)\beta(hZ)-\beta(\nu)\eta(X)\varphi hZ\\ &-\nu\eta(X)\beta(\varphi hZ)+2n\kappa\gamma(Z)\eta(X)\\ &+\delta(\xi)S(X,Z)-\rho(\kappa)(g(X,Z)\xi-\eta(Z)X)\\ &-\kappa(g(X,Z)\rho(\xi)-\eta(Z)\rho(X))\\ &-\rho(\mu)(g(hZ,X)\xi-\eta(Z)hX)\\ &-\mu(g(hZ,X)\rho(\xi)-\eta(Z)\rho(hX))\\ &-\rho(\nu)(g(\varphi hZ,X)\xi-\eta(Z)\varphi hX)\\ &-\nu(g(\varphi hZ,X)\rho(\xi)-\eta(Z)\rho(\varphi hX)).\end{aligned}$$

Putting $X=Z=\xi$ in (3.4) and using (2.1), (2.2), (2.3) and (2.6) we get

$$2n\kappa[\alpha(\xi)+\gamma(\xi)+\delta(\xi)]=2n\xi(\kappa).$$

Since $n>1$ and $\kappa\neq 0$, we obtain

$$(3.5)\qquad \alpha(\xi)+\gamma(\xi)+\delta(\xi)=\frac{\xi(\kappa)}{\kappa}.$$

So, vanishing of the 1-form $\alpha+\gamma+\delta$ over the vector field ξ necessary in order that M^{2n+1} be an almost Kenmotsu (κ,μ,ν)-space. Now we will show that

$$\alpha(X)+\gamma(X)+\delta(X)=\frac{X(\kappa)}{\kappa}$$

holds for all vector fields on M^{2n+1}.

In (1.5) $Z=\xi$, similar to the previous calculations it follows that

$$\begin{aligned}
&2nX(\kappa)\eta(W)+2n\kappa g(W,\nabla_X\xi)-S(\nabla_X\xi,W)=\\
&2n\kappa\alpha(X)\eta(W)+\beta(\kappa)(g(X,W)\xi-\eta(W)X)\\
&-\kappa(g(X,W)\beta(\xi)-\eta(W)\beta(X))\\
&-\beta(\mu)(g(hW,X)\xi-\eta(W)hX)\\
&-\mu(g(hW,X)\beta(\xi)-\eta(W)\beta(hX))\\
&-\beta(\nu)(g(\varphi hW,X)\xi-\eta(W)\varphi hX)\\
&-\nu(g(\varphi hW,X)\beta(\xi)-\eta(W)\beta(\varphi hX))+\gamma(\xi)S(X,W)\\
&+2n\kappa\delta(W)\eta(X)+\rho(\kappa)\eta(W)X\\
&+\kappa\eta(W)\rho(X)+\rho(\mu)\eta(W)hX\\
&+\mu\eta(W)\rho(hX)+\rho(\nu)\eta(W)\varphi hX\\
&+\nu\eta(W)\rho(\varphi hX)-\rho(\kappa)\eta(X)W\\
&-\kappa\eta(X)\rho(W)-\rho(\mu)\eta(X)hW\\
&-\mu\eta(X)\rho(hW)-\rho(\nu)\eta(X)\varphi hW-\nu\eta(X)\rho(\varphi hW).
\end{aligned}\tag{3.6}$$

Replacing W with ξ in (3.6) and by making use of (1.1) 2.1), (2.2), (2.3) and (2.6) we have

$$\begin{aligned}
2nX(\kappa)=\ &2n\kappa\alpha(X)-\beta(\kappa)(\eta(X)\xi-X)\\
&-\kappa(\eta(X)\beta(\xi)-\beta(X))\\
&+\beta(\mu)hX+\mu\beta(hX)\\
&+\beta(\nu)\varphi hX+\nu\beta(\varphi hX)+2n\kappa\gamma(\xi)\eta(X)\\
&+2n\kappa\delta(\xi)\eta(X)+\rho(\kappa)\nu+\kappa\rho(X)\\
&+\rho(\mu)hX+\mu\rho(hX)+\rho(\nu)\varphi hX\\
&+\nu\rho(\varphi hX)-\rho(\kappa)\eta(X)\xi-\kappa\eta(X)\rho(\xi).
\end{aligned}\tag{3.7}$$

Taking $X=\xi$ in (3.6) and replacing W with X we get

$$\begin{aligned}
2n\xi(\kappa)\eta(X)=\ &2n\kappa\alpha(\xi)\eta(X)+2n\kappa\gamma(\xi)\eta(X)+2n\kappa\delta(X)\\
&+\rho(\kappa)\eta(X)\xi+\kappa\rho(\xi)\eta(X)-\rho(\kappa)X-\kappa\rho(X)\\
&-\rho(\mu)hX-\mu\rho(hX)-\rho(\nu)\varphi hX-\nu\rho(\varphi hX).
\end{aligned}\tag{3.8}$$

Now putting $X=\xi$ in (3.4) and replacing Z with X we have

$$\begin{aligned}
2n\xi(\kappa)\eta(X)=\ &2n\kappa\alpha(\xi)\eta(X)+\beta(\kappa)\eta(X)\xi+\kappa\eta(X)\beta(\xi)\\
&-\beta(\kappa)X-\kappa\beta(X)-\beta(\mu)hX-\mu\beta(hX)-\beta(\nu)\varphi hX\\
&-\nu\beta(\varphi hX)+2n\kappa\gamma(X)+2n\kappa\eta(X)\delta(\xi)
\end{aligned}\tag{3.9}$$

Taking (3.7), (3.8) and (3.9) we get

$$2nX(\kappa)=2n\kappa\left[\alpha(X)+\gamma(X)+\delta(X)\right],$$

for all X. This implies

$$\alpha(X)+\gamma(X)+\delta(X)=\frac{X(\kappa)}{\kappa},$$

which completes the proof of the theorem. □

3.2. Theorem. *Let M be an almost Kenmotsu (κ,μ,ν)-space. If M is weakly Ricci symmetric then $\varepsilon+\sigma+\rho=\dfrac{X(\kappa)}{\kappa}$.*

Proof. Assume that M^{2n+1} is a weakly Ricci symmetric almost Kenmotsu (κ,μ,ν)-space. Putting $Z=\xi$ in (1.4) and using (2.6) we have

$$(\nabla_X S)(Y,\xi)=2n\kappa\varepsilon(X)\eta(Y)+2n\kappa\sigma(Y)\eta(X)+\rho(\xi)S(X,Y). \tag{3.10}$$

Replacing Z with Y in (3.3) and comparing the right hand sides of the equations (3.10) and (3.3) we obtain

$$\begin{aligned}2nX(\kappa)\eta(Y)+2n\kappa g(Y,\nabla_X\xi)-S(Y,\nabla_X\xi)= \quad & 2n\kappa\varepsilon(X)\eta(Y)+2n\kappa\sigma(Y)\eta(X)\\ & +\rho(\xi)S(X,Y).\end{aligned} \tag{3.11}$$

Taking $X=Y=\xi$ in (3.11) by making use of (2.1), (2.2), (2.3) and (2.6) we get

$$2n\xi(\kappa)=2n\kappa\left[\varepsilon(\xi)+\sigma(\xi)+\rho(\xi)\right],$$

which gives, (since $n>1$ and $\kappa\neq 0$),

$$\varepsilon(\xi)+\sigma(\xi)+\rho(\xi)=\frac{\xi(\kappa)}{\kappa}. \tag{3.12}$$

Putting $X=\xi$ in (3.11) we have

$$2n\xi(\kappa)\eta(Y)=2n\kappa\eta(Y)\left[\varepsilon(\xi)+\rho(\xi)\right]+2n\kappa\sigma(Y).$$

So by virtue of (3.12) this yields

$$\sigma(Y)=\sigma(\xi)\eta(Y).$$

Replacing Y with X we get

$$\sigma(X)=\sigma(\xi)\eta(X). \tag{3.13}$$

Similarly taking $Y=\xi$ in (3.11) we also have

$$\varepsilon(X)=\varepsilon(\xi)\eta(X). \tag{3.14}$$

Since $(\nabla_\xi S)(\xi,X)=0$,we obtain

$$\rho(X)=\rho(\xi)\eta(X). \tag{3.15}$$

Therefore the summation of the equations (3.13), (3.14) and (3.15) give us

$$\varepsilon(X)+\sigma(X)+\rho(X)=\frac{X(\kappa)}{\kappa}$$

for all X. Our theorem is proved. □

By virtue of the above theorems we have the following corollaries:

3.3. Corollary. *Let M be an almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, and the functions κ, μ ν only vary in the direction of ξ. There exists no weakly symmetric almost Kenmotsu (κ,μ,ν)-space M^{2n+1}, $(\kappa\leq -1)$, if $\alpha+\gamma+\delta$ is not everywhere zero.*

3.4. Corollary. *Let M be an almost Kenmotsu (κ,μ,ν)-space of dimension greater than or equal to 5, and the functions κ, μ ν only vary in the direction of ξ. There exists no weakly Ricci-symmetric almost Kenmotsu (κ,μ,ν)-space M^{2n+1}, $(\kappa\leq -1)$, if $\varepsilon+\sigma+\rho$ is not everywhere zero.*

References

[1] Pitis, G. *A remark on Kenmotsu manifolds.* Buletinul Universitatii Din Brasov. **30**, 1988.

[2] De, U.C. and Pathak,G. *On 3-dimensional Kenmotsu manifolds.* Ind. J. Pure Applied Math. **35** 159-165, 2004.

[3] Jun, J. B., De, U. C. and Pathak, G. *On Kenmotsu manifolds*, J.Korean Math.Soc. **42,** 435-445, 2005.

[4] Binh T.Q., Tamassy, L., De, U. C. and Tarafder, M. *Some Remarks on almost Kenmotsu manifolds*, Math Pannon. **13**, 31-39, 2002.

[5] Bagewadi, C.S. and Venkatesha *Some curvature tensor on a Kenmotsu manfold*, Tensor, **68**, 140-147,2007.

[6] Bagewadi, C.S. and Venkatesha *On pseudo projective ϕ-recurrent Kenmotsu manifolds*, Soochow J. of Mathematics **32**, 1-7, 2006.

[7] Bagewadi, C.S. and Venkatesha *On projective ϕ-recurrent Kenmotsu manifolds*, MSJ,Bangalore **4**, 15-21, 2005.

[8] Özgür, C *On weakly symmetric Kenmotsu manifolds*, Di er. Geom. Dyn. Syt. **8**, 204-209, 2006.

[9] Özgür, C. *On generalized recurrent Kenmotsu manifolds*, World Applied Sciences Journal **2**, 29-33, 2007.

[10] Tamassy L. and Binh T. Q. *On weakly symmetric and weakly projective symmetric Riemannian manifolds*, Coll. Math. Soc. J. Bolyai, **56**, 663-670, 1992.

[11] Tamassy L. and Binh T. Q. *On weak symmetries of Einstein and Sasakian manifolds*, Tensor N. S., **53**, 140-148, 1993.

[12] Chaki M.C. *On pseudo Ricci-symmetric manifolds*, Bulgar J. Phys., **15**, 526-531, 1988.

[13] De U. C., Binh T.Q. and Shaikh A.A. *On weakly symmetric and weakly Ricci-symmetric K-contact manifolds*, Acta Mathematica Acadeiae Paedagogicae Nyiregyhaziensis, **16**, 65-71, 2000.

[14] Özgür C. *On weak symmetries of Lorentzian para Sasakian manifolds*, Radovi Matamaticki **11**, 263-270, 2002.

[15] Aktan N. and Görgülü A. *On weak symmetries of almost r-paracontact Riemannian manifold of P-Sasakian type*, Differential Geometry - Dynamical Systems, **9**, 1-8, 2007.

[16] Kenmotsu K. *A class of almost contact Riemannian manifolds*, Tôhoku Math. J. **24** 93–103, 1972.

[17] Koufogiorgos T., Markellos M. and Papantoniou V. J. T*he harmonicity of the Reeb vector fields on contact metric* 3*-manifolds*, Pacific J. Math. **234** (2), 325–344, 2008.

[18] Öztürk H., Aktan N. and Murathan C. *Almost α-cosymplectic (κ, μ, ν)-spaces,* arXiv:1007.0527v1.