

DYNAMICS OF PERFORMANCE AND TECHNOLOGY IN HIGHER EDUCATION: AN APPLIED STOCHASTIC MODEL AND A CASE STUDY

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Abstract

The purpose of this paper is to develop a stochastic-dynamic model of performance and technology in education sector and bring into light the presence, in a particular subset of the Turkish higher education sector, of stochastically-evolving equilibria moving towards a low performance trap over time. The dynamics of the movement in question hinges, in part, on two factors, namely, (1) the productivity growth and (2) student population growth. We formulate a stochastically-driven, technology-based policy option that could help the sector to escape the trap, moving the sector towards high performance equilibria. The proposed policy option illustrates that technological transformation in educational practices could solve a structural problem (a low performance trap) in developing-country education sectors.

Keywords: Education, universities, low performance trap, economic dimensions, technology, transformation, stochastic-dynamic models.

2000 AMS Classification: 37N40

1. Introduction

Education is unarguably one of the most important forms of investment shaping the modern economies in the twenty-first century. Skills, knowledge and capabilities (i.e., various dimensions of human capital) acquired or developed through education have been among the key determinants of the micro performance of economic actors, institutions and sectors as well as the macro performance of contemporary economies. Studies

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demonstrating the positive effect of human capital on economic growth have been sufficiently indicative of the powerful influence of education on the economies in question [1].[†]

Studies on education are not, however, limited to the education (or human capital)-induced economic growth. The issues covered by a wide spectrum of works in the literature range from efficient provision of educational services to quality management practices in higher education. Some of these works, such as [2] and [3] examine issues that center around funding, equity and efficiency of higher education. Some, such as [8], [12] and [10], analyze the possibility of optimal or strategic educational subsidies. There are other works studying the quality assurance programs and their effects on the demand for education [16]. Topics covered by the works in the literature are indeed rich and multidimensional, and yet many dimensions (issues) of this fascinatingly complex area still remain under-explored. Among these issues is the stochastic economic dynamics in education (university) sector which we will examine in this paper. We will present an applied stochastic model of the dynamics in question and demonstrate that a subset of the higher education sector in Turkey has been trapped into low performance-stochastic-equilibria. We will propose an information technology-driven stochastic policy rule, which helps the sector to get out of such equilibria and reach a stable high performance-target.

In the second section of the paper, we develop the model. The third section *presents* the empirical results. The policy implications are articulated in the fourth section. The concluding remarks follow in the fifth section.

2. The Model [‡]

Consider an education sector where suppliers (such as universities) provide a service, say x , to the customers. [§] For the sake of simplicity we will analyze the case of a typical supplier in the market. Let D_t denote the *quantity demanded* for service x supplied by the firm, which indicates the degree to which customers are willing to buy the service at time t . [¶] D_t depends on the relative price of the service at time t (R_t), customers' income at time t (M_t), the service performance at time t (P_t), and the degree to which information technology is used in educational services at time t (T_t).

$$i.e., D_t = f^D(R_t, M_t, P_t, T_t),$$

which is a demand function for the educational service. $R_t \in (0, \infty)$, $M_t \in (0, \infty)$. By virtue of the particular way of measuring performance and technology utilization, explained in Section III, P_t and T_t take on values between 1 and 7, i.e., $P_t \in [1, 7]$, and $T_t \in [1, 7]$. $D_t \in (0, \infty)$. Among the variables in the equation, relative price and income are standard variables that appear in the conventional specification of demand functions. Depending on the empirical case, however, one or both of these variables may turn out to be statistically insignificant, rendering their presence in the equation "unnecessary/dispensable". In the empirical case examined (in the next section) in this paper, relative price turns out to be such a dispensable variable and is left out of the

[†]For an example of "growth-centered" works on human capital, with a different focus, see Tatoglu [18]. There are also works studying the effects of human capital on microeconomic processes. An example of such works is Danchev [6], which examines the influence of human capital/social capital on the sustainable behavior of the firm.

[‡]The model and policy implications benefit from some of the structures presented in Kara [12].

[§]Educational firms could provide multiple services, in which case x could be conceived as "a composite service" representing these services.

[¶]This is a "degree-based" concept of quantity demanded. The concept of quantity supplied defined below is degree-based as well.

equation. The other two variables in the equation, namely the use of technology and performance, are of more specific nature and have contextual relevance to the issue under consideration. The use of technology, which is studied in a number of works in the literature such as Ellis [9] and Goffe and Sosin [11] is key to modern educational practices and could reasonably affect the demand for educational services. For instance, teaching accounting software for practical purposes in business and economics departments could increase the job performance of the graduates. Same holds for computer-based skills involving statistical, optimization and simulation software. Increased job performance of graduates increases the demand for graduates in question and hence the demand for higher education. Similarly, service performance - which is rated on the basis of such tangible factors as the number of students taught/reached (quantitative teaching performance), the number of articles published (quantitative research performance), number of social activities organized by the institution, logistic services (such as dormitory) etc., and such intangible factors as reliable and effective teaching, responsiveness of personnel to students' needs etc. - is a potentially relevant variable for the study of demand (as well as supply, as indicated below) of educational services (Kara [13]).

Let S_t denote the *quantity supplied* for the service, which indicates the degree to which the supplier is willing to supply the service at time t . Suppose that S_t depends on the relative price of the service (R_t) as well as on the present and past performances (P_t , P_{t-1}), i.e., the supply function for the educational service is:

$$S_t = f^S(R_t, P_t, P_{t-1}). \parallel$$

$R_t \in (0, \infty)$, $P_t \in [1, 7]$, $P_{t-1} \in [1, 7]$, and $S_t \in (0, \infty)$.

For analytical purposes, we will assume that the demand and supply functions have the following explicit (log-linear) forms: **

$$\ln D_t = \alpha_0 + \alpha_1 \ln P_t + \alpha_2 \ln M_t + \alpha_3 \ln R_t + \alpha_4 \ln T_t + u_t$$

and

$$\ln S_t = \beta_0 + \beta_1 \ln P_t + \beta_2 \ln P_{t-1} + \beta_3 \ln R_t + v_t$$

where u_t and v_t are independent normally distributed white noise stochastic terms uncorrelated over time. They have zero means and constant variances σ_u^2 and σ_v^2 respectively.

Here a specific feature of the service supply of the higher education institutions in Turkey needs to be noted: Even at low performance levels, many of these institutions do end up supplying services. The level of these services at time t depends on the level of these services at $t = 0$, and the growth rate of these services reflecting roughly the growth of student population in the system. Let, in the absence of stochastic shocks, and at the minimal performance levels, S_t have the value of A , which grows at a rate of g over time. Thus, at $P_t = 1$ and $P_{t-1} = 1$, $S_t = A(1+g)^t \Rightarrow \ln S_t = t \ln A(1+g) = \beta_0$ (By the argument presented in the subsection on supply behavior below, the effects of prices have been left out).

To theorize about the movements over time (i.e., the dynamic trajectory) of service performance, we will make two reasonable assumptions: First, the relative strength (or magnitude) of the demand compared to the supply provides an impetus for performance to be adjusted upwards over time. Second, productivity growth contributes to performance improvements over time. These assumptions are relevant to the Turkish educational system in the following respects. Regarding the first assumption, in Turkey, the gap between the demand for higher educational services and the supply has been a key source of pressure for the increased volume of higher educational services, which is a key

**Needless to say, of course, that log linear forms are extensively used throughout economic literature.

determinant of service performance. The wider is the gap between demand and supply, the stronger is the pressure on higher education institutions to increase services. Thus, it is reasonable to assume that the relative strength of the demand compared to supply provides an impetus for performance to be adjusted upwards over time. To exemplify the relevance of the second assumption, suppose, for instance, that due to asupplydvances in information technology (such as e-learning), universities are able to reach a higher number of students with the same number of teachers. This is an increase in the average productivity of teachers, which, of course, increases the overall teaching performance of the universities.

Taking these factors into account, we formulate the following adjustment dynamic for performance.

$$P_{t+1}/P_t = (D_t/S_t)^k (1 + \delta)^t,$$

where k is the coefficient of adjustment and δ is a productivity growth at t .

Taking the logarithmic transformation of both sides, we get:

$$\ln P_{t+1} = \ln P_t + k(\ln D_t - \ln S_t) + t(\ln(1 + \delta))$$

We will call this the dynamic adjustment equation. Substituting the functional expressions (forms) for $\ln D_t$ and $\ln S_t$ specified above, setting the values of M_t , R_t and T_t to their average values M^{avr} , R^{avr} and T^{avr} and rearranging the terms in the equation, we get,

$$\ln P_{t+1} + (k\beta_1 - k\alpha_1 - 1) \ln P_t + k\beta_2 \ln P_{t-1} = k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr}) + k(u_t - v_t) - k[\ln A(1 + g)]t + [\ln(1 + \delta)]t,$$

which is a second order stochastic difference equation, the solution of which is provided in Appendix A.

The solution in Appendix A shows that the intertemporal equilibrium performance, P^* is:

$$P^* = \exp \left\{ \frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)} + \frac{(k \ln A(1 + g) - \ln(1 + \delta))(1 - k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1 + g) - \ln(1 + \delta)}{k(\beta_1 + \beta_2 - \alpha_1)} t + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j} \right\}$$

where $z_t = k(u_t - v_t)$

$$\begin{aligned} \lambda_1 \lambda_2 &= k\beta_2 \\ \lambda_1 + \lambda_2 &= 1 - k\beta_1 + k\alpha_1 \end{aligned}$$

In case where λ_1 and λ_2 are conjugate complex numbers, i.e., $\lambda_1, \lambda_2 = h \mp vi = r(\cos \theta \mp i \sin \theta)$, the intertemporal equilibrium performance is:

$$P^* = \exp \left\{ \frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)} + \frac{(k \ln A(1 + g) - \ln(1 + \delta))(1 - k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1 + g) - \ln(1 + \delta)}{k(\beta_1 + \beta_2 - \alpha_1)} t + \sum_{j=0}^{\infty} r^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j} \right\}$$

where r is the absolute value of the complex number, and $\sin \theta = v/r$ and $\cos \theta = h/r$.

To study whether this intertemporal equilibrium performance is high or low, and whether it remains stable over time, we need to empirically estimate the parameters involved. This is done in the next section.

3. Empirical Analysis

3.1. The sample. Data for this study was gathered using a questionnaire including questions about demand, supply, income, prices, performance and the use technology in the educational services in Turkey. 100 respondents were asked to answer the questions. 66 useable questionnaires were returned giving a response rate of 66 percent, which was considered satisfactory for the analysis in the paper. Some responses with considerable missing information were excluded. Each question (item) was rated on a seven-point Likert scale with 1 representing the lowest score that can be assigned, and 7 representing the highest. (The reason for choosing a seven-point scale is simple. In the literature, researchers often use a five-point scale or a seven-point scale in the questionnaires. I have chosen to use a seven-point scale in order to capture the differences between the answers to the questions in a more sensitive manner)[12].

The information in the questionnaire is used to estimate the parameters in the regression equations in the following manner: Each variable in the model is represented by a question in the questionnaire. Thus responses to questions will be the values for the variables. For instance, T_t represents the degree to which technology is used in educational practices - with 1 representing the lowest use and 7 the highest. Values between 1 and 7 represent varying degrees to which the developments in technology (such as in information technology) are put in practice in classroom instruction in particular, and knowledge, capacity and skill formation in general. M_t represents customer incomes, which are translated into bands. Bands are, in turn, rated on a seven point scale, with 1 representing the lowest income interval and 7 representing the highest income interval. Other questions (variables) are directly rated on a seven point scale. Thus, our sample that consists of answers to the questions in the questionnaire contains integer values (from 1 to 7) for the variables associated with demand, performance, income, use of technology etc. In order to run a regression relating to, for instance, demand, we regress quantity demanded for the educational service on performance, income, the use of technology etc.

3.2. Estimation of the parameters. To estimate the parameters involved, we make use of the demand and supply equations for the educational service, i.e., equations (4) and (5).

3.2.1. Demand. In the particular empirical case under consideration, public provision of the educational service could be considered free, and the differences between the relative prices of private educational institutions in the sample are insignificant, thus relative prices do not appear to play a deciding role in the demand for the educational service in question, therefore the relative price variable is left out of the demand equation. The regression-results are as follows:

$$\ln D_t = -0.756 + 0.428 \ln P_t + 0.358 \ln M_t + 0.598 \ln T_t$$

(-2.008) (1.827) (2.195) (2.939)

$R^2 = 0.58$. t -statistics are given in the parentheses. Thus,

$$\begin{aligned}\alpha_0 &= -0.756 \\ \alpha_1 &= 0.428 \\ \alpha_2 &= 0.358 \\ \alpha_3 &= 0 \\ \alpha_4 &= 0.598\end{aligned}$$

3.2.2. Supply. Supply is largely determined by central bureaucratic authorities whose decisions are based on certain criteria, such as the adequacy and quality of physical infrastructure and human resources, rather than prices. Thus prices could be conveniently left out of the supply function. R_t drops out of the log-linear formulation of the supply equation. To estimate the other parameters of the supply equation, we asked officials of the relevant institutions questions, the answers of which were designed to give the values of the elasticities of supply with respect to the present and past performances. The answers indicate that a 1% increase in the past performance would increase the quantity supplied by about 0.25 %, but a 1% increase in the present performance would increase the quantity supplied by about 0.75%. However, by virtue of the enrollment constraints placed by the Higher Education Council, what the institutions under examination could supply was 90 % of what they were willing to supply, Thus,

$$\begin{aligned}\beta_1 &= 0.9 \cdot 0.75 = 0.675 \\ \beta_2 &= 0.9 \cdot 0.25 = 0.225 \\ \beta_3 &= 0\end{aligned}$$

The value of A is normalized to 1.

3.2.3. The coefficient of adjustment (k). For simplicity, we will assume that P_{t+1}/P_t is proportional to the ratio of demand to supply, and hence, $k = 1$.

Given the empirical values of the parameters obtained above, we get,

$$\lambda_1 = 0.376 + 0.288i \quad \text{and} \quad \lambda_2 = 0.376 - 0.288i$$

We will now consider a particularly interesting case where the student population growth is equal to the productivity growth, i.e., $g = \delta$. This special case is interesting and worthwhile to consider because the equality between the growth rates in question ensures the sustainable provision of the educational service. Student population growth rates roughly represent "increases in the demand for the educational service" while productivity growth rates represent "increases in the supply (provision) of the educational service". Starting from an equilibrium, equal demand and supply growth rates enable supply to meet demand, in a sustainable manner, over time.

With this assumption and with all the needed parameter values at hand, the intertemporal equilibrium performance is [12]:

$$P^* = \exp \left\{ 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j} \right\}$$

For analytical convenience, we will carry out some of our analysis in terms of logarithmically transformed performance, $\ln P^*$, rather than P^* . Since function is an order-preserving transformation, analysis in terms of $\ln P^*$ and P^* will yield the same qualitative results; and the quantitative results could be transformed into one another. The

expected value of the logarithmically transformed intertemporal equilibrium performance is:

$$E(\ln P^*) = 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} E(z_{t-j})$$

Since, by virtue of the assumptions about u_t , and v_t , $E(u_t) = 0$, and $E(v_t) = 0$, $E(z_t) = E(u_t) - E(v_t) = 0$. Thus,

$$E(\ln P^*) = 1.29$$

In view of the logarithmically transformed performance scale of $\ln 1 = 0$ to $\ln 7 \cong 1.95$, an intertemporal equilibrium expected performance of 1.29 is low (or, at best, mediocre). As proven in Appendix A, this low performance is also stable over time in the particular sense that it has a stationary distribution with a constant mean and variance. This indicates a low-performance trap facing the sector across time.^{††}

To elaborate on the stability property of the equilibrium obtained, we will present a general description of the stability conditions in terms of k , α_1 , β_1 , β_2 , and then focus on the specific case that is relevant to the numerical values of the parameters estimated above. Consider the three different cases concerning the roots of the complementary function associated with the stochastic difference equation (8), i.e.,

$$\ln P_{t+1} + (k\beta_1 - k\alpha_1 - 1) \ln P_t + k\beta_2 \ln P_{t-1} = 0.$$

Stability conditions for the three different cases are as follows:

(i) The case of distinct real roots.

Stability conditions:

$$\left| \frac{-(k\beta_1 - k\alpha_1 - 1) + ((k\beta_1 - k\alpha_1 - 1)^2 - 4k\beta_2)^{\frac{1}{2}}}{2} \right| < 1$$

and

$$\left| \frac{-(k\beta_1 - k\alpha_1 - 1) - ((k\beta_1 - k\alpha_1 - 1)^2 - 4k\beta_2)^{\frac{1}{2}}}{2} \right| < 1$$

(ii) The case of repeated real roots.

The stability condition:

$$\left| \frac{-(k\beta_1 - k\alpha_1 - 1)}{2} \right| < 1$$

(iii) The case of complex roots.

Stability condition:

$$0 < R = (k\beta_2)^{\frac{1}{2}} < 1.$$

Since the parameter estimates we obtained above lead to complex roots, the case relevant to our work is (iii), which requires that the square root of $k\beta_2$, hence $k\beta_2$ itself, be less than 1. Since $k=1$, $\beta_2 < 1$. What is the meaning and implication of this condition? β_2 is nothing but the elasticity of supply with respect to past performance. The condition requiring that $\beta_2 < 1$ implies that supply of educational services should not be “too sensitive” to past performances. More explicitly, a 1% increase in past performance should,

^{††}It is possible to formulate the problems in terms of a concept of quality as well. An example of such formulation in the context of a different sector is presented by Kara and Kurtulmus [14]. There are other studies that deal with the quality issues differently. Among the works that explore the issues of quality in education, for instance, from a “total quality management” perspective are Dahlgaard, Kristensen and Kanji [4, 5].

in the future, lead to a less than 1 % increase in the quantity supplied of educational services. The condition, in a way, limits the effect of the “past” on the “present”. In the absence of such a condition, the educational sector under examination would exhibit a destabilizing dynamic similar to the ones observed in some agricultural sectors.

Since our estimate of β_2 is 0.225, the condition stipulated in (iii) is met. The low performance equilibrium is deterministically stable. It is also stochastically stable in the particular sense mentioned (and proved) in Appendix A.

The following section will formulate a stochastic policy rule, which will enable the sector to escape the low performance equilibrium by helping the sector to reach a performance-target, and which will stabilize the sector around that target.

4. Policy Implications ^{††}: An Example

Suppose that the educational service providers in Turkey aim to reach a stable (sustainable) performance-target in the presence of stochastic shocks. Consider a stochastic shock to demand, in the magnitude of u_t , which may have come, for instance, from a sudden reduction in the demand for education induced by the recent economic slowdown. Let us design the following demand-side stochastic policy response (rule):

$$R = \eta_0 + \eta_1 u_t$$

where η_0 and η_1 represent the non-stochastic and stochastic components respectively. Suppose that the non-stochastic component will take the form of a governmental support for the use of technology in education. Let $\eta_0 = \ln \Delta T$. The government will provide support so as to increase the use of technology in education by ΔT .

Reaching a stable (“minimally varying”) expected (logarithmically expressed) performance target in the presence of stochastic shocks turns out to be equivalent to minimizing the expected loss function of the following kind at the intertemporal equilibrium:

$$E \left[(\ln P_t - \ln P^{**})^2 \right],$$

where $\ln P^{**}$ is the (logarithmically expressed) performance target. Decomposing the expected loss function, we get,

$$\begin{aligned} E \left[(\ln P_t - \ln P^{**})^2 \right] &= E \left[((\ln P_t - E(\ln P_t)) + (E(\ln P_t) - \ln P^{**}))^2 \right] \\ &= E \left((\ln P_t - E(\ln P_t))^2 \right) + E \left((E(\ln P_t) - \ln P^{**})^2 \right) \\ &\quad + 2E(\ln P_t - E(\ln P_t))(E(\ln P_t) - \ln P^{**}) \end{aligned}$$

Since $(E(\ln P_t) - \ln P^{**})$ is not random and since $E(\ln P_t - E(\ln P_t)) = E(\ln P_t) - E(\ln P_t) = 0$, the decomposition will take the form of:

$$E \left[(\ln P_t - \ln P^{**})^2 \right] = E \left((\ln P_t - E(\ln P_t))^2 \right) + (E(\ln P_t) - \ln P^{**})^2.$$

The first term represents the variance of (logarithmically expressed) performance and the second term denotes the “squared deviation” around $\ln P^{**}$. Thus minimizing expected loss is equivalent to minimizing the squared deviation, which requires that expected (logarithmically expressed) performance be equal to the (logarithmically expressed) performance target, and minimizing the variance of (logarithmically expressed) performance, enabling the educational service provider to reach a stable (minimally varying) logarithmically expressed performance target [12].

^{††}In the literature, there are a number of works exploring various policy issues which range from comprehensive school reform (Desimone [7]) to cost reduction strategies in higher education (Leonard [15]).

For a similar decomposition, though in a different context, see Sargent [17].

To find, for the special case where $g = \delta$, the parameters of the stochastic policy rule which minimize the expected loss function, let us incorporate the rule into the function.

$$E[(\ln P_t - \ln P^{**})^2] = \left\{ \sum_{j=0}^{\infty} 0.47^{2j} \left(\frac{\sin \theta(j+1)}{\sin \theta} \right)^2 \right\} ((1 + \eta_1) \sigma_u^2 + \sigma_v^2) \\ + \left\{ \frac{\alpha_0 + \alpha_2 \ln M^{avr} + \alpha_4 \ln T^{avr}}{\beta_1 + \beta_2 - \alpha_1} + \frac{\eta_0}{\beta_1 + \beta_2 - \alpha_1} - P^{**} \right\}^2$$

The values of η_0 and η_1 that minimize the expected loss function are:

$$\eta_0 = \left\{ P^{**} - \frac{\alpha_0 + \alpha_2 \ln M^{avr} + \alpha_4 \ln T^{avr}}{\beta_1 + \beta_2 - \alpha_1} \right\} (\beta_1 + \beta_2 - \alpha_1) \\ \eta_1 = -1.$$

For instance, for $\ln P^{**} = 1.5$, the value of η_0 is calculated to be 0.099. This implies that $\Delta T = 1.1$, which represents the required policy-induced increase in the use of technology for the performance target in question. The fact that $\eta_1 = -1$ implies that, for stabilization against the kind of demand shock exemplified here, demand should be increased by the magnitude of the stochastic shock.

In sum, the designed policy response requires an increase in the use of technology at the intertemporal equilibrium, captured by the non-stochastic component of the policy, and an increase in the stochastic component so as to meet the temporary reduction in the demand for services.

5. Concluding Remarks

The paper develops an applied stochastic model of higher education sector in Turkey, and shows that the sector could, under certain conditions, slide into a low-performance trap over time. The paper presents a stochastic policy option that could help the sector to avoid the trap in question. The designed stochastic resolution and the model, however, focus on the overall performance of the educational institutions, and as such, do not take into account the micro components of the overall performance. Decomposing the overall performance into its micro components could more precisely reveal the sources of potential improvements in overall performance, opening up new possibilities for policy formulations. The decomposition in question could be done in a number of ways. For instance, a functional decomposition of the overall performance into the components of “teaching performance” and “research performance” would enable the institutions to exactly identify their teaching-related or research-related strengths and weaknesses, and hence would help them to take concrete/measurable steps towards their teaching-and-research-related-targets. Alternatively, the overall performance could be decomposed into its departmental components, identifying the contributions of each department to the overall performance. Dynamic modeling based on such decomposition could provide key information about the strategic decisions about how departmental priorities should be arranged and revised over time.

Finally, we will elaborate on one feature of the paper that can be relaxed, namely that supply and demand functions are of log-linear form. There are two main reasons for choosing the log-linear form for the functions used in the paper. First, the log-linear form is one of the most extensively used forms in the economic literature, which made it the prime candidate for our work as well. Second, the log-linear form has a peculiar feature that the coefficients of the variables represent the “elasticities” of the dependent variable with respect to the relevant independent variables. This has made it possible to directly estimate the parameters of the supply function in Section 3.

A similar trap in the banking sector is studied in the literature (e.g., Kara and Kurtulmus [14]).

Do we obtain similar results when the supply and demand functions are nonlinear? Our conjecture is that, with certain types of nonlinear functions, the central result concerning the low performance trap could still be obtained. For instance, in certain equilibrium models with particularly specified quadratic functions, a similar result could be obtained through computer-based simulations. However, in view of the wide range of possible nonlinear functions, it would be difficult to make general definitive statements for all cases where functions are nonlinear. Nonlinearity could, in some cases, lead to multiple equilibria with different stability properties. Such cases, which may not lend themselves to analytical solutions and which could be studied through computer-based simulations or other means, are worthy of future research.

6. Appendix A

The solution for the second order stochastic difference equation,

$$\begin{aligned} & \ln P_{t+1} + (k\beta_1 - k\alpha_1 - 1)\ln P_t + k\beta_2 \ln P_{t-1} \\ & = k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3)\ln R^{avr} + \alpha_4 \ln T^{avr}) \\ & + k(u_t - v_t) - [k \ln A(1 + g)]t + [(\ln(1 + \delta))t, \end{aligned}$$

has two components, namely a particular solution and a complementary function. We will find these components for $\ln P_t$ and then take the anti-log of $\ln P_t$ so as to find the solution for P_t .

- (1) **Particular Solution:** Letting $x_t = \ln P_t$, using the lag operator L (defined as $L^i P_t = P_{t-i}$, for $i = 1, 2, 3, \dots$), and rearranging the terms, we get the following form of the second order stochastic difference equation above,

$$\begin{aligned} [1 - (1 - k\beta_1 + k\alpha_1)L - (-k\beta_2)L^2]x_t = & k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3)\ln R^{avr} \\ & + \alpha_4 \ln T^{avr}) + k(u_t - v_t) - [k \ln A(1 + g)]t + [(\ln(1 + \delta))t, \end{aligned}$$

which could be transformed into,

$$\begin{aligned} (1 - \lambda_1 L)(1 - \lambda_2 L)x_t = & k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3)\ln R^{avr} + \alpha_4 \ln T^{avr}) \\ & + k(u_t - v_t) - [k \ln A(1 + g)]t + [(\ln(1 + \delta))t, \end{aligned}$$

where

$$\lambda_1 \lambda_2 = k\beta_2$$

$$\lambda_1 + \lambda_2 = 1 - k\beta_1 + k\alpha_1$$

Thus, we get,

$$\begin{aligned} x_t = & (1 - \lambda_1 L)^{-1}(1 - \lambda_2 L)^{-1}[k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3)\ln R^{avr} + \alpha_4 \ln T^{avr}) \\ & + k(u_t - v_t) - [k \ln A(1 + g) - \ln(1 + \delta)]t] \end{aligned}$$

Using the properties of partial fractions,

$$(1 - \lambda_1 L)^{-1}(1 - \lambda_2 L)^{-1} = \frac{\lambda_1}{\lambda_1 - \lambda_2}(1 - \lambda_1 L)^{-1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - \lambda_2 L)^{-1}$$

Thus,

$$\begin{aligned} x_t = & \frac{\lambda_1}{\lambda_1 - \lambda_2}(1 - \lambda_1 L)^{-1} + \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - \lambda_2 L)^{-1} \\ & [k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3)\ln R^{avr} + \alpha_4 \ln T^{avr}) \\ & + k(u_t - v_t) - [k \ln A(1 + g) - \ln(1 + \delta)]t] \end{aligned}$$

Using the properties of some series and lag operators, and doing some algebraic manipulations, we get,

$$\begin{aligned} x_t = & \frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)} \\ & + \frac{(k \ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1+g) - \ln(1+\delta)}{k(\beta_1 + \beta_2 - \alpha_1)} t \\ & + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j} \end{aligned}$$

where $z_t = k(u_t - v_t)$

This is the parametric expression of $x_t = \ln P_t$, at the intertemporal equilibrium. Let P^* denote the intertemporal equilibrium performance. Thus,

$$\begin{aligned} P^* = & \exp\left[\frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)}\right. \\ & + \frac{(k \ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1+g) - \ln(1+\delta)}{k(\beta_1 + \beta_2 - \alpha_1)} t \\ & \left. + \frac{\lambda_1}{\lambda_1 - \lambda_2} \sum_{j=0}^{\infty} \lambda_1^j z_{t-j} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \sum_{j=0}^{\infty} \lambda_2^j z_{t-j}\right] \end{aligned}$$

In case where λ_1 and λ_2 are conjugate complex numbers, i.e., $\lambda_1, \lambda_2 = h \pm vi = r(\cos\theta \pm i\sin\theta)$, the intertemporal equilibrium performance is:

$$\begin{aligned} P^* = & \exp\left[\frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)}\right. \\ & + \frac{(k \ln A(1+g) - \ln(1+\delta))(1-k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1+g) - \ln(1+\delta)}{k(\beta_1 + \beta_2 - \alpha_1)} t \\ & \left. + \sum_{j=0}^{\infty} r^j \frac{\sin\theta(j+1)}{\sin\theta} z_{t-j}\right] \end{aligned}$$

where r is the absolute value of the complex number, and $\sin\theta = v/r$ and $\cos\theta = h/r$.

- (2) **Complementary Function:** To find this component of the solution, we need to consider the following reduced form of the second order difference equation.

$$\ln P_{t+1} + (k\beta_1 - k\alpha_1 - 1)\ln P_t + k\beta_2 \ln P_{t-1} = 0.$$

A possible general solution could take the form $\ln P_t = Ay^t$. Through the standard procedure, we obtain,

$$y^2 + (k\beta_1 - k\alpha_1 - 1)y + k(\beta_2) = 0.$$

This quadratic equation could have at most two roots. Suppose that $k = 1$. Substituting $\alpha_1 = 0.428$, $\beta_1 = 0.675$ and $\beta_2 = 0.225$ into the quadratic equation and solving it for the roots, we get,

$$y_1 = 0.376 + 0.288i$$

and

$$y_2 = 0.376 - 0.288i.$$

Thus, the solution for the reduced equation is

$$A_1 y_1^t + A_2 y_2^t = A_1(0.376 + 0.288i)^t + A_2(0.376 - 0.288i)^t$$

where A_1 and A_2 are non-zero constants. This could be shown to be equivalent to:

$$0.47^t (A_3 \cos \theta t + A_4 \sin \theta t)$$

where $A_3 = A_1 + A_2$ and $A_4 = (A_1 - A_2)i$. $\sin \theta = 0.288/0.47$ and $\cos \theta = 0.376/0.47$.

- (3) **The general solution:** The general solution for the equation is the sum of the two solutions obtained in (a) and (b),

$$\begin{aligned} \ln P^* = & \left[\frac{k(\alpha_0 + \alpha_2 \ln M^{avr} + (\alpha_3 - \beta_3) \ln R^{avr} + \alpha_4 \ln T^{avr})}{k(\beta_1 + \beta_2 - \alpha_1)} \right. \\ & + \frac{(k \ln A(1+g) - \ln(1+\delta))(1 - k\beta_2)}{(k(\beta_1 + \beta_2 - \alpha_1))^2} - \frac{k \ln A(1+g) - \ln(1+\delta)}{k(\beta_1 + \beta_2 - \alpha_1)} \Big]_t \\ & + \sum_{j=0}^{\infty} r^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j} \Big] + 0.47^t (A_3 \cos \theta t + A_4 \sin \theta t). \end{aligned}$$

In the paper, we analyze the case where $g = \delta$. Substituting the values of the parameters involved, we get, for this special case,

$$\ln P^* = 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j} + 0.47^t (A_3 \cos \theta t + A_4 \sin \theta t)$$

The values of A_3 and A_4 could be obtained by specifying two initial conditions. However, for the purposes of our analysis, we do not need to know the values of those constants. Since the absolute value of the complex number involved is 0.47, which is less than 1, as $t \rightarrow \infty$, $0.47^t (A_3 \cos \theta t + A_4 \sin \theta t)$ will converge toward zero, and hence the general solution converges toward the particular solution,

$$\ln P^* = 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j}$$

Thus,

$$E(\ln P^*) = 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} E(z_{t-j})$$

Since, by virtue of the assumptions about u_t , and v_t , $E(u_t) = 0$, and $E(v_t) = 0$, $E(z_t) = k(E(u_t) - E(v_t)) = 0$. Thus,

$$E(\ln P^*) = 1.29.$$

which is nothing but the intertemporal expected equilibrium performance. Note that u_t , and v_t are uncorrelated over time, and so is z_t . They have zero covariances. Thus, the variance (V) of $\ln P^*$ is,

$$\begin{aligned} V(\ln P^*) &= V \left\{ 1.29 + \sum_{j=0}^{\infty} 0.47^j \frac{\sin \theta(j+1)}{\sin \theta} z_{t-j} \right\} \\ V(\ln P^*) &= \left\{ \sum_{j=0}^{\infty} 0.47^{2j} \left(\frac{\sin \theta(j+1)}{\sin \theta} \right)^2 \right\} (\sigma_u^2 + \sigma_v^2) \end{aligned}$$

which is constant. (Please note the value of $\sin \theta$ specified above). Needless to say, taking the limits of mean and variance as $t \rightarrow \infty$ yields the same results. For the sake of exposition,

$$\lim_{t \rightarrow \infty} E(\ln P^*) = 1.29,$$

and

$$\lim_{t \rightarrow \infty} V(\ln P^*) = \left\{ \sum_{j=0}^{\infty} 0.47^{2j} \left(\frac{\sin \theta(j+1)}{\sin \theta} \right)^2 \right\} (\sigma_u^2 + \sigma_v^2)$$

Thus, logarithmically transformed intertemporal performance has a stationary distribution in the sense that it has a constant mean and variance.

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