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COMMON FIXED POINT THEOREM FOR A FAMILY OF NON-SELF MAPPINGS IN CONE METRIC SPACES[‡]

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Abstract

In this paper, we prove a common fixed point theorem for a family of non-self mappings in cone metric spaces (over a cone which is not necessarily normal). Our result generalizes and extends some recent results of Radenovic and Rhoades.

Keywords: Cone metric spaces, Common fixed point, Non-self mappings.

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1. Introduction and preliminaries

The existing literature of fixed point theory contains many results enunciating fixed point theorems for self-mappings in metric and Banach spaces. Recently, Huang and Zhang [11] generalized the concept of a metric space, replacing the set of real numbers by ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions. Subsequently, the study of fixed point theorems in such spaces is followed by some other mathematicians, see [1-3, 5-7, 9-10, 12-13, 16-22, 24-28, 30-31]. However, fixed point theorems for non-self mappings are not frequently discussed and so form a natural subject for further investigation. The study of fixed point theorems for non-self mappings in metrically convex metric spaces was initiated by Assad and Kirk[4]. Recently, Radenovic and Rhoades [22] obtained a fixed point theorem for two non-self mappings in cone metric spaces. Motivated by Radenovic and Rhoades [22], we

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prove a common fixed point theorem for a family of non-self mappings in cone metric spaces in which the cone need not be normal.

Consistent with Huang and Zhang [11], the following definitions and results will be needed in the sequel.

Let E be a real Banach space. A subset P of E is called a *cone* if and only if:

(a) P is closed, nonempty and $P \neq \{\theta\}$;
(b) $a, b \in R, a, b \geq 0, x, y \in P$ implies $ax + by \in P$;
(c) $P \cap (-P) = \{\theta\}$.

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. A cone P is called *normal* if there is a number $K > 0$ such that for all $x, y \in E$,

$$\theta \leq x \leq y \text{ implies } \| x \| \leq K \| y \| .$$

The least positive number K satisfying the above inequality is called the *normal constant* of P, while $x \ll y$ stands for $y - x \in \mathrm{int}P$ (interior of P).

Rezapour and Hamlbarani [28] proved that there is no normal cone with normal constant $K < 1$ and for each $k > 1$ there are cones with normal constants $K > k$.

1.1. Definition. [11] Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies:

(d1) $\theta \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
(d2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(d3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a *cone metric on* X and (X, d) is called a *cone metric space*.

The concept of a cone metric space is more general than that of a metric space.

1.2. Definition. [11] Let (X, d) be a cone metric space. We say that $\{x_n\}$ is:

(e) a *Cauchy sequence* if for every $c \in E$ with $\theta \ll c$, there is an N such that for all $n, m > N, d(x_n, x_m) \ll c$;
(f) a *convergent sequence* if for every $c \in E$ with $\theta \ll c$, there is an N such that for all $n > N, d(x_n, x) \ll c$ for some fixed $x \in X$.

The cone metric space (X, d) is called *complete* if every Cauchy sequence in X is convergent in X. It is known that $\{x_n\}$ converges to $x \in X$ if and only if $d(x_n, x) \to \theta$ as $n \to \infty$. It is a Cauchy sequence if and only if $d(x_n, x_m) \to \theta (n, m \to \infty)$.

1.3. Remark. [32] Let E be an ordered Banach (normed) space. Then c is an interior point of P, if and only if $[-c, c]$ is a neighborhood of θ.

1.4. Corollary. [30]

(1) *If $a \leq b$ and $b \ll c$, then $a \ll c$.*
Indeed, $c - a = (c - b) + (b - a) \geq c - b$ implies $[-(c - a), c - a] \supseteq [-(c - b), c - b]$.
(2) *If $a \ll b$ and $b \ll c$, then $a \ll c$.*
Indeed, $c - a = (c - b) + (b - a) \geq c - b$ implies $[-(c - a), c - a] \supseteq [-(c - b), c - b]$.
(3) *If $\theta \leq u \ll c$ for each $c \in intP$ then $u = \theta$.* □

1.5. Remark. [22] If $c \in \mathrm{int}P$, $\theta \leq a_n$ and $a_n \to \theta$, then there exists an n_0 such that for all $n > n_0$ we have $a_n \ll c$.

1.6. Remark.

If E is a real Banach space with cone P and if $a \leq ka$ where $a \in P$ and $0 < k < 1$, then $a = \theta$.

We find it convenient to introduce the following definition.

1.8. Definition. [22] Let (X, d) be a cone metric space, C a nonempty closed subset of X, and $f, g : C \to X$. If f and g satisfy the condition

$$d(fx, fy) \leq \lambda u$$

where

$$u \in \Big\{\frac{d(gx, gy)}{2}, d(fx, gx), d(fy, gy), \frac{d(fx, gy) + d(fy, gx)}{q}\Big\}, \tag{1.1}$$

for all $x, y \in C, 0 < \lambda < \frac{1}{2}, q \geq 2 - \lambda$, then f is called a *generalized g-contractive mapping* of C into X.

1.9. Definition. [1] Let f and g be self maps of a set X (i.e., $f, g : X \to X$). If $w = fx = gx$ for some x in X, then x is called a *coincidence point* of f and g, and w is called a *point of coincidence* of f and g. Self maps f and g are said to be *weakly compatible* if they commute at their coincidence point; i.e., if $fx = gx$ for some $x \in X$, then $fgx = gfx$.

2. Main result

Radenovic and Rhoades [22] obtained the following theorem which is a generalization of the corresponding result of Imdad and Kumar [15] in cone metric spaces.

2.1. Theorem. *Let (X, d) be a complete cone metric space, C a nonempty closed subset of X such that for each $x \in C$ and $y \notin C$ there exists a point $z \in \partial C$ (the boundary of C) such that*

$$d(x, z) + d(z, y) = d(x, y).$$

Suppose that $f, g : C \to X$ are such that f is a generalized g-contractive mapping of C into X, and

(i) *$\partial C \subseteq gC, fC \cap C \subseteq gC$,*
(ii) *$gx \in \partial C$ implies that $fx \in C$,*
(iii) *gC is closed in X.*

Then the pair (f, g) has a coincidence point. Moreover, if the pair (f, g) is weakly compatible, then f and g have a unique common fixed point. □

The purpose of this paper is to extend the above theorem to a family of non-self mappings in cone metric spaces whose cone need not be normal. It is worth pointing out that some fixed point results are not real generalizations [10], but our main result is a real generalization, since Theorem 2.3 cannot be considered as a consequence of Theorem 2.1. We begin with the following definition.

2.2. Definition. Let (X, d) be a complete cone metric space, C a nonempty closed subset of X, and $\{F_n\}_{n=1}^{\infty}, S, T : C \to X$ non-self mappings. If there exist $\lambda \in (0, \frac{1}{2})$ and $q \geq 2 - \lambda$ such that for all $x, y \in C$ with $x \neq y$,

$$d(F_i x, F_j y) \leq \lambda u, \tag{2.1}$$

where

$$u \in \{\frac{d(Tx, Sy)}{2}, d(Tx, F_i x), d(Sy, F_j y), \frac{d(Tx, F_j y) + d(Sy, F_i x)}{q}\},$$

$i = 2n - 1, j = 2n$ for some $n \in N$, then (F_i, F_j) is called a *generalized (T, S)-contractive mapping pair of C into X.*

Notice that by setting $F_i = F_j = f$ and $T = S = g$ in (2.1), one deduces a slightly generalized form of (1.1).

We state and prove our main result as follows.

2.3. Theorem. *Let (X, d) be a complete cone metric space, C a nonempty closed subset of X such that for each $x \in C$ and $y \notin C$ there exists a point $z \in \partial C$ (the boundary of C) such that*

$$d(x,z) + d(z,y) = d(x,y).$$

Suppose that $\{F_n\}_{n=1}^{\infty}, S, T : C \to X$ are such that (F_i, F_j) is a generalized (T, S)-contractive mapping pair on C for all $i = 2n-1, j = 2n (n \in N)$, and

(I) $\partial C \subseteq SC \cap TC, F_iC \cap C \subseteq SC, F_jC \cap C \subseteq TC$,
(II) $Tx \in \partial C$ *implies that* $F_ix \in C, Sx \in \partial C$ *implies that* $F_jx \in C$,
(III) SC *and* TC *(or* F_iC *and* F_jC*) are closed in* X.

Then

(IV) (F_i, T) *has a point of coincidence,*
(V) (F_j, S) *has a point of coincidence.*

Moreover, if (F_i, T) and (F_j, S) are weakly compatible pairs for all $i = 2n-1$, $j = 2n$ $(n \in N)$, then $\{F_n\}_{n=1}^{\infty}, S$ and T have a unique common fixed point.

Proof. Firstly, we proceed to construct two sequences $\{x_n\}$ and $\{y_n\}$ in the following way.

Let $x \in \partial C$ be arbitrary. Then (due to $\partial C \subseteq TC$) there exists a point $x_0 \in C$ such that $x = Tx_0$. Since $Tx_0 \in \partial C$, from (I) and (II), we have $F_1x_0 \in F_1C \cap C \subseteq SC$. Thus, there exists $x_1 \in C$ such that $y_1 = Sx_1 = F_1x_0 \in C$. Since $y_1 = F_1x_0$ there exists a point $y_2 = F_2x_1$ such that

$$d(y_1, y_2) = d(F_1x_0, F_2x_1).$$

Suppose $y_2 \in C$. Then $y_2 \in F_2C \cap C \subseteq TC$, which implies that there exists a point $x_2 \in C$ such that $y_2 = Tx_2$. Otherwise, if $y_2 \notin C$, then there exists a point $p \in \partial C$ such that

$$d(Sx_1, p) + d(p, y_2) = d(Sx_1, y_2).$$

Since $p \in \partial C \subseteq TC$, there exists a point $x_2 \in C$ with $p = Tx_2$ such that

$$d(Sx_1, Tx_2) + d(Tx_2, y_2) = d(Sx_1, y_2).$$

Let $y_3 = F_3x_2$ be such that $d(y_2, y_3) = d(F_2x_1, F_3x_2)$. Thus, repeating the foregoing arguments, one obtains two sequences $\{x_n\}$ and $\{y_n\}$ such that

(a) $y_{2n} = F_{2n}x_{2n-1}$ for every $n \in N$, $y_{2n+1} = F_{2n+1}x_{2n}$ for every $n \in N_0 = N \cup \{0\}$,
(b) $y_{2n} \in C$ implies that $y_{2n} = Tx_{2n}$ or $y_{2n} \notin C$ implies that $Tx_{2n} \in \partial C$ and

$$d(Sx_{2n-1}, Tx_{2n}) + d(Tx_{2n}, y_{2n}) = d(Sx_{2n-1}, y_{2n}).$$

(c) $y_{2n+1} \in C$ implies that $y_{2n+1} = Sx_{2n+1}$ or $y_{2n+1} \notin C$ implies that $Sx_{2n+1} \in \partial C$ and

$$d(Tx_{2n}, Sx_{2n+1}) + d(Sx_{2n+1}, y_{2n+1}) = d(Tx_{2n}, y_{2n+1}).$$

We set

$$P_0 = \{Tx_{2i} \in \{Tx_{2n}\} : Tx_{2i} = y_{2i}\},$$
$$P_1 = \{Tx_{2i} \in \{Tx_{2n}\} : Tx_{2i} \neq y_{2i}\},$$
$$Q_0 = \{Sx_{2i+1} \in \{Sx_{2n+1}\} : Sx_{2i+1} = y_{2i+1}\},$$
$$Q_1 = \{Sx_{2i+1} \in \{Sx_{2n+1}\} : Sx_{2i+1} \neq y_{2i+1}\}.$$

Note that $(Tx_{2n}, Sx_{2n+1}) \notin P_1 \times Q_1$, as if $Tx_{2n} \in P_1$, then $y_{2n} \neq Tx_{2n}$ and one infers that $Tx_{2n} \in \partial C$ which implies that $y_{2n+1} = F_{2n+1}x_{2n} \in C$. Hence $y_{2n+1} = Sx_{2n+1} \in Q_0$. Similarly, one can argue that $(Sx_{2n-1}, Tx_{2n}) \notin Q_1 \times P_1$.

Now, we distinguish the following three cases.

Case 1. If $(Tx_{2n}, Sx_{2n+1}) \in P_0 \times Q_0$, then from (2.1)

$$d(Tx_{2n}, Sx_{2n+1}) = d(F_{2n+1}x_{2n}, F_{2n}x_{2n-1}) \leq \lambda u_{2n},$$

where

$$\begin{aligned} u_{2n} \in \{ &\frac{d(Sx_{2n-1}, Tx_{2n})}{2}, d(Sx_{2n-1}, F_{2n}x_{2n-1}), d(Tx_{2n}, F_{2n+1}x_{2n}), \\ &\frac{d(Tx_{2n}, F_{2n}x_{2n-1}) + d(Sx_{2n-1}, F_{2n+1}x_{2n})}{q}\} \\ &= \{\frac{d(y_{2n-1}, y_{2n})}{2}, d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \frac{d(y_{2n-1}, y_{2n+1})}{q}\}. \end{aligned}$$

Clearly, there are infinitely many n such that at least one of the following four cases holds:

(1) $d(Tx_{2n}, Sx_{2n+1}) \leq \lambda \frac{d(y_{2n-1}, y_{2n})}{2} \leq \lambda d(Sx_{2n-1}, Tx_{2n})$;
(2) $d(Tx_{2n}, Sx_{2n+1}) \leq \lambda d(y_{2n-1}, y_{2n}) = \lambda d(Sx_{2n-1}, Tx_{2n})$;
(3) $d(Tx_{2n}, Sx_{2n+1}) \leq \lambda d(y_{2n}, y_{2n+1}) \Rightarrow d(Tx_{2n}, Sx_{2n+1}) = \theta \leq \lambda d(Sx_{2n-1}, Tx_{2n})$;
(4) $$\begin{aligned} d(Tx_{2n}, Sx_{2n+1}) &\leq \lambda \frac{d(y_{2n-1}, y_{2n+1})}{q} \leq \lambda \frac{d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})}{q} \\ &= \lambda \frac{d(Sx_{2n-1}, Tx_{2n}) + d(Tx_{2n}, Sx_{2n+1})}{q}, \end{aligned}$$

which implies that $(1 - \frac{\lambda}{q}) d(Tx_{2n}, Sx_{2n+1}) \leq \frac{\lambda}{q} d(Sx_{2n-1}, Tx_{2n})$, that is,

$$d(Tx_{2n}, Sx_{2n+1}) \leq \frac{\lambda}{q - \lambda} d(Sx_{2n-1}, Tx_{2n}) \leq \lambda d(Sx_{2n-1}, Tx_{2n}).$$

It follows from (1), (2), (3), (4) that

$$d(Tx_{2n}, Sx_{2n+1}) \leq \lambda d(Sx_{2n-1}, Tx_{2n}). \tag{2.2}$$

Similarly, if $(Sx_{2n+1}, Tx_{2n+2}) \in Q_0 \times P_0$, we have

$$d(Sx_{2n+1}, Tx_{2n+2}) = d(F_{2n+1}x_{2n}, F_{2n+2}x_{2n+1}) \leq \lambda d(Tx_{2n}, Sx_{2n+1}). \tag{2.3}$$

If $(Sx_{2n-1}, Tx_{2n}) \in Q_0 \times P_0$, we have

$$d(Sx_{2n-1}, Tx_{2n}) = d(F_{2n-1}x_{2n-2}, F_{2n}x_{2n-1}) \leq \lambda d(Tx_{2n-2}, Sx_{2n-1}). \tag{2.4}$$

Case 2. If $(Tx_{2n}, Sx_{2n+1}) \in P_0 \times Q_1$, then $Sx_{2n+1} \in Q_1$ and

$$d(Tx_{2n}, Sx_{2n+1}) + d(Sx_{2n+1}, y_{2n+1}) = d(Tx_{2n}, y_{2n+1}) \tag{2.5}$$

which in turns yields

$$d(Tx_{2n}, Sx_{2n+1}) \leq d(Tx_{2n}, y_{2n+1}) = d(y_{2n}, y_{2n+1}) \tag{2.6}$$

and hence

$$d(Tx_{2n}, Sx_{2n+1}) \leq d(y_{2n}, y_{2n+1}) = d(F_{2n+1}x_{2n}, F_{2n}x_{2n-1}). \tag{2.7}$$

Now, proceeding as in Case 1, we have that (2.2) holds.

If $(Sx_{2n+1}, Tx_{2n+2}) \in Q_1 \times P_0$, then $Tx_{2n} \in P_0$. We show that

$$d(Sx_{2n+1}, Tx_{2n+2}) \leq \lambda d(Tx_{2n}, Sx_{2n-1}). \tag{2.8}$$

Using (2.5), we get

$$\begin{aligned} d(Sx_{2n+1}, Tx_{2n+2}) &\leq d(Sx_{2n+1}, y_{2n+1}) + d(y_{2n+1}, Tx_{2n+2}) \\ &= d(Tx_{2n}, y_{2n+1}) - d(Tx_{2n}, Sx_{2n+1}) + d(y_{2n+1}, Tx_{2n+2}). \end{aligned} \tag{2.9}$$

By noting that $Tx_{2n+2}, Tx_{2n} \in P_0$, one can conclude that

$$\begin{aligned} d(y_{2n+1}, Tx_{2n+2}) &= d(y_{2n+1}, y_{2n+2}) = d(F_{2n+1}x_{2n}, F_{2n+2}x_{2n+1}) \\ &\leq \lambda d(Tx_{2n}, Sx_{2n+1}), \end{aligned} \tag{2.10}$$

and

$$d(Tx_{2n}, y_{2n+1}) = d(y_{2n}, y_{2n+1}) = d(F_{2n}x_{2n-1}, F_{2n+1}x_{2n}) \leq \lambda d(Sx_{2n-1}, Tx_{2n}), \tag{2.11}$$

in view of Case 1.

Thus,

$$\begin{aligned} d(Sx_{2n+1}, Tx_{2n+2}) &\leq \lambda d(Sx_{2n-1}, Tx_{2n}) - (1-\lambda) d(Tx_{2n}, Sx_{2n+1}) \\ &\leq \lambda d(Sx_{2n-1}, Tx_{2n}), \end{aligned}$$

and we have proved (2.8).

Case 3. If $(Tx_{2n}, Sx_{2n+1}) \in P_1 \times Q_0$, then $Sx_{2n-1} \in Q_0$. We show that

$$d(Tx_{2n}, Sx_{2n+1}) \leq \lambda d(Sx_{2n-1}, Tx_{2n-2}). \tag{2.12}$$

Since $Tx_{2n} \in P_1$, then

$$d(Sx_{2n-1}, Tx_{2n}) + d(Tx_{2n}, y_{2n}) = d(Sx_{2n-1}, y_{2n}) \tag{2.13}$$

From this, we get

$$\begin{aligned} d(Tx_{2n}, Sx_{2n+1}) &\leq d(Tx_{2n}, y_{2n}) + d(y_{2n}, Sx_{2n+1}) \\ &= d(Sx_{2n-1}, y_{2n}) - d(Sx_{2n-1}, Tx_{2n}) + d(y_{2n}, Sx_{2n+1}). \end{aligned} \tag{2.14}$$

By noting that $Sx_{2n+1}, Sx_{2n-1} \in Q_0$, one can conclude that

$$d(y_{2n}, Sx_{2n+1}) = d(y_{2n}, y_{2n+1}) = d(F_{2n+1}x_{2n}, F_{2n}x_{2n-1}) \leq \lambda d(Sx_{2n-1}, Tx_{2n}), \tag{2.15}$$

and

$$\begin{aligned} d(Sx_{2n-1}, y_{2n}) &= d(y_{2n-1}, y_{2n}) = d(F_{2n-1}x_{2n-2}, F_{2n}x_{2n-1}) \\ &\leq \lambda d(Sx_{2n-1}, Tx_{2n-2}), \end{aligned} \tag{2.16}$$

in view of Case 1. Thus,

$$\begin{aligned} d(Tx_{2n}, Sx_{2n+1}) &\leq \lambda d(Sx_{2n-1}, Tx_{2n-2}) - (1-\lambda) d(Sx_{2n-1}, Tx_{2n}) \\ &\leq \lambda d(Sx_{2n-1}, Tx_{2n-2}), \end{aligned}$$

and we have proved (2.12).

Similarly, if $(Sx_{2n+1}, Tx_{2n+2}) \in Q_0 \times P_1$, then $Tx_{2n+2} \in P_1$, and

$$d(Sx_{2n+1}, Tx_{2n+2}) + d(Tx_{2n+2}, y_{2n+2}) = d(Sx_{2n+1}, y_{2n+2}).$$

From this, we have

$$\begin{aligned} d(Sx_{2n+1}, Tx_{2n+2}) &\leq d(Sx_{2n+1}, y_{2n+2}) + d(y_{2n+2}, Tx_{2n+2}) \\ &\leq d(Sx_{2n+1}, y_{2n+2}) + d(Sx_{2n+1}, y_{2n+2}) - d(Sx_{2n+1}, Tx_{2n+2}) \\ &= 2d(Sx_{2n+1}, y_{2n+2}) - d(Sx_{2n+1}, Tx_{2n+2}), \end{aligned}$$

thus

$$d(Sx_{2n+1}, Tx_{2n+2}) \le d(Sx_{2n+1}, y_{2n+2}).$$

By noting that $Sx_{2n+1} \in Q_0$, one can conclude that

$$\begin{aligned} d(Sx_{2n+1}, Tx_{2n+2}) &\le d(Sx_{2n+1}, y_{2n+2}) = d(F_{2n+1}x_{2n}, F_{2n+2}x_{2n+1}) \\ &\le \lambda d(Tx_{2n}, Sx_{2n+1} \end{aligned} \tag{2.17}$$

in view of Case 1. Thus, in all the cases 1-3, there exists $w_{2n} \in \{d(Sx_{2n-1}, Tx_{2n}), d(Tx_{2n-2}, Sx_{2n-1})\}$ such that

$$d(Tx_{2n}, Sx_{2n+1}) \le \lambda w_{2n},$$

and there exists $w_{2n+1} \in \{d(Sx_{2n-1}, Tx_{2n}), d(Tx_{2n}, Sx_{2n+1})\}$ such that

$$d(Sx_{2n+1}, Tx_{2n+2}) \le \lambda w_{2n+1}.$$

Following the procedure of Assad and Kirk [4], it can be easily shown by induction that, for $n \ge 1$, there exists $w_2 \in \{d(Tx_0, Sx_1), d(Sx_1, Tx_2)\}$, such that

$$d(Tx_{2n}, Sx_{2n+1}) \le \lambda^{n-\frac{1}{2}} w_2 \text{ and } d(Sx_{2n+1}, Tx_{2n+2}) \le \lambda^n w_2. \tag{2.18}$$

From (2.18) and by the triangle inequality, for $n > m$ we have

$$\begin{aligned} d(Tx_{2n}, Sx_{2m+1}) &\le d(Tx_{2n}, Sx_{2n-1}) + d(Sx_{2n-1}, Tx_{2n-2}) \\ &\qquad + \cdots + d(Tx_{2m+2}, Sx_{2m+1}) \\ &\le (\lambda^m + \lambda^{m+\frac{1}{2}} + \cdots + \lambda^{n-1}) w_2 \\ &\le \frac{\lambda^m}{1-\sqrt{\lambda}} w_2 \to \theta, \text{ as } m \to \infty. \end{aligned}$$

From Remark 1.5 and Corollary 1.4 (1), we, obtain $d(Tx_{2n}, Sx_{2m+1}) \ll c$.

Thus, the sequence $\{Tx_0, Sx_1, Tx_2, Sx_3, \ldots, Sx_{2n-1}, Tx_{2n}, Sx_{2n-1}, \ldots\}$ is a Cauchy sequence and hence converges to some point z in C (say). Then as noted in [8], there exists at least one subsequence $\{Tx_{2n_k}\}$ or $\{Sx_{2n_k+1}\}$ which is contained in P_0 or Q_0 respectively having as limit point z. Furthermore, the subsequences $\{Tx_{2n_k}\}$ and $\{Sx_{2n_k+1}\}$ both converge to $z \in C$ as C is a closed subset of the complete cone metric space (X, d). We assume that there exists a subsequence $\{Tx_{2n_k}\} \subseteq P_0$ for each $k \in N$ and TC as well as SC are closed in X. Since $\{Tx_{2n_k}\}$ is Cauchy in TC, it converges to a point $z \in TC$. Let $w \in T^{-1}z$, then $Tw = z$. Similarly, $\{Sx_{2n_k+1}\}$ being a subsequence of Cauchy sequence $\{Tx_0, Sx_1, Tx_2, Sx_3, \ldots, Sx_{2n-1}, Tx_{2n}, Sx_{2n-1}, \ldots\}$ also converges to z as SC is closed.

Using (2.1), one can write

$$d(F_i w, z) \le d(F_i w, F_j x_{2n_k-1}) + d(F_j x_{2n_k-1}, z) \le \lambda u_{2n_k-1} + d(F_j x_{2n_k-1}, z),$$

where

$$\begin{aligned} u_{2n_k-1} \in &\Big\{ \frac{d(Tw, Sx_{2n_k-1})}{2}, d(Tw, F_i w), d(Sx_{2n_k-1}, F_j x_{2n_k-1}), \\ &\qquad \frac{d(Tw, F_j x_{2n_k-1}) + d(F_i w, Sx_{2n_k-1})}{q} \Big\} \\ = &\Big\{ \frac{d(z, Sx_{2n_k-1})}{2}, d(z, F_i w), d(Sx_{2n_k-1}, F_j x_{2n_k-1}), \\ &\qquad \frac{d(z, F_i x_{2n_k-1}) + d(F_j w, Sx_{2n_k-1})}{q} \Big\}, \end{aligned}$$

for any odd integer $i \in N$ and even integer $j \in N$.

Let $\theta \ll c$. Clearly at least one of the following four cases holds for infinitely many n.

(1) $d(F_iw, z) \leq \lambda \frac{d(z, Sx_{2n_k-1})}{2} + d(F_jx_{2n_k-1}, z) \ll \lambda \frac{c}{2\lambda} + \frac{c}{2} = c$;

(2) $d(F_iw, z) \leq \lambda d(z, F_iw) + d(F_jx_{2n_k-1}, z) \implies$

$$d(F_iw, z) \leq \frac{1}{1-\lambda} d(F_jx_{2n_k-1}, z) \ll \frac{1}{1-\lambda}(1-\lambda)c = c;$$

(3)
$$\begin{aligned} d(F_iw, z) &\leq \lambda d(Sx_{2n_k-1}, F_jx_{2n_k-1}) + d(F_jx_{2n_k-1}, z) \\ &\leq \lambda(d(Sx_{2n_k-1}, z) + d(z, F_jx_{2n_k-1})) + d(F_jx_{2n_k-1}, z) \\ &\leq (\lambda+1) d(F_jx_{2n_k-1}, z) + \lambda d(Sx_{2n_k-1}, z) \\ &\ll (\lambda+1)\frac{c}{2(\lambda+1)} + \lambda \frac{c}{2\lambda} = c; \end{aligned}$$

(4)
$$\begin{aligned} d(F_iw, z) &\leq \lambda \frac{d(z, F_jx_{2n_k-1}) + d(F_iw, Sx_{2n_k-1})}{q} + d(F_jx_{2n_k-1}, z) \\ &\leq \lambda \frac{d(z, F_jx_{2n_k-1}) + d(F_iw, z) + d(z, Sx_{2n_k-1})}{q} + d(F_jx_{2n_k-1}, z) \implies \end{aligned}$$

$$\begin{aligned} d(F_iw, z) &\leq \frac{q+\lambda}{q-\lambda} d(F_jx_{2n_k-1}, z) + \frac{\lambda}{q-\lambda} d(z, Sx_{2n_k-1}) \\ &\ll \frac{q+\lambda}{q-\lambda} \frac{c}{2\frac{q+\lambda}{q-\lambda}} + \frac{\lambda}{q-\lambda} \frac{c}{2\frac{\lambda}{q-\lambda}} = c \end{aligned}$$

In all the cases we obtain $d(F_iw, z) \ll c$ for each $c \in intP$. Using Corollary 1.4 (3) it follows that $d(F_iw, z) = \theta$ or $F_iw = z$. Thus, $F_iw = z = Tw$, that is z is a coincidence point of F_i, T for any odd integer $i \in N$.

Furthermore, since the Cauchy sequence $\{T_{x_{2n_k}}\}$ converges to $z \in C$ and $z = F_iw, z \in F_iC \cap C \subseteq SC$, there exists $v \in C$ such that $Sv = z$. Again using (2.1), we get

$$d(Sv, F_jv) = d(z, F_jv) = d(F_iw, F_jv) \leq \lambda u,$$

where

$$\begin{aligned} u &\in \left\{ \frac{d(Tw, Sv)}{2}, d(Tw, F_iw), d(Sv, F_jv), \frac{d(Tw, F_jv) + d(F_iw, Sv)}{q} \right\} \\ &= \left\{ \theta, \theta, d(Sv, F_jv), \frac{d(z, F_jv) + \theta}{q} \right\} = \left\{ \theta, d(Sv, F_jv), \frac{d(Sv, F_jv)}{q} \right\} \end{aligned}$$

for any odd integer $i \in N$ and even integer $j \in N$.

Hence, we get the following cases:

$$d(Sv, F_jv) \leq \lambda\theta = \theta, d(Sv, F_jv) \leq \lambda d(Sv, F_jv) \text{ and } d(Sv, F_jv) \leq \frac{\lambda}{q} d(Sv, F_jv).$$

Since $\frac{\lambda}{q} \leq \frac{\lambda}{2-\lambda} = \frac{\lambda}{1+(1-\lambda)} < \lambda$, using Remark 1.6 and Corollary 1.4 (3), it follows that $Sv = F_jv$, therefore, $Sv = z = F_jv$, that is z is a coincidence point of (F_j, S) for any even integer $j \in N$.

In the case F_iC and F_jC are closed in X, then $z \in F_iC \cap C \subseteq SC$ or $z \in F_jC \cap C \subseteq TC$. Analogous arguments establish (IV) and (V). If we assume that there exists a subsequence $\{Sx_{2n_k+1}\} \subseteq Q_0$ with TC as well as SC closed in X, then noting that $\{Sx_{2n_k+1}\}$ is a Cauchy sequence in SC, the foregoing arguments establish (IV) and (V).

Suppose now that (F_i, T) and (F_j, S) are weakly compatible pairs, then

$$z = F_iw = Tw \text{ implies that } F_iz = F_iTw = TF_iw = Tz$$

and

$$z = F_j v = Sv \text{ implies that } F_j z = F_j Sv = SF_j v = Sz.$$

Thus, from (2.1),

$$d(F_i z, z) = d(F_i z, F_j v) \leq \lambda u,$$

where

$$\begin{aligned} u \in & \left\{ \frac{d(Sv, Tz)}{2}, d(Tz, F_i z), d(Sv, F_j v), \frac{d(Tz, F_j v) + d(Sv, F_i z)}{q} \right\} \\ &= \left\{ \frac{d(z, F_i z)}{2}, d(z, F_i z), d(z, z), \frac{d(F_i z, z) + d(z, F_i z)}{q} \right\} \\ &= \left\{ \frac{d(z, F_i z)}{2}, d(z, F_i z), \theta, \frac{2d(z, F_i z)}{q} \right\}. \end{aligned}$$

Hence, we get the following cases:

$$d(F_i z, z) \leq \lambda \frac{d(z, F_i z)}{2}, d(F_i z, z) \leq \lambda d(z, F_i z),$$

$$d(F_i z, z) \leq \lambda \theta = \theta, \quad \text{and} \quad d(F_i z, z) \leq \frac{2\lambda d(z, F_i z)}{q}.$$

Since $\frac{2\lambda}{q} \leq \frac{2\lambda}{2-\lambda} = \frac{2\lambda}{1+(1-\lambda)} < 2\lambda < 1$, using Remark 1.6 and Corollary 1.4 (3), it follows that $F_i z = z$. Thus, $F_i z = z = Tz$.

Similarly, we can prove $F_j z = z = Sz$. Therefore $z = F_i z = F_j z = Sz = Tz$, that is, z is a common fixed point of $\{F_n\}_{n=1}^{\infty}, S$ and T.

The uniqueness of the common fixed point follows easily from (2.1). □

The following example shows that in general F_i, F_j, S and T satisfying the hypotheses of Theorem 2.3 need not have a common point of coincidence so justifying the two separate conclusions (IV) and (V).

2.4. Example. Let $E = C^1([0,1], R), P = \{\varphi \in E : \varphi(t) \geq 0, t \in [0,1]\}, X = [0, +\infty), C = [0,2]$ and $d : X \times X \to E$ defined by $d(x,y) = |x-y|\varphi$, where $\varphi \in P$ is a fixed function, e.g., $\varphi(t) = e^t$. Then (X,d) is a complete cone metric space with a non-normal cone having a nonempty interior. Define F_i, F_j, S and $T : C \to X$ as

$$F_i x = x + \frac{4}{5}, i = 2n-1, \ F_j x = x^2 + \frac{4}{5}, j = 2n, \ Tx = 5x \text{ and } Sx = 5x^2, \ x \in C.$$

Note that $\partial C = \{0, 2\}$. Clearly, for each $x \in C$ and $y \notin C$ there exists a point $z = 2 \in \partial C$ such that $d(x,z) + d(z,y) = d(x,y)$. Furthermore,

$$\begin{aligned} &SC \cap TC = [0,20] \cap [0,10] = [0,10] \supset \{0,2\} = \partial C, \\ &F_i C \cap C = \left[\tfrac{4}{5}, \tfrac{14}{5}\right] \cap [0,2] = \left[\tfrac{4}{5}, 2\right] \subset C, \\ &F_j C \cap C = \left[\tfrac{4}{5}, \tfrac{24}{5}\right] \cap [0,2] = \left[\tfrac{4}{5}, 2\right] \subset TC, \end{aligned}$$

and, $SC, TC, F_i C$ and $F_j C$ are closed in X.
Also,

$$\begin{aligned} &T0 = 0 \in \partial C \implies F_i 0 = \frac{4}{5} \in C, \ S0 = 0 \in \partial C \implies F_j 0 = \frac{4}{5} \in C. \\ &T(\tfrac{2}{5}) = 2 \in \partial C \implies F_i(\tfrac{2}{5}) = \tfrac{6}{5} \in C, \\ &S\Big(\sqrt{\tfrac{2}{5}}\Big) = 2 \in \partial C \implies F_j\Big(\sqrt{\tfrac{2}{5}}\Big) = \tfrac{6}{5} \in C. \end{aligned}$$

Moreover, for each $x, y \in C$,

$$d(F_i x, F_j y) = |x - y^2|\varphi = \tfrac{2}{5}(\tfrac{1}{2} d(Tx, Sy)),$$

that is (2.1) is satisfied with $\lambda = \frac{2}{5}$.

Evidentally, $1 = T\big(\frac{1}{5}\big) = F_i\big(\frac{1}{5}\big) \neq \frac{1}{5}$ and $1 = S\big(\frac{1}{\sqrt{5}}\big) = F_j\big(\frac{1}{\sqrt{5}}\big) \neq \frac{1}{\sqrt{5}}$. Notice that the two separate coincidence points are not common fixed points as $F_iT\big(\frac{1}{5}\big) \neq TF_i\big(\frac{1}{5}\big)$ and $SF_j\big(\frac{1}{\sqrt{5}}\big) \neq F_jS\big(\frac{1}{\sqrt{5}}\big)$, which shows the necessity of the weakly compatible property in Theorem 2.3.

Next, We furnish an illustrate example in support of our result. In doing so, we are essentially inspired by Imdad and Kumar [15].

2.5. Example. Let $E = C^1([0,1], R)$, $P = \{\varphi \in E : \varphi(t) \geq 0,\ t \in [0,1]\}$, $X = [1, +\infty)$, $C = [1,3]$ and $d : X \times X \to E$ defined by $d(x,y) = |x-y|\varphi$, where $\varphi \in P$ is a fixed function, e.g., $\varphi(t) = e^t$. Then (X,d) is a complete cone metric space with a non-normal cone having a nonempty interior. Define $F_i,\ F_j,\ S$ and $T : C \to X$ as

$$F_ix = \begin{cases} \frac{x^2-1+n}{n} & \text{if } 1 \leq x \leq 2, \\ \frac{n+1}{n} & \text{if } 2 < x \leq 3, \end{cases} i = 2n-1\,(n \geq 1) \quad Tx = \begin{cases} 4x^4 - 3 & \text{if } 1 \leq x \leq 2, \\ 13 & \text{if } 2 < x \leq 3, \end{cases}$$

$$F_jx = \begin{cases} \frac{x^3-1+n}{n} & \text{if } 1 \leq x \leq 2, \\ \frac{n+1}{n} & \text{if } 2 < x \leq 3, \end{cases} j = 2n\,(n \geq 1) \qquad Sx = \begin{cases} 4x^6 - 3 & \text{if } 1 \leq x \leq 2, \\ 13 & \text{if } 2 < x \leq 3. \end{cases}$$

Note that $\partial C = \{1,3\}$. Clearly, for each $x \in C$ and $y \notin C$ there exists a point $z = 3 \in \partial C$ such that $d(x,z) + d(z,y) = d(x,y)$. Further,

$$SC \cap TC = [1,253] \cap [1,61] = [1,61] \supset \{1,3\} = \partial C,$$

$$F_iC \cap C = \Big[1, \frac{n+3}{n}\Big] \cap [1,3] \subset SC \text{ and } F_jC \cap C = \Big[1, \frac{n+7}{n}\Big] \cap [1,3] \subset TC.$$

Also,

$$T1 = 1 \in \partial C \implies F_i1 = 1 \in C,\ S1 = 1 \in \partial C \implies F_j1 = 1 \in C.$$

$$T\bigg(\sqrt[4]{\frac{3}{2}}\bigg) = 3 \in \partial C \implies F_i\bigg(\sqrt[4]{\frac{3}{2}}\bigg) = \frac{\sqrt{\frac{3}{2}} - 1}{n} + 1 \in C,$$

$$S\bigg(\sqrt[6]{\frac{3}{2}}\bigg) = 3 \in \partial C \implies F_j\bigg(\sqrt[6]{\frac{3}{2}}\bigg) = \frac{\sqrt{\frac{3}{2}} - 1}{n} + 1 \in C.$$

Moreover, if $x \in [1,2]$ and $y \in [2,3]$, then

$$d(F_ix, F_jy) = \frac{1}{n}|x^2-2|\varphi = \frac{|x^4-4|}{n|x^2+2|}\varphi = \frac{4|x^4-4|/2}{2n|x^2+2|}\varphi = \frac{1}{2n(x^2+2)}\frac{d(Tx,Sy)}{2}.$$

Next, if $x, y \in (2,3]$, then

$$d(F_ix, F_jy) = 0 = \lambda\frac{d(Tx,Sy)}{2}.$$

Finally, if $x, y \in [1,2]$, then

$$\begin{aligned} d(F_ix, F_jy) &= \frac{1}{n}|x^2-y^3|\varphi = \frac{|x^4-y^6|}{n|x^2+y^3|}\varphi = \frac{4|x^4-y^6|/2}{2n|x^2+y^3|}\varphi \\ &= \frac{1}{2n(x^2+y^3)}\frac{d(Tx,Sy)}{2}. \end{aligned}$$

Therefore, condition (2.1) is satisfied if we choose $\lambda = \max\big\{\frac{1}{2n(x^2+2)}, \frac{1}{2n(x^2+y^3)}\big\} \in \big(0, \frac{1}{2}\big)$. Moreover 1 is a point of coincidence as $T1 = F_i1$ as well as $S1 = F_j1$, whereas both pairs (F_i, T) and (F_j, S) are weakly compatible as $TF_i1 = 1 = F_iT1$ and $SF_j1 = 1 = F_jS1$. Also, SC, TC, F_iC and F_jC are closed in X. Thus, all the conditions of

Theorem 2.3 are satisfied and 1 is the unique common fixed point of F_i, F_j, S and T. One may note that 1 is also a point of coincidence for both pairs (F_i, T) and (F_j, S).

2.6. Remark. Setting $F_i = F$ and $F_j = G$ for all $i = 2n-1, j = 2n (n \in N)$ in Theorem 2.3, we obtain the following result:

2.7. Corollary. [12] *Let (X, d) be a complete cone metric space, C a nonempty closed subset of X such that for each $x \in C$ and $y \notin C$ there exists a point $z \in \partial C$ such that*

$$d(x, z) + d(z, y) = d(x, y).$$

Suppose that $F, G, S, T : C \to X$ are such that (F, G) is a generalized (T, S)-contractive mapping pair of C into X, and

(I) $\partial C \subseteq SC \cap TC,\ FC \cap C \subseteq SC,\ GC \cap C \subseteq TC,$

(II) $Tx \in \partial C$ *implies that* $Fx \in C,\ Sx \in \partial C$ *implies that* $Gx \in C,$

(III) SC *and* TC *(or* FC *and* GC*) are closed in* X.

Then

(IV) (F, T) *has a point of coincidence,*

(V) (G, S) *has a point of coincidence.*

Moreover, if (F, T) and (G, S) are weakly compatible pairs, then F, G, S and T have a unique common fixed point.

2.8. Remark. Setting $F_i = F_j = f$ for all $i = 2n-1, j = 2n (n \in N)$ and $T = S = g$ in Theorem 2.3, one deduces Theorem 2.1 due to Radenovic and Rhoades [22].

Setting $F_i = F_j = f$ for all $i = 2n-1, j = 2n (n \in N)$ and $T = S = I_X$ in Theorem 2.3, we obtain the following result:

2.9. Corollary. *Let (X, d) be complete cone metric space, and C a nonempty closed subset of X such that for each $x \in C$ and $y \notin C$ there exists a point $z \in \partial C$ such that*

$$d(x, z) + d(z, y) = d(x, y).$$

Suppose that $f : C \to X$ satisfies the condition

$$d(fx, fy) \leq \lambda u(x, y),$$

where

$$u(x, y) \in \Big\{\frac{d(x, y)}{2}, d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{q}\Big\}$$

for all $x, y \in C, 0 < \lambda < \frac{1}{2}, q \geq 2 - \lambda$, and f has the additional property that for each $x \in \partial C$, $fx \in C$. Then f has a unique fixed point.

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