Hacettepe Journal of Mathematics and Statistics
Volume 41 (5) (2012), 615 – 625

MEANS OF FP-SOFT SETS AND THEIR APPLICATIONS

Naim Çağman*† and Irfan Deli‡

Received 08 : 06 : 2011 : Accepted 19 : 06 : 2012

Abstract

Soft set theory was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with problems that contain uncertainties. In this paper, we first give most of the fundamental definitions of FP-soft set theory. We secondly define means of FP-soft sets and study their related properties. We then define decision making methods on FP-soft set theory. We finally apply the method successfully to problems that contain fuzzy objects.

Keywords: Soft sets, Fuzzy sets, FP-soft sets, *AND*-means, *OR*-means, Decision making.

2000 AMS Classification: 91 A 35, 62 C 86, 03 E 72.

1. Introduction

In 1999, the concept of soft sets was introduced by Molodtsov [22] for modeling problems that contain vagueness and uncertainty. After Molodtsov, Maji *et al.* [20] gave the operations of soft sets and their properties. Since then, based on these operations, soft set theory has developed in many directions and found its applications in a wide variety of fields. For instance; on the theory of soft sets [2, 4, 5, 9, 17, 20, 21, 25], on soft decision making [13, 14, 15, 18, 19, 24], on fuzzy soft sets [7, 10, 11], soft rough sets [13] are some of the selected works. Some authors, such as [1, 3, 6, 16, 23, 26, 27] have also studied the algebraic properties of soft sets.

The present expository paper is a condensation of part of the dissertation [12]. The presentation of the rest of the paper is organized as follows. In the next section, most of the fundamental definitions of the operations of fuzzy sets and soft sets are presented. In Section 3, we give FP-soft set-sets and their operations, which are more functional, to make theoretical studies of soft set theory in greater detail and improve several results.

*Department of Mathematics, Faculty of Arts and Sciences, Gaziosmanpaşa University, 60250 Tokat, Turkey. E-mail: naim.cagman@gop.edu.tr

†Corresponding Author.

‡Department of Mathematics, Faculty of Arts and Sciences, University of 7 Aralık, 79000 Kilis, Turkey. E-mail: irfandeli@kilis.edu.tr

In Section 4, we define means of FP-soft sets and their properties. In Section 5, we define soft fuzzification operators for And means and Or means of two FP-soft sets. In Section 6, we construct a FP-soft set-decision making method. We also give an application which shows that these methods work successfully. In the final section, some concluding comments are presented.

2. Preliminary

In this section, we present the basic definitions and results of soft set theory [22] and fuzzy set theory [28] that are useful for subsequent discussions. Detailed explanations related to soft sets and fuzzy sets can be found in [4, 20, 22] and [28, 29], respectively.

2.1. Definition. [22] Let U be an initial universe, $P(U)$ the power set of U, E a set of parameters and $A \subseteq E$. Then, a soft set F_A over U is defined as follows:

$$F_A = \{(x, f_A(x)) : x \in E\},$$

where $f_A : E \to P(U)$ is such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called the *approximate function* of the soft set F_A, and the value $f_A(x)$ is a set called the *x-element* of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary.

2.2. Example. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ a set of parameters. If $A = \{x_2, x_3, x_4\}$ and $f_A(x_2) = \{u_2, u_4\}$, $f_A(x_3) = \emptyset$, $f_A(x_4) = U$, then the soft-set F_A is written as

$$F_A = \{(x_2, \{u_2, u_4\}), (x_4, U)\},$$

F_A	x_1	x_2	x_3	x_4	x_5
u_1	0	0	0	1	0
u_2	0	1	0	1	0
u_3	0	0	0	1	0
u_4	0	1	0	1	0
u_5	0	0	0	1	0
u_6	0	0	0	0	0

or

F_A	x_2	x_4
u_1	0	1
u_2	1	1
u_3	0	1
u_4	1	1
u_5	0	1

2.3. Definition. [28] Let U be a universe. Then a fuzzy set X over U is a function defined as follows:

$$X = \{(\mu_X(u)/u) : u \in U\},$$

where $\mu_X : U \to [0, 1]$.

Here, μ_X is called the *membership function of* X, and the value $\mu_X(u)$ is called the *grade of membership of* $u \in U$. This value represents the *degree of* u *belonging to the fuzzy set* X.

3. FP-soft sets

In this section, we recall FP-soft sets and their operations [8]. The basic ideas of FP-soft set theory and its extensions, as well as its applications, can be found in [8].

In section 2, subsets of E are classical sets, denoted by the letters $A, B, C, \ldots$, but in this section the subsets of E will be fuzzy and denoted by the letters $X, Y, Z, \ldots$ to avoid confusion and complexity of the symbols.

3.1. Definition. [8] Let U be an initial universe, $P(U)$ be the power set of U, E a set of parameters and X a fuzzy set over E. Then a *FP-soft set* (f_X, E) *on the universe* U is defined as follows:

$$(f_X, E) = \{(\mu_X(x)/x, f_X(x)) : x \in E\},$$

where $\mu_X : E \to [0.1]$ and $f_X : E \to P(U)$ are such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$.

Here f_X is called the *approximate function* and μ_X the *membership function* of the FP-soft set.

Note that the set of all FP-soft sets over U will be denoted by $FPS(U)$.

3.2. Definition. [8] Let $F_X \in FPS(U)$. If $\mu_X(x) = 0$ for all $x \in E$, then F_X is called the *empty FP-soft set*, denoted by $F_\emptyset$.

3.3. Definition. [8] Let $F_X \in FPS(U)$. If $\mu_X(x) = 1$ and $f_X(x) = U$ for all $x \in X$, then F_X is called the X*-universal FP-soft set*, denoted by $F_{\tilde{X}}$.

If $X = E$, then the X-universal FP-soft set is called the *universal FP-soft set*, denoted by $F_{\tilde{E}}$.

3.4. Example. [8] Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ a set of parameters.

If $X = \{0.2/x_2, 0.5/x_3, 1/x_4\}$ and $f_X(x_2) = \{u_2, u_4\}$, $f_X(x_3) = \emptyset$, $f_X(x_4) = U$, then the FP-soft set F_X is written as

$$F_X = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \emptyset), (1/x_4, U)\}.$$

If $Y = \emptyset$, then the FP-soft set F_Y is the empty soft set. That is, $F_Y = F_\emptyset$.

If $Z = \{1/x_1, 1/x_2\}$ and $f_Z(x_1) = U$, $f_Z(x_2) = U$, then the FP-soft set F_Z is the Z-universal FP-soft set, that is, $F_Z = F_{\tilde{Z}}$.

If $X = E$, and $f_X(x_i) = U$ for all $x_i \in E$, $i = 1, 2, 3, 4$, then the FP-soft set F_X is the universal FP-soft set, that is, $F_X = F_{\tilde{E}}$.

3.5. Definition. [8] Let $F_X, F_Y \in FPS(U)$. Then F_X is a *FP-soft set-subset of* F_Y, denoted by $F_X \tilde{\subseteq} F_Y$, if $\mu_X(x) \le \mu_Y(x)$ and $f_X(x) \subseteq f_Y(x)$ for all $x \in E$.

3.6. Remark. [8] $F_X \tilde{\subseteq} F_Y$ does not imply that every element of F_X is an element of F_Y as in the definition of a classical subset.

For example, assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ the set of parameters. If $X = \{0.5/x_1\}$ and $Y = \{0.9/x_1, 0.1/x_3\}$, and $F_X = \{(0.5/x_1, \{u_2, u_4\})\}$, $F_Y = \{(0.9/x_1, \{u_2, u_3, u_4\}), (0.1/x_3, \{u_1, u_5\})\}$, then for all $x \in E$, $\mu_X(x) \le \mu_Y(x)$ and $f_X(x) \subseteq f_Y(x)$ is valid. Hence $F_X \tilde{\subseteq} F_Y$. It is clear that $(0.5/x_1, \{u_2, u_4\}) \in F_X$ but $(0.5/x_1, \{u_2, u_4\}) \notin F_Y$.

3.7. Definition. [8] Let $F_X, F_Y \in FPS(U)$. Then F_X and F_Y are *FP-soft set-equal*, written as $F_X = F_Y$, if and only if $\mu_X(x) = \mu_Y(x)$ and $f_X(x) = f_Y(x)$ for all $x \in E$.

3.8. Definition. [8] Let $F_X \in FPS(U)$. Then the *complement* of F_X, denoted by F_X^c, is the FP-soft set defined by the approximate and membership functions

$$\mu_{X^c}(x) = 1 - \mu_X(x) \text{ and } f_{X^c}(x) = U \setminus f_X(x),$$

respectively.

3.9. Definition. [8] Let $F_X, F_Y \in FPS(U)$. Then the *union of* F_X *and* F_Y, denoted by $F_X \tilde{\cup} F_Y$, is defined by

$$\mu_{X \tilde{\cup} Y}(x) = \max\{\mu_X(x), \mu_Y(x)\} \text{ and } f_{X \tilde{\cup} Y}(x) = f_X(x) \cup f_Y(x), \text{ for all } x \in E$$

3.10. Definition. [8] Let $F_X, F_Y \in FPS(U)$. Then the *intersection of* F_X *and* F_Y, denoted by $F_X \tilde{\cap} F_Y$, is the FP-soft set defined by the approximate and membership functions

$$\mu_{X\tilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\} \text{ and } f_{X\tilde{\cap}Y}(x) = f_X(x) \cap f_Y(x),$$

respectively.

3.11. Remark. [8] Let $F_X \in FPS(U)$. If $F_X \neq F_\emptyset$ or $F_X \neq F_{\tilde{E}}$, then $F_X \tilde{\cup} F_X^c \neq F_{\tilde{E}}$ and $F_X \tilde{\cap} F_X^c \neq F_\emptyset$.

4. Means of FP-soft sets

In this section, we define logical means, that is the AND-means and OR-means of two FP-soft sets. We then study some of their desired properties.

4.1. Definition. Let $F_{X_1}, F_{X_2} \in FPS(U)$. Then for ($p \in \mathbb{R} - \{0\}$), the AND-mean of F_{X_1} and F_{X_2}, denoted by $(F_{X_1} \widehat{\wedge} F_{X_2})$, is the FP-soft set defined by the approximate and membership functions,

$$\mu_{F_{X_1}\widehat{\wedge}F_{X_2}} : E \to [0,1], \quad \mu_{F_{X_1}\widehat{\wedge}F_{X_2}}(a) = \left(\frac{1}{2}\sum_{i=1}^{2} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},$$

and

$$f_{X_1\widehat{\wedge}X_2} : E \to P(U), \quad f_{X_1\widehat{\wedge}X_2}(a) = f_{X_1}(a) \cap f_{X_2}(a),$$

respectively.

4.2. Definition. Let $F_{X_1}, F_{X_2} \in FPS(U)$. Then for $p \in \mathbb{R} - \{0\}$ the *OR-mean of* F_{X_1} *and* F_{X_2}, denoted by $(F_{X_1} \widehat{\vee} F_{X_1})$, is the FP-soft set defined by the approximate and membership functions

$$\mu_{F_{X_1}\widehat{\vee}F_{X_2}} : E \to [0,1], \quad \mu_{F_{X_1}\widehat{\vee}F_{X_2}}(a) = \left(\frac{1}{2}\sum_{i=1}^{2} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},$$

$$f_{X_1\widehat{\vee}X_2} : E \to P(U), \quad f_{X_1\widehat{\vee}X_2}(a) = f_{X_1}(a) \cup f_{X_2}(a),$$

respectively.

Note that for $F_{X_1}, F_{X_2}, \ldots, F_{X_n} \in \mu SS(U)$ the OR-mean of F_{X_1}, $F_{X_2}, \ldots, F_{X_n}$, denoted by $(F_{X_1} \widehat{\vee} F_{X_2} \widehat{\vee} \cdots \widehat{\vee} F_{X_n})$, is a FP-soft set defined by the approximate and membership functions

$$\mu_{F_{X_1}\widehat{\vee}F_{X_2}\widehat{\vee}\cdots\widehat{\vee}F_{X_n}} : E \to [0,1], \quad \mu_{F_{X_1}\widehat{\vee}F_{X_2}\widehat{\vee}\cdots\widehat{\vee}F_{X_n}}(a) = \left(\frac{1}{n}\sum_{i=1}^{n} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},$$

$$f_{F_{X_1}\widehat{\vee}F_{X_2}\widehat{\vee}\cdots\widehat{\vee}F_{X_n}} : E \to P(U), \quad f_{X_1\widehat{\vee}F_{X_2}\widehat{\vee}\cdots\widehat{\vee}F_{X_n}}(a) = f_{X_1}(a) \cup \cdots \cup f_{X_n}(a),$$

respectively. Similarly, $F_{X_1} \widehat{\wedge} F_{X_2} \widehat{\wedge} \cdots \widehat{\wedge} F_{X_2}$ can easily be defined.

Note that the symbols $\widehat{\wedge}$ and $\widehat{\vee}$ used in the subscripts of the approximate and membership functions are not operations on fuzzy sets. They indicate that $f_{X_1\widehat{\wedge}X_2}$ and $f_{X_1\widehat{\vee}X_2}$ are the approximate functions of $F_{X_1} \widehat{\wedge} F_{X_2}$ and $F_{X_1} \widehat{\vee} F_{X_2}$, respectively, and that $\mu_{X_1\widehat{\wedge}X_2}$ and $\mu_{X_1\widehat{\vee}X_2}$ are the membership functions of $F_{X_1} \widehat{\wedge} F_{X_2}$ and $F_{X_1} \widehat{\vee} F_{X_2}$, respectively.

4.3. Example. Let p=1. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ is a universal set and $E = \{a_1, a_2, a_3, a_4, a_5\}$ a set of parameters. If $X_1 = \{0.5/a_2, 0.2/a_3, 1/a_4\}$ and

$X_2 = \{0.9/a_1, 0.3/a_3, 0.4/a_4, 0.8/a_5\}$ are two fuzzy sets over E, then we can write the following FP-soft sets,

$$F_{X_1} = \big\{(0.5/a_2, \{u_2, u_3, u_4, u_5, u_7\}), (0.2/a_3, \{u_1, u_2, u_3, u_4\}), (1/a_4, \{u_1, u_2, u_5, u_7, u_8\})\big\}$$

F_{X_1}	$0.25/a_2$	$0.25/a_3$	$0.70/a_4$
u_1	0	1	1
u_2	1	1	1
u_3	1	1	0
u_4	1	1	0
u_5	0	0	1
u_6	0	0	0
u_7	1	0	1
u_8	0	0	1

$$F_{X_2} = \big\{(0.9/a_1, \{u_1, u_2, u_5, u_6\}), (0.3/a_3, \{u_3, u_4, u_5, u_8\}), (0.4/a_4, U), (0.8/a_5, \{u_3, u_4, u_6, u_7, u_8\})\big\},$$

F_{X_2}	$0.9/a_1$	$0.3/a_3$	$0.4/a_4$	$0.8/a_5$
u_1	1	0	1	0
u_2	1	0	1	0
u_3	0	1	1	1
u_4	0	1	1	1
u_5	1	1	1	0
u_6	1	0	1	1
u_7	0	0	1	1
u_8	0	1	1	1

$$F_{X_1} \widehat{\vee} F_{X_2} = \big\{(0.45/a_1, \{u_1, u_2, u_5, u_6\}), (0.25/a_2, \{u_2, u_3, u_4, u_5, u_7\}), (0.25/a_3, \{u_1, u_2, u_3, u_4, u_5, u_8\}), (0.70/a_4, U), (0.4/a_5, \{u_3, u_4, u_6, u_7, u_8\})\big\}$$

$F_{X_1} \widehat{\vee} F_{X_2}$	$0.25/a_2$	$0.25/a_3$	$0.70/a_4$	$0.4/a_5$	
u_1	1	0	1	1	0
u_2	1	1	1	1	0
u_3	0	1	1	1	1
u_4	0	1	1	1	1
u_5	1	1	1	1	0
u_6	1	0	0	1	1
u_7	0	1	0	1	1
u_8	0	0	1	1	1

Similarly $F_{X_1} \widehat{\wedge} F_{X_2}$ easily can be found.

4.4. Proposition. *Let* $F_{X_1}, F_{X_2} \in FPS(U)$. *Then*

i. $(F^c_{X_1})^c = F_{X_1}$,
ii. $F^c_\emptyset = F_{\tilde{E}}$.

Proof. For all $a \in E$, since $\mu_{X_1^c}(a) = 1 - \mu_{X_1}(a)$ and $f_{X_1{}^c}(a) = U \setminus f_{X_1}(a)$;
i.

$$\mu_{(X_1{}^c)^c}(a) = 1 - \mu_{X_1{}^c}(a) = 1 - (1 - \mu_{X_1}(a)) = \mu_{X_1}(a)$$

and

$$f_{(X_1{}^c)^c}(a) = U \setminus (U \setminus f_{X_1}(a)) = f_{X_1}(a).$$

Therefore we get

$$(F^c_{X_1})^c = F_{X_1}.$$

ii. $F^c_\emptyset = F_{\tilde{E}}$ can be shown similarly. □

4.5. Proposition. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*
i. $F_{X_1} \widehat{\wedge} F_{X_2} = F_{X_1} \widehat{\wedge} F_{X_3} \iff F_{X_2} = F_{X_3}$,
ii. $F_{X_1} \widehat{\vee} F_{X_2} = F_{X_1} \widehat{\vee} F_{X_3} \iff F_{X_2} = F_{X_3}$.

Proof. For all $a \in E$,
i.

$$\begin{aligned} F_{X_1} \widehat{\wedge} F_{X_2} = F_{X_1} \widehat{\wedge} F_{X_3} &\Longrightarrow \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}} \\ &\qquad = \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_3}(a)^p\right)^{\frac{1}{p}} \\ &\Longrightarrow \mu_{X_2}(a)^p = \mu_{X_3}(a)^p \\ &\Longrightarrow \mu_{X_2}(a) = \mu_{X_3}(a) \end{aligned}$$

and

$$f_{X_1}(a) \cap f_{X_2}(a) = f_{X_1}(a) \cap f_{X_3}(a) \iff f_{X_2}(a) = f_{X_3}(a).$$

Hence we obtain

$$F_{X_1} \widehat{\wedge} F_{X_2} = F_{X_1} \widehat{\wedge} F_{X_3} \Longrightarrow F_{X_2} = F_{X_3}.$$

ii.

$$\begin{aligned} F_{X_1} \widehat{\vee} F_{X_2} = F_{X_1} \widehat{\vee} F_{X_3} &\Longrightarrow \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}} \\ &\qquad = \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_3}(a)^p\right)^{\frac{1}{p}} \\ &\Longrightarrow \mu_{X_2}(a)^p = \mu_{X_3}(a)^p \\ &\Longrightarrow \mu_{X_2}(a) = \mu_{X_3}(a) \end{aligned}$$

and

$$f_{X_1}(a) \cup f_{X_2}(a) = f_{X_1}(a) \cup f_{X_3}(a) \iff f_{X_2}(a) = f_{X_3}(a).$$

Therefore we obtain

$$F_{X_1} \widehat{\vee} F_{X_2} = F_{X_1} \widehat{\vee} F_{X_3} \iff F_{X_2} = F_{X_3}.$$

□

4.6. Proposition. *Let $F_X, F_Y, F_Z \in FPS(U)$. Then*
i. $F_{X_1} \widehat{\vee} F_{X_1} = F_{X_1}$,
ii. $F_{X_1} \widehat{\wedge} F_{X_1} = F_{X_1}$,

Proof. Follows easily from Definition 4.1 and Definition 4.2. □

4.7. Proposition. *Let $F_{X_1}, F_{X_2}, F_{X_3} \in FPS(U)$. Then*
i. $(F_{X_1} \widehat{\vee} F_{X_2}) \widehat{\vee} F_{X_3} = F_{X_1} \widehat{\vee} (F_{X_2} \widehat{\vee} F_{X_3})$,

ii. $(F_{X_1}\widehat{\wedge}F_{X_2})\widehat{\wedge}F_{X_3} = F_{X_1}\widehat{\wedge}(F_{X_2}\widehat{\wedge}F_{X_3})$.

Proof. Follows easily from Definition 4.1 and Definition 4.2. □

4.8. Proposition. *Let* $F_{X_1}, F_{X_2}, F_{X_3} \in FPS(U)$. *Then*

i. $F_{X_1}\widehat{\vee}(F_{X_2} \cap F_{X_3}) = (F_{X_1}\widehat{\vee}F_{X_2}) \cap (F_{X_1}\widehat{\vee}F_{X_3})$
ii. $F_{X_1}\widehat{\vee}(F_{X_2} \cup F_{X_3}) = (F_{X_1}\widehat{\vee}F_{X_2}) \cup (F_{X_1}\widehat{\vee}F_{X_3})$
iii. $F_{X_1}\widehat{\wedge}(F_{X_2} \cap F_{X_3}) = (F_{X_1}\widehat{\wedge}F_{X_2}) \cap (F_{X_1}\widehat{\wedge}F_{X_3})$
iv. $F_{X_1}\widehat{\wedge}(F_{X_2} \cup F_{X_3}) = (F_{X_1}\widehat{\wedge}F_{X_2}) \cup (F_{X_1}\widehat{\wedge}F_{X_3})$

Proof.

i. $$\begin{aligned} F_{X_1}\widehat{\vee}(F_{X_2} \cap F_{X_3}) &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \left(\min\left\{\mu_{X_2}(a), \mu_{X_3}(a)\right\}\right)^p\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \min\left\{\left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}}, \left(\mu_{X_1}(a)^p + \mu_{X_3}(a)^p\right)\right\}^{\frac{1}{p}} \end{aligned}$$

and

$$\begin{aligned} f_X(x) \cap (f_Y(y) \cap f_Z(z)) &= f_X(x) \cap f_Y(y) \cap f_Z(z) \\ &= (f_X(x) \cap f_Y(y)) \cap (f_X(x) \cap f_Z(z)) \end{aligned}$$

so we get $(F_{X_1}\widehat{\vee}F_{X_2}) \cap (F_{X_1}\widehat{\vee}F_{X_3})$.

ii. Similarly, $F_{X_1}\widehat{\vee}(F_{X_2} \cup F_{X_3}) = (F_{X_1}\widehat{\vee}F_{X_2}) \cup (F_{X_1}\widehat{\vee}F_{X_3})$ easily follows.

iii. $$\begin{aligned} F_{X_1}\widehat{\wedge}(F_{X_2} \cap F_{X_3}) &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \min\left\{\mu_{X_2}(a), \mu_{X_3}(a)^p\right\}\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \min\left\{\left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}}, \left(\mu_{X_1}(a)^p + \mu_{X_3}(a)^p\right)^{\frac{1}{p}}\right\} \end{aligned}$$

and

$$\begin{aligned} f_X(x) \cap (f_Y(y) \cap f_Z(z)) &= f_X(x) \cap f_Y(y) \cap f_Z(z) \\ &= (f_X(x) \cap f_Y(y)) \cap (f_X(x) \cap f_Z(z)), \end{aligned}$$

so we obtain

$$(F_{X_1}\widehat{\wedge}F_{X_2}) \cap (F_{X_1}\widehat{\wedge}F_{X_3}).$$

iv. Similarly, $F_{X_1}\widehat{\wedge}(F_{X_2} \cup F_{X_3}) = (F_{X_1}\widehat{\wedge}F_{X_2}) \cup (F_{X_1}\widehat{\wedge}F_{X_3})$ can easily be proved. □

4.9. Proposition. *Let* $F_{X_1}, F_{X_2}, F_{X_3} \in FPS(U)$. *Then*

i. $F_{X_1}\widehat{\vee}F_{X_2} = F_{X_2}\widehat{\vee}F_{X_1}$,
ii. $F_{X_1}\widehat{\wedge}F_{X_2} = F_{X_2}\widehat{\wedge}F_{X_1}$.

Proof. For all $a \in E$,

i. $$\begin{aligned} F_{X_1}\widehat{\vee}F_{X_2} &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_2}(a)^p + \mu_{X_1}(a)^p\right)^{\frac{1}{p}} \end{aligned}$$

and

$$f_{X_1}(a) \cap f_{X_2}(a) = f_{X_2}(a) \cap f_{X_1}(a).$$

Therefore, we obtain

$$F_{X_1}\widehat{\vee}F_{X_2} = F_{X_2}\widehat{\vee}F_{X_1}.$$

ii. $$\begin{aligned} F_{X_1}\widehat{\wedge}F_{X_2} &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_1}(a)^p + \mu_{X_2}(a)^p\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{2}\right)^{\frac{1}{p}} \left(\mu_{X_2}(a)^p + \mu_{X_1}(a)^p\right)^{\frac{1}{p}} \end{aligned}$$

and

$$f_{X_1}(a) \cup f_{X_2}(a) = f_{X_2}(a) \cup f_{X_1}(a),$$

therefore we get

$$F_{X_1} \widehat{\wedge} F_{X_2} = F_{X_2} \widehat{\wedge} F_{X_1}. \qquad \square$$

4.10. Proposition. *Let* $F_{X_1}, F_{X_2}, F_{X_3} \in FPS(U)$. *Then*

i. $F_{X_1} \widehat{\wedge} (F_{X_2} \widehat{\vee} F_{X_3}) = (F_{X_1} \widehat{\wedge} F_{X_2}) \widehat{\vee} (F_{X_1} \widehat{\wedge} F_{X_3})$,
ii. $F_{X_1} \widehat{\vee} (F_{X_2} \widehat{\wedge} F_{X_3}) = (F_{X_1} \widehat{\vee} F_{X_2}) \widehat{\wedge} (F_{X_1} \widehat{\vee} F_{X_3})$.

Proof. Follows easily from Definition 4.1 and Definition 4.2. □

5. Soft fuzzification operators

In this section, we give soft fuzzification operators by using the AND-means, OR-means that convert two FP-soft sets to a fuzzy set.

5.1. Definition. In this definition, for the shake of brevity, we use T instead of $F_{X_1 \widehat{\wedge}} F_{X_2}$. Let f_T be an approximation function and μ_T a membership function of T. Then, an *AND-soft fuzzification operator*, denoted by $s_{\widehat{\wedge}}$ is defined by

$$s_{\widehat{\wedge}}\mathrm{T} = \{\mu_{\widehat{\wedge}}(u)/u : u \in f_{\mathrm{T}}(a), \mu_{\widehat{\wedge}}(u) \in [0,1]\}, \ F_{X_1 \widehat{\wedge}} F_{X_2} = T,$$

where

$$\mu_{\widehat{\wedge}}(u) = \frac{1}{|U|} \sum_{i,j} \mu_{\mathrm{T}}(a_i) \chi_{\mathrm{T}(a_i)}(u_j)$$

and where

$$\chi_{\mathrm{T}(a_i)}(u_j) = \begin{cases} 1, & u_j \in f_{\mathrm{T}}(a_i), \\ 0, & u_j \notin f_{\mathrm{T}}(a_i). \end{cases}$$

The set $s_{\widehat{\wedge}}\mathrm{T}$ is a fuzzy set called an *OR-decision fuzzy set over* U.

Note that $s_{\widehat{\vee}}(F_{X_1} \widehat{\vee} F_{X_2})$ is defined in a similar way.

5.2. Proposition. *Let* $F_{X_1}, F_{X_2} \in FPS(U)$. *Then*

i. $s_{\widehat{\wedge}}(F_{X_1} \widehat{\wedge} F_{X_2}) = s_{\widehat{\wedge}}(F_{X_2} \widehat{\wedge} F_{X_1})$,
ii. $s_{\widehat{\vee}}(F_{X_1} \widehat{\vee} F_{X_2}) = s_{\widehat{\vee}}(F_{X_2} \widehat{\vee} F_{X_1})$.

Proof.

i. $F_{X_1} \widehat{\wedge} F_{X_2} = F_{X_2} \widehat{\wedge} F_{X_1} \implies s_{\widehat{\wedge}}(F_{X_1} \widehat{\wedge} F_{X_2}) = s_{\widehat{\wedge}}(F_{X_2} \widehat{\wedge} F_{X_1})$.
ii. $F_{X_1} \widehat{\vee} F_{X_2} = F_{X_2} \widehat{\vee} F_{X_1} \implies s_{\widehat{\vee}}(F_{X_1} \widehat{\vee} F_{X_2}) = s_{\widehat{\vee}}(F_{X_2} \widehat{\vee} F_{X_1})$. □

6. FP-soft set-decision making methods

In this section we construct FP-soft set-decision making methods. Assume that U is an initial universe that contains alternatives and that E is a set of parameters. We then construct an *OR-FP-soft set decision making method* by the following algorithm to produce a decision fuzzy set from U:

i. Choose feasible fuzzy subsets X_1 and X_2 over E;
ii. Construct the FP-soft sets F_{X_1} and F_{X_2} over U;
iii. Find the OR-means $F_{X_1} \widehat{\vee} F_{X_2}$;
iv. Compute the OR-decision fuzzy set $s_{\widehat{\vee}}(F_{X_1} \widehat{\vee} F_{X_2})$.

Note that, in a similar way, we can construct an *AND-FP-soft set-decision making method*. Now, we can give an example for the FP-soft set-decision making method.

6.1. Example. Assume that a country has a set of different places for industrial plants:

$$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}.$$

For $i = 1, 2, 3, 4, 5, 6, 7, 8$ the parameters u_i will stand for "Istanbul", "Denizli", "Ankara ", "Izmir", "Adana","Mersin", "Antalya" and "Bursa" respectively, which may be characterized by a set of parameters $E = \{a_1, a_2, a_3, a_4, a_5\}$. For $i = 1, 2, 3, 4, 5$ the parameters a_i stand for "transportation", "productive power", "marketing", "profit" and "sale" respectively.

Let p=1, and suppose that a company planning to build a plant in that country sends two administrators Mr X and Mr Y to the country to decide the best place to built the plant. If each member is to consider his own fuzzy set of parameters, then we can select a place for the plant on the basis of the sets of members' parameters by using the Or-means-FP-soft set-decision making method as follows.

Step i : Mr X and Mr Y construct fuzzy sets, say

$$X_1 = \{0.9/a_1, 0.1/a_2, 0.5/a_3, 0.9/a_5\}, \text{ and}$$
$$X_2 = \{0.2/a_1, 0.3/a_3, 0.7/a_4, 0.8/a_5\},$$

over E, respectively.

Step ii : F_{X_1} and F_{X_2} over U are written by

$$F_{X_1} = \big\{(0.9/a_2, \{u_3, u_4, u_5\}), (0.1/a_3, \{u_1, u_2, u_3, u_4\}), (0.5/a_4, \{u_5, u_7, u_8\}), (0.9/a_5, \{u_3, u_4, u_5\})\big\},$$
$$F_{X_2} = \big\{(0.2/a_1, \{u_1, u_2, u_5, u_6\}), (0.3/a_3, \{u_3, u_4, u_5, u_8\}), (0.7/a_4, \{u_5, u_8\}), (0.8/a_5, \{u_3, u_4, u_6, u_7, u_8\})\big\}.$$

Step iii : Calculate the OR-means of F_{X_1} and F_{X_2} as

$$F_{X_1} \widehat{\triangledown} F_{X_2} = \big\{(0.10/a_1, \{u_1, u_2, u_5, u_6\}), (0.45/a_2, \{u_3, u_4, u_5\}), (0.20/a_3, \{u_1, u_2, u_3, u_4, u_5, u_8\}), (0.60/a_4, \{u_5, u_7, u_8\}), (0.85/a_5, \{u_3, u_4, u_5, u_6, u_7, u_8\})\big\}$$

Step vi : The OR-decision fuzzy set $s_{\widehat{\triangledown}}(F_{X_1} \widehat{\triangledown} F_{X_2})$ is computed as

$$\{0.037/u_1, 0.037/u_2, 0.187/u_3, 0.187/u_4, 0.275/u_5, 0.118/u_6, 0.181/u_7, 0.206/u_8\},$$

where the alternative u_5 is the decision-maker's best choice since it has the largest degree of membership, 0.2750, among the others.

7. Conclusion

Molodtsov, in 1999, proposed soft set theory as a general mathematical tool for dealing with the problems that contain uncertainties. In this paper, after giving most of the fundamental definitions of FP-soft set theory needed to develop applications of soft set theory we defined means in FP-soft theory and studied their properties. We then defined decision making methods in FP-soft set theory. We also give an application which shows that they can be successfully applied to problems that contain uncertainties. In the future, these results can also be applied similarly using other theories such as fuzzy sets and rough sets.

Acknowledgement

The authors are grateful for financial support from the Research Fund of Gaziosmanpaşa University under grand no: 2009/47.

References

[1] Acar, U., Koyuncu, F. and Tanay, B. *Soft sets and soft rings*, Comput. Math. Appl. **59**, 3458–3463, 2010.

[2] Ali, M. I., Feng, F., Liu, X., Min, W. K. and Shabir, M. *On some new operations in soft set theory*, Comput. Math. Appl. **57**, 1547–1553, 2009.

[3] Aktaş, H. and Çağman, N. *Soft sets and soft groups*, Inform. Sci. **177**, 2726–2735, 2007.

[4] Çağman, N. and Enginoğlu, S. *Soft set theory and uni-int decision making*, Eur. J. Oper. Res. **207**, 848–855, 2010.

[5] Çağman, N. and Enginoğlu, S. *Soft matrix theory and its decision making*, Comput. Math. Appl. **59**, 3308–3314, 2010.

[6] Çağman, N., Çıtak, F. and Aktaş, H. *Soft int-group and its applications to group theory*, Neural Computing and Applications **21**, 151–158, 2012.

[7] Çağman, N. and Enginoğlu, S. *Fuzzy soft matrix theory and its applications in decision making*, Iranian Journal of Fuzzy Systems **9** (1), 109–119, 2012.

[8] Çağman, N. Çıtak, F. and Enginoğlu, S. *FP-soft set theory and its applications*, Annals of Fuzzy Mathematics and Informatics **2** (2), 219–226, 2011.

[9] Çağman, N., Karataş, S. and Enginoğlu, S. *Soft topology*, Computers and Mathematics with Applications **62**, 351–358, 2011.

[10] Çağman, N., Çıtak, F. and Enginoğlu, S. *Fuzzy parameterized fuzzy soft set theory and its applications*, Turkish Journal of Fuzzy Systems **1**, 21–35, 2010.

[11] Çağman, N. Enginoğlu, S. and Çıtak, F. *Fuzzy soft set theory and its applications*, Iran. J. Fuzzy Syst. **8** (3), 137–147, 2011.

[12] Deli, I. *Means of Fuzzy Parameterized Soft Sets and its Applications* (In Turkish) (Masters Thesis, Gaziosmanpasa University, Graduate School of Natural and Applied Science, 2010).

[13] Feng, F., Liu, X. Y., Leoreanu-Fotea, V. and Jun, Y. B. *Soft sets and soft rough sets*, Inform. Sci. **181** (6), 1125–1137, 2011.

[14] Feng, F., Jun, Y. B., Liu, X. and Li, L. *An adjustable approach to fuzzy soft set based decision making*, J. Comput. App. Math. **234**, 10–20, 2010.

[15] Feng, F., Li, L. and Çağman, N. *Generalized uni-int decision making schemes based on choice value soft sets*, European Journal of Operational Research **220**, 162–170, 2012.

[16] Jun, Y. B., Lee, K. J. and Khan, A. *Soft ordered semigroups*, Math. Logic Q. **56** (1), 42–50, 2010.

[17] Karaaslan, F., Cağman N. and Enginoğlu, S. *Soft lattices*, Journal of New Results in Science **1**, 5–17, 2012.

[18] Kovkov, D. V., Kolbanov, V. M. and Molodtsov, D. A. *Soft sets theory-based optimization*, J. Comput. Sys. Sc. Int. **46** (6), 872–880, 2007.

[19] Maji, P. K., Roy, A. R. and Biswas, R. *An application of soft sets in a decision making problem*, Comput. Math. Appl. **44**, 1077–1083, 2002.

[20] Maji, P. K., Biswas, R. and Roy, A. R. *Soft set theory*, Comput. Math. Appl. **45**, 555–562, 2003.

[21] Majumdar, P. and Samanta, S. K. *Generalised fuzzy soft sets*, Comput. Math. Appl. **59**, 1425–1432, 2010.

[22] Molodtsov, D. A. *Soft set theory-first results*, Comput. Math. Appl. **37**, 19–31, 1999.

[23] Park, C. H., Jun, Y. B. and Öztürk, M. A. *Soft WS-algebras*, Commun. Korean Math. Soc. **23** (3), 313–324, 2008.

[24] Roy, A. R. and Maji, P. K. *A fuzzy soft set theoretic approach to decision making problems*, J. Comput. App. Math. **203**, 412–418, 2007.

[25] Sezgin, A. and Atagün, A. O. *On operations of soft sets*, Comput. Math. Appl. **61** (5), 1457–1467, 2011.

[26] Sezgin, A. Atagün, A. O. and Çağman N. *Soft intersection near-rings with its applications*, Neural Computing and Applications **21**, 221–229, 2012.

[27] Sezgin, A., Atagün, A. O. and Çağman N. *Union soft substructures of near-rings and N-groups*, Neural Computing and Applications **21**, 133–143, 2012.

[28] Zadeh, L. A. *Fuzzy sets*, Inform. and Control **8**, 338–353, 1965.

[29] Zimmermann, H. J. *Fuzzy Set Theory and its Applications* (Kluwer Academic, 2th edition, Dordrecht, 1991).