

## A NEW APPROACH TO SOFT TOPOLOGY

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### Abstract

The aim of this work is to define a soft topology and an L-fuzzy soft topology with respect to a parameter set  $E$  using a new approach and to study the concept of soft compactness and L-fuzzy soft compactness.

**Keywords:** Soft set, L-fuzzy soft set, Soft topology, L-soft topology, L-fuzzifying soft topology, L-fuzzy soft topology, Soft compactness.

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### 1. Introduction

In 1999, Russian researcher Molodtsov [13] introduced the concept of soft set theory and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. Editing the original definition, by a soft set over  $X$  we mean a triple  $(M, E, X)$ , where  $E$  is a set of parameters and the mapping  $M : E \rightarrow 2^X$  is referred to as a soft structure on the set  $X$ . Soft set theory has a rich potential for applications in several directions. Maji *et al.*[11] presented some new definitions on soft sets. Pei *et al.* [17] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems.

In 2001, Maji *et al.* [10] expanded the soft set to fuzzy soft set theory. To continue the investigation on fuzzy soft sets, Ahmad and Kharal [1] presented some more properties of fuzzy soft sets and introduced the notion of a mapping on the class of fuzzy soft sets. Yang *et al.*[22] combined the interval-valued fuzzy set and soft set models and introduced the concept of interval-valued fuzzy soft set. Majumdar and Samanta [12] introduced the concept of generalized fuzzy soft sets.

Algebraic structures of fuzzy soft sets and soft sets have been studied by many authors. Aktaş and Çağman [2] gave the notion of soft groups. In 2009, Aygünöglu and Aygün [3] introduced fuzzy soft groups by using a t-norm. Feng [8] defined soft semirings and several related concepts. Varol *et al.* [14] studied fuzzy soft rings.

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Topological structures of soft set and fuzzy soft set have been studied by some authors in recent years. Shabir and Naz [20] gave the definition of soft topological spaces and studied soft neighborhoods of a point, soft separation axioms and their basic properties. At the same time, Aygünoğlu and Aygün [4] introduced soft topological spaces and soft continuity of soft mappings. They also investigated initial soft topologies and soft compactness. Fuzzy soft topology was studied by Varol and Aygün [15]. They showed that a fuzzy soft topological space gives a parametrized family of fuzzy topological spaces and studied fuzzy soft continuity. As a different approach to soft topology Varol *et al.* [16] interpreted categories related to categories of topological spaces as special categories of soft sets.

In the present paper we consider the soft interpretation of topological spaces. Firstly we give some basic concepts related to soft sets and L-fuzzy soft sets. We define soft topology and L-fuzzy soft topology, which are mappings from the parameter set  $E$  to  $2^{2^X}$  and from  $E$  to  $L^{L^X}$  respectively (where  $L$  is a fuzzy lattice). With respect to this idea, the soft topology  $\mathcal{T}$  is a soft set on  $2^X$  and we can say that this set is open according to the parameter set  $E$ . The L-fuzzy soft topology  $\mathcal{T}$  is an L-fuzzy soft set on the family of all L-fuzzy sets on  $X$ . Finally we present the concepts of soft compactness and L-fuzzy soft compactness for soft topological spaces and L-fuzzy soft topological spaces, respectively.

## 2. Preliminaries

Throughout this paper let  $X$  be a nonempty set referred to as the universe and let  $E$  be the set of all convenient parameters for the universe  $X$ .

**2.1. Definition.** [13] A pair  $(M, E)$  is called a *soft set over  $X$*  if  $M$  is a mapping from  $E$  into the set of all subsets of the set  $X$ , i.e.,  $M : E \rightarrow 2^X$ , where  $2^X$  is the power set of  $X$ . In what follows we denote a soft set  $(M, E)$  over  $X$  as a triple  $(M, E, X)$ . Sometimes the mapping  $M : E \rightarrow 2^X$  is referred to as a *soft structure* on the set  $X$ .

A soft set is a parameterized family of subsets of the set  $X$ . For  $e \in E$ ,  $M(e)$  can be considered as the set of  $e$ -approximate elements of the soft set  $(M, E)$ . According to this manner, we can view a soft set  $(M, E, X)$  as a consisting of collection of approximations:  $(M, E) = \{M(e) : e \in E\}$ .

**2.2. Example.** Let a soft set  $(M, E, X)$  describe the attractiveness of the skirts with respect to the parameters, which Mrs. A is going to wear. Suppose that there are four skirts in the universe  $X = \{x_1, x_2, x_3, x_4\}$  under consideration and  $E = \{e_1 = \text{cheap}, e_2 = \text{expensive}, e_3 = \text{colorful}\}$  is the set of parameters. To define a soft set means to point out cheap skirts, expensive skirts and colorful skirts.

Suppose that  $M(e_1) = \{x_1, x_2\}$ ,  $M(e_2) = \{x_3, x_4\}$ ,  $M(e_3) = \{x_1, x_3, x_4\}$ . Then the family  $\{M(e_i) : i = 1, 2, 3\}$  of  $2^X$  is a soft set  $(M, E, X)$ .

**2.3. Definition.** Given two soft structures  $M_1 : E \rightarrow 2^X$ ,  $M_2 : E \rightarrow 2^X$  over the set  $X$  we say that  $M_1$  *weaker* than  $M_2$  if  $M_1(e) \subseteq M_2(e)$  for every  $e \in E$ . We write in this case  $M_1 \preceq M_2$ .

**2.4. Definition.** For two soft sets  $(M, E, X)$  and  $(N, E, X)$ , we say that  $(M, E, X)$  is a *soft subset* of  $(N, E, X)$  and write  $(M, E, X) \sqsubseteq (N, E, X)$  if for each  $e \in E$ ,  $M(e) \subseteq N(e)$  [6].

$(M, E, X)$  is called a *soft super set* of  $(N, E, X)$  if  $(N, E, X)$  is a soft subset of  $(M, E, X)$ , and we write  $(M, E, X) \supseteq (N, E, X)$ .

**2.5. Definition.** [6] Two soft sets  $(M, E, X)$  and  $(N, E, X)$  are said to be *equal* if  $(M, E, X) \sqsubseteq (N, E, X)$  and  $(N, E, X) \sqsubseteq (M, E, X)$ .

**2.6. Definition.** [6] The *union* of two soft sets  $(M, E, X)$  and  $(N, E, X)$  is the soft set  $(K, E, X)$ , where  $K(e) = M(e) \cup N(e), \forall e \in E$ .

We write  $(M, E, X) \sqcup (N, E, X) = (K, E, X)$ .

**2.7. Definition.** [6] The *intersection* of two soft sets  $(M, E, X)$  and  $(N, E, X)$  is the soft set  $(K, E, X)$ , where  $K(e) = M(e) \cap N(e), \forall e \in E$ .

We write  $(M, E, X) \cap (N, E, X) = (K, E, X)$ .

**2.8. Definition.** [6] The *complement* of a soft set  $(M, E, X)$  is denoted by  $(M, E, X)^c$ , where  $M^c : E \rightarrow 2^X$  is the mapping given by  $M^c(e) = X \setminus M(e), \forall e \in E$ .

**2.9. Proposition.** [20] *Let  $(M, E, X)$  and  $(N, E, X)$  be soft sets. Then we have the following:*

- (1)  $((M, E, X) \sqcup (N, E, X))^c = (M, E, X)^c \cap (N, E, X)^c,$
- (2)  $((M, E, X) \cap (N, E, X))^c = (M, E, X)^c \sqcup (N, E, X)^c.$  □

**2.10. Definition.** [6] Let  $(M, E, X)$  be a soft set. If  $M(e) = \emptyset, \forall e \in E$ , then  $(M, E, X)$  is called the *null soft set* and is denoted by  $\tilde{\Phi}$ .

**2.11. Definition.** [6] Let  $(M, E, X)$  be a soft set. If  $M(e) = X, \forall e \in E$ , then  $(M, E, X)$  is called the *universal soft set* and is denoted by  $\tilde{E}$ .

**2.12. Theorem.** [16] [**Lattice of soft structures**]

*Let  $\mathcal{M}$  be the set of all soft structures on  $(E, X)$  equipped with the partial order  $\preceq$ .*

*Then  $(\mathcal{M}, \preceq)$  is a complete lattice where the supremum and the infimum of a family  $\{M_i \mid i \in I\}$  are defined respectively by  $\bigvee_{i \in I} M_i(e) = \bigcup_{i \in I} M_i(e)$  and  $\bigwedge_{i \in I} M_i(e) = \bigcap_{i \in I} M_i(e)$ . In particular the top and the bottom elements in the lattice  $\mathcal{M}$  are given respectively by  $\mathcal{M}_\top(e) = X \forall e \in E$  and  $\mathcal{M}_\perp(e) = \emptyset \forall e \in E$ . □*

To consider soft sets as a category we have to define morphisms between two soft sets.

**2.13. Definition.** [16] Let  $\psi : E \rightarrow F$  and  $\varphi : X \rightarrow Y$  be two functions. Then the pair  $(\psi, \varphi)$  is called a *soft mapping* from  $(M, E, X)$  to  $(N, F, Y)$ , denoted as  $(\psi, \varphi) : (M, E, X) \rightarrow (N, F, Y)$ , whenever  $\varphi^\rightarrow \circ M \geq N \circ \psi$ :

$$\begin{array}{ccc}
 E & \xrightarrow{M} & 2^X \\
 \psi \downarrow & & \downarrow \varphi^\rightarrow \\
 F & \xrightarrow{N} & 2^Y
 \end{array}$$

where  $\varphi^\rightarrow : 2^X \rightarrow 2^Y$  is the forward powerset operator (see e.g. [19]), that is  $\varphi^\rightarrow(A) := \varphi(A)$  for all  $A \in 2^X$ .

Since the componentwise composition of two soft functions  $(\psi, \varphi) : (M, E, X) \rightarrow (N, F, Y)$  and  $(\psi', \varphi') : (N, F, Y) \rightarrow (P, G, Z)$  is obviously a soft function

$$(\psi' \circ \psi, \varphi' \circ \varphi) : (M, E, X) \rightarrow (P, G, Z),$$

and the pair of identities  $(id_E, id_X) : (M, E, X) \rightarrow (M, E, X)$  is the identical morphism, soft sets and soft mappings form a category which will be denoted by **SOFSSET**.

Let  $(M, E, X)$  and  $(N, F, Y)$  be two soft sets and  $(\psi, \varphi)$  a soft function from  $(M, E, X)$  to  $(N, F, Y)$ . The image of  $(M, E, X)$  under the soft function  $(\psi, \varphi)$  is defined by

$(\psi, \varphi)(M, E, X) := (\varphi(M), \psi(E), Y)$ , where  $\psi(E)$  is the image of  $E$  in the category **SET** and  $\varphi(M)$  is defined by the Zadeh extension principle, that is

$$\varphi(M)(f) = \bigcup_{\psi(e)=f} \varphi(M(e)), \quad \forall f \in F,$$

see e.g. [4].

The pre-image of  $(N, F, Y)$  under the soft function  $(\psi, \varphi)$  is defined by

$$(\psi, \varphi)^{-1}(N, F, Y) := (\varphi^{\leftarrow} \circ N \circ \psi, \psi^{-1}(F), X),$$

where  $\psi^{-1}(F)$  is the preimage of  $F$  in the category of sets and the mapping  $\varphi^{\leftarrow}$  is the backward operator induced by the mapping  $\varphi : X \rightarrow Y$ , (see e.g. [19]), that is

$$(\varphi^{\leftarrow} \circ N \circ \psi)(e) = \varphi^{\leftarrow}(N(\psi(e))), \quad \forall e \in \psi^{-1}(F),$$

cf e.g. [4].

**2.14. Definition.** [16] For two soft sets  $(M, E, X)$  and  $(N, F, Y)$ , we say that  $(M, E, X)$  is a *soft subset* of  $(N, F, Y)$ , and write  $(M, E, X) \sqsubseteq (N, F, Y)$ , if

- (i)  $X \subseteq Y$ ,
- (ii)  $E \subseteq F$  and
- (iii) For each  $e \in E$ ,  $M(e) = N(e) \cap X$ .

Two soft sets  $(M, E, X)$  and  $(N, F, Y)$  are said to be *equal* if  $(M, E, X) \sqsubseteq (N, F, Y)$  and  $(N, F, Y) \sqsubseteq (M, E, X)$ .

**2.15. Corollary.** [16] *From Definition 2.14 and Theorem 2.12 it is clear that the soft subset  $(M, E, X)$  of a soft set  $(N, F, Y)$  can be characterized as the initial soft structure  $M : E \rightarrow 2^X$  for the mapping  $(\psi, \varphi) : (E, X) \rightarrow (N, F, Y)$  determined by the pair of inclusion functions  $\varphi : X \rightarrow Y$  and  $\psi : E \rightarrow F$  in the category **SOFSET**. Hence a soft subset  $(M, E, X)$  of a soft set  $(N, F, Y)$  is indeed its subobject in the category **SOFSET**.  $\square$*

**2.16. Definition.** [16] Given two soft sets  $(M, E, X)$  and  $(N, F, Y)$  we consider the triple  $(M \times N, E \times F, X \times Y)$  where the mapping  $M \times N : E \times F \rightarrow 2^{X \times Y}$  is defined by

$$(M \times N)(e, f) = M(e) \times N(f) \in 2^X \times 2^Y \subseteq 2^{X \times Y}.$$

One can easily see that the pairs of projections  $p_E : E \times F \rightarrow E$ ,  $q_X : X \times Y \rightarrow X$  and  $p_F : E \times F \rightarrow F$ ,  $q_Y : X \times Y \rightarrow Y$  determine morphisms

$$(p_E, q_X) : (M \times N, E \times F, X \times Y) \rightarrow (M, E, X)$$

and

$$(p_F, q_Y) : (M \times N, E \times F, X \times Y) \rightarrow (N, F, Y).$$

Further, let  $\mathcal{P}$  be the family of all soft structures  $P$  on  $(E \times F, X \times Y)$  for which

$$(p_E, q_X) : (P, E \times F, X \times Y) \rightarrow (M, E, X)$$

and

$$(p_F, q_Y) : (P, E \times F, X \times Y) \rightarrow (N, F, Y)$$

are morphisms in the category **SOFSET** and let  $P_0 := \bigwedge \mathcal{P}$ . Then  $(P_0, E \times F, X \times Y)$  is the *product* of the soft sets  $(M, E, X)$  and  $(N, F, Y)$  in the category **SOFSET**.

### 3. Soft topological spaces and compactness

**3.1. Soft Topological Spaces.** In this subsection, by a soft topology we mean a soft set on  $2^X$ .

**3.1. Definition.** A soft topology on a set  $X$  with respect to parameters  $E$  is a mapping  $\mathcal{T} : E \rightarrow 2^{2^X}$  such that for all  $e \in E$ ,  $\mathcal{T}(e) = \mathcal{T}_e \in 2^{2^X}$  is a classical topology on  $X$ .

The soft topology is denoted by  $\mathcal{T}(X, E)$ . The triple  $(X, \mathcal{T}, E)$  is called a *soft topological space*.

**3.2. Example. (1)** Let  $E = \{*\}$  and let  $(\mathcal{T}, \{*\})$  be a soft set on  $2^X$ , i.e.,  $\mathcal{T} : \{*\} \rightarrow 2^{2^X}$ . If  $\mathcal{T}(\{*\}) \subset 2^{2^X}$  is a topology on  $X$ , then  $\mathcal{T}$  is a soft topology on  $X$  with respect to  $\{*\}$ .

**(2)** Let  $E = \{0, 1\}$  and let  $(\mathcal{T}, \{0, 1\})$  be a soft set on  $2^X$ , i.e.,  $\mathcal{T} : \{0, 1\} \rightarrow 2^{2^X}$ . If  $\mathcal{T}(0)$  and  $\mathcal{T}(1)$  are topologies on  $X$ , then  $\mathcal{T}$  is a soft topology on  $X$  with respect to  $\{0, 1\}$ .

**(3)** Let  $E = [0, 1] = I$  and let  $(\mathcal{T}, I)$  be a soft set on  $2^X$ , i.e.,  $\mathcal{T} : I \rightarrow 2^{2^X}$ . If  $\mathcal{T}(\alpha)$ ,  $\forall \alpha \in I$  is a topology on  $X$ , then  $\mathcal{T}$  is a soft topology on  $X$  with respect to  $I$ .

**3.3. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological spaces and let  $(M, E, X)$  be a soft set.

$(M, E, X)$  is called an *open soft set* if for all  $e \in E$ ,  $M(e) \in \mathcal{T}(e)$ .

$(M, E, X)$  is called a *closed soft set* if for all  $e \in E$ ,  $X \setminus M(e) \in \mathcal{T}(e)$ .

**3.4. Definition.** A soft topology  $\mathcal{T}(X, E)$  is called *coarser* than a soft topology  $\mathcal{T}^*(X, E)$  if for all  $e \in E$ ,  $\mathcal{T}(e) \subset \mathcal{T}^*(e)$ .

**3.5. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $(M, E, X)$  a soft set.

- (1) The *closure* of  $(M, E, X)$  is a soft set  $\text{cl}(M, E, X)$  with the same of parameters, that is  $\text{cl}(M, E, X) = (\text{cl}M, E, X)$ , where  $\text{cl}M : E \rightarrow 2^X$  and  $\text{cl}M(e) = \text{cl}(M(e)) = \cap \{K \subset X : K \text{ is closed in } \mathcal{T}(e) \text{ and } M(e) \subset K\}$ .

Clearly,  $\text{cl}(M, E, X)$  is the smallest closed soft set which includes  $(M, E, X)$ .

- (2) The *interior* of  $(M, E, X)$  is  $\text{int}(M, E, X) = (\text{int}M, E, X)$  where  $\text{int}M : E \rightarrow 2^X$  and  $\text{int}M(e) = \text{int}(M(e)) = \cup \{G \subset X : G \text{ is open in } \mathcal{T}(e) \text{ and } G \subset M(e)\}$ .

Clearly,  $\text{int}(M, E, X)$  is the biggest open soft set which is a subset of  $(M, E, X)$ .

The following are results of (1) and (2):

$\text{cl}(M, E, X)$  is a closed soft set and  $\text{int}(M, E, X)$  is an open soft set on  $X$ , and

- (1)  $(M, E, X)$  is closed soft set iff  $\text{cl}(M, E, X) = (M, E, X)$ ,
- (2)  $(M, E, X)$  is open soft set iff  $\text{int}(M, E, X) = (M, E, X)$ .

**3.6. Theorem.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $(M, E, X)$ ,  $(N, E, X)$  two soft sets. Then the following hold:

- (1)  $\text{int}\tilde{\Phi} = \tilde{\Phi}$ ,  $\text{int}\tilde{E} = \tilde{E}$  and  $\text{cl}\tilde{\Phi} = \tilde{\Phi}$ ,  $\text{cl}\tilde{E} = \tilde{E}$ ;
- (2)  $\text{int}(M, E, X) \sqsubseteq (M, E, X) \sqsubseteq \text{cl}(M, E, X)$ ;
- (3)  $(M, E, X) \sqsubseteq (N, E, X)$  implies  $\text{int}(M, E, X) \sqsubseteq \text{int}(N, E, X)$  and  $\text{cl}(M, E, X) \sqsubseteq \text{cl}(N, E, X)$ ;
- (4)  $\text{int}(\text{int}(M, E, X)) = \text{int}(M, E, X)$  and  $\text{cl}(\text{cl}(M, E, X)) = \text{cl}(M, E, X)$ ;
- (5)  $\text{int}((M, E, X) \cap (N, E, X)) = \text{int}(M, E, X) \cap \text{int}(N, E, X)$  and  $\text{int}(M, E, X) \sqcup \text{int}(N, E, X) \sqsubseteq \text{int}((M, E, X) \sqcup (N, E, X))$ ;
- (6)  $\text{cl}((M, E, X) \sqcup (N, E, X)) = \text{cl}(M, E, X) \sqcup \text{cl}(N, E, X)$  and  $\text{cl}((M, E, X) \cap (N, E, X)) \sqsubseteq \text{cl}(M, E, X) \cap \text{cl}(N, E, X)$ ;
- (7)  $\text{cl}((M, E, X)^c) = (\text{int}(M, E, X))^c$  and  $\text{int}((M, E, X)^c) = (\text{cl}(M, E, X))^c$ .

*Proof.* Straightforward.  $\square$

**3.7. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $E_0 \subset E$ ,  $X_0 \subset X$ . Consider the mapping  $\mathcal{T}_0 : E_0 \rightarrow 2^{2^{X_0}}$ . The family  $\mathcal{T}_0(e') = \{X_0 \cap \mathcal{T}(e) : \mathcal{T}(e) \in 2^X\}$  for each  $e' \in E_0$ , is a topology on  $X_0$ .  $\mathcal{T}_0(X_0, E_0)$  is called the *soft subspace topology*.

$(\mathcal{T}_0(e'))$  is a subspace of  $\mathcal{T}(e)$ ,  $\forall e \in E$ .

**3.8. Definition.** Let  $(X, \mathcal{T}, E)$  and  $(Y, \mathcal{T}^*, F)$  be two soft topological spaces.  $(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  is called *soft continuous* if for all  $e \in E$  and  $f = \psi(e) \in F$ ,  $\varphi : (X, \mathcal{T}(e)) \rightarrow (Y, \mathcal{T}^*(f))$  is continuous.

(Here,  $\mathcal{T}(e)$  and  $\mathcal{T}^*(f)$  are topologies on  $X$  and  $Y$ , respectively).

$(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  is soft continuous iff  $(\varphi^{-1})^{-1}(\mathcal{T}(e)) \supseteq \mathcal{T}^*(\psi(e))$ :

$$\begin{array}{ccc} E & \xrightarrow{\mathcal{T}} & 2^{2^X} \\ \psi \downarrow & & \downarrow (\varphi^{-1})^{-1} \\ F & \xrightarrow{\mathcal{T}^*} & 2^{2^Y} \end{array}$$

**3.9. Proposition.** Let  $(X, \mathcal{T}_1, E_1)$ ,  $(Y, \mathcal{T}_2, E_2)$  and  $(Z, \mathcal{T}_3, E_3)$  be soft topological spaces. If  $(\psi_1, \varphi_1) : (X, \mathcal{T}_1, E_1) \rightarrow (Y, \mathcal{T}_2, E_2)$  and  $(\psi_2, \varphi_2) : (Y, \mathcal{T}_2, E_2) \rightarrow (Z, \mathcal{T}_3, E_3)$  are soft continuous functions, then the composition  $(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1) = (\psi_2 \circ \psi_1, \varphi_2 \circ \varphi_1)$  is also soft continuous.

*Proof.* Since  $(\psi_1, \varphi_1)$  is soft continuous, for each  $e_1 \in E_1$  and  $e_2 = \psi_1(e_1) \in E_2$ ,  $\varphi_1 : (X, \mathcal{T}_1(e_1)) \rightarrow (Y, \mathcal{T}_2(e_2))$  is continuous.

Since  $(\psi_2, \varphi_2)$  is soft continuous, for each  $e_2 \in E_2$  and  $e_3 = \psi_2(e_2) \in E_3$ ,  $\varphi_2 : (Y, \mathcal{T}_2(e_2)) \rightarrow (Z, \mathcal{T}_3(e_3))$  is continuous.

Hence,  $(\varphi_2 \circ \varphi_1)$  is continuous, then  $(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1)$  is soft continuous.  $\square$

**3.10. Theorem.** The following are equivalent to each other:

- (1)  $(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  is soft continuous.
- (2)  $(\psi, \varphi)^{-1}(\text{int}(N, F, Y)) \subseteq \text{int}((\psi, \varphi)^{-1}(N, F, Y))$  for each soft set  $(N, F, Y)$ .
- (3)  $\text{cl}((\psi, \varphi)^{-1}(N, F, Y)) \subseteq (\psi, \varphi)^{-1}(\text{cl}(N, F, Y))$  for each soft set  $(N, F, Y)$ .

*Proof.* Straightforward.  $\square$

**3.11. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $(\mathcal{B}, E)$  a soft set on  $2^X$ . If for each  $e \in E$  a subcollection  $\mathcal{B}(e)$  of  $\mathcal{T}(e)$  is a base for  $\mathcal{T}(e)$ , then  $\mathcal{B}(X, E)$  is called a *soft base* for  $\mathcal{T}(X, E)$ .

**3.12. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $(\mathcal{S}, E)$  a soft set on  $2^X$ . If for each  $e \in E$  a subcollection  $\mathcal{S}(e)$  of  $\mathcal{T}(e)$  is a subbase for  $\mathcal{T}(e)$ , then  $\mathcal{S}(X, E)$  is called *soft subbase* for  $\mathcal{T}(X, E)$ .

**3.13. Definition.** [Construction of the product]

Let  $(X, \mathcal{T}, E)$  and  $(Y, \mathcal{T}^*, F)$  be soft topological spaces. Consider the triple  $(X \times Y, \mathcal{T} \times \mathcal{T}^*, E \times F)$  where the mapping  $\mathcal{T} \times \mathcal{T}^* : E \times F \rightarrow 2^{2^{X \times Y}}$  is defined by

$$(\mathcal{T} \times \mathcal{T}^*)(e, f) = \mathcal{T}(e) \times \mathcal{T}^*(f) \in 2^{2^X} \times 2^{2^Y} \subseteq 2^{2^X \times 2^Y} \subseteq 2^{2^{X \times Y}}.$$

Here  $\mathcal{T}(e)$  and  $\mathcal{T}^*(f)$  are classical topologies on  $X$  and  $Y$ , respectively.

The pairs of projections  $p_E : E \times F \rightarrow E$ ,  $(q_X^\rightarrow)^\rightarrow : 2^{2^{X \times Y}} \rightarrow 2^{2^X}$  and  $p_F : E \times F \rightarrow F$ ,  $(q_Y^\rightarrow)^\rightarrow : 2^{2^{X \times Y}} \rightarrow 2^{2^Y}$  determine morphisms

$$(p_E, (q_X^\rightarrow)^\rightarrow) : (X \times Y, \mathcal{T} \times \mathcal{T}^*, E \times F) \rightarrow (X, \mathcal{T}, E)$$

and

$$(p_F, (q_Y^\rightarrow)^\rightarrow) : (X \times Y, \mathcal{T} \times \mathcal{T}^*, E \times F) \rightarrow (Y, \mathcal{T}^*, F).$$

**3.14. Definition.** Let  $\{(X_i, \mathcal{T}_i, E_i)\}_{i \in J}$  be a family of soft topological spaces. Then the *initial soft topology on  $X (= \prod_{i \in J} X_i)$*  generated by the family  $\{(p_{E_i}, q_{X_i})\}_{i \in J}$  is called the *product soft topology on  $X$* .

**3.2. Soft compactness.**

**3.15. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space. If for all  $e \in E$ ,  $(X, \mathcal{T}(e))$  is compact (nearly compact, almost compact), then  $(X, \mathcal{T}, E)$  is called a *soft compact (soft nearly compact, soft almost compact) space*.

It is easy to see that we have the following assertion:

$$\text{soft compactness} \implies \text{soft nearly compactness} \implies \text{soft almost compactness}$$

But the reverse assertion does not hold always.

**3.16. Example.** Let  $E = \{*\}$  and let  $(X, \mathcal{T}, E)$  be a soft topological space as, given in Example 3.2 (1). Here  $\mathcal{T}(*)$  is a topology on  $X$  and  $(X, \mathcal{T}(*))$  is a topological space. It is well known that in classical topological spaces the reverse assertion does not hold in general.

**3.17. Definition.** Let  $(X, \mathcal{T}, E)$  be a soft topological space and  $(M, E, X)$  a soft set.  $(M, E, X)$  is called a *soft compact set* if for all  $e \in E$ ,  $M(e)$  is a compact set on  $X$ .

**3.18. Theorem.** Let  $(X, \mathcal{T}, E)$ ,  $(Y, \mathcal{T}^*, F)$  be two soft topological spaces and  $(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  a soft continuous and onto mapping. If  $(X, \mathcal{T}, E)$  is soft compact (soft nearly compact, soft almost compact), then  $(Y, \mathcal{T}^*, F)$  is soft compact (soft nearly compact, soft almost compact).

*Proof.* Let  $(X, \mathcal{T}, E)$  be soft compact space. Then for all  $e \in E$ ,  $(X, \mathcal{T}(e))$  is compact.

Since  $(\psi, \varphi)$  is soft continuous, for all  $\psi(e) = f$ , we have  $\varphi : (X, \mathcal{T}(e)) \rightarrow (Y, \mathcal{T}^*(f))$  is continuous. Hence,  $(Y, \mathcal{T}^*(f))$  is compact. Consequently,  $(Y, \mathcal{T}^*, F)$  is soft compact.  $\square$

**3.19. Theorem.** Let  $(X, \mathcal{T}, E)$  be a soft compact space and  $(M, E, X)$  a closed soft set, then  $(M, E, X)$  is a soft compact set.

*Proof.* Straightforward.  $\square$

**3.20. Theorem.** Let  $\{(X_i, \mathcal{T}_i, E_i)\}_{i \in J}$  be a family of soft topological spaces. Then  $\{(X_i, \mathcal{T}_i, E_i)\}_{i \in J}$  is soft compact space if and only if the product  $(\prod_{i \in J} X_i, \prod_{i \in J} \mathcal{T}_i, \prod_{i \in J} E_i)$  of these soft topological spaces is a soft compact space.

*Proof.* Let  $S(X, E)$  be a subbase of the product soft topology, i.e., for each  $e \in E$ ,  $S(e) \subset \mathcal{T}(e)$  is subbase for  $\mathcal{T}(e)$ . By the Alexander Subbase Theorem, we know that every cover of  $X = \prod_{i \in J} X_i$  from  $S(e)$  has a finite subcover.

Conversely, since the projection mappings  $(p_{E_i}, (q_{X_i}^\rightarrow)^\rightarrow)_i : (\prod_{i \in J} X_i, \prod_{i \in J} \mathcal{T}_i, \prod_{i \in J} E_i) \rightarrow (X_i, \mathcal{T}_i, E_i)$  are soft continuous and by Theorem 3.18 we obtain that  $(X_i, \mathcal{T}_i, E_i)$  is soft compact.  $\square$

#### 4. L-fuzzy soft topological spaces and compactness

**4.1. L-fuzzy soft topological spaces.** In this subsection,  $L = L(\leq, \vee, \wedge, \prime)$  denotes a fuzzy lattice, i.e., a completely distributive complete lattice with an order-reversing involution  $\prime$ .  $1_X$  and  $0_X$  denotes the greatest and the least elements of  $L$ , respectively. We denote by  $L^X$  the set of all L-fuzzy sets on  $X$ .

**4.1. Definition.** [16] A triple  $(m, E, X)$  is called an *L-fuzzy soft set over  $X$*  if  $m$  is a mapping from  $E$  into  $L^X$ , i.e.,  $m : E \rightarrow L^X$ . That is, for each  $e \in E$ ,  $m(e) = m_e : E \rightarrow L$  is an L-fuzzy set on  $X$ . Sometimes the mapping  $m : E \rightarrow L^X$  is referred to as a *fuzzy soft structure over the pair  $(E, X)$* .

If we take  $L = I = [0, 1]$  and  $A \subset E$ , then  $(m, A, X)$  is an I-fuzzy soft set on  $X$  as defined by Maji *et.al.* [10].

**4.2. Definition.** [16] Given two fuzzy soft structures  $m_1, m_2$  over the pair  $(E, X)$  we say that  $m_1$  is weaker than  $m_2$  if  $m_1(e) \leq m_2(e)$  for every  $e \in E$ . We write in this case  $m_1 \preceq m_2$ .

**4.3. Definition.** Let  $(m, E, X)$  and  $(n, E, X)$  be L-fuzzy soft sets. Then  $(m, E, X)$  is called an *L-fuzzy soft subset of  $(n, E, X)$* , and we write  $(m, E, X) \sqsubseteq (n, E, X)$ , if for each  $e \in E$ ,  $m_e \leq n_e$ .

**4.4. Definition.** Two L-fuzzy soft sets  $(m, E, X)$  and  $(n, E, X)$  are called *equal* if  $(m, E, X) \sqsubseteq (n, E, X)$  and  $(n, E, X) \sqsubseteq (m, E, X)$ .

**4.5. Definition.** The *union* of two L-fuzzy soft sets  $(m, E, X)$  and  $(n, E, X)$  is the L-fuzzy soft set  $(k, E, X)$ , where  $k_e = m_e \vee n_e, \forall e \in E$ .

**4.6. Definition.** The *intersection* of two L-fuzzy soft sets  $(m, E, X)$  and  $(n, E, X)$  is the L-fuzzy soft set  $(k, E, X)$ , where  $k_e = m_e \wedge n_e, \forall e \in E$ .

**4.7. Definition.** The *complement* of an L-fuzzy soft set  $(m, E, X)$  is denoted by  $(m, E, X)^c$ , where  $m^c : E \rightarrow L^X$  is a mapping given by  $m_e^c = (m_e)^\prime, \forall e \in E$ .

**4.8. Definition.** Let  $(m, E, X)$  be an L-fuzzy soft set. If  $m_e = 0_X, \forall e \in E$ , then  $(m, E, X)$  is called the *null L-fuzzy soft set* and is denoted by  $\tilde{0}_E$ .

**4.9. Definition.** Let  $(m, E, X)$  be an L-fuzzy soft set. If  $m_e = 1_X, \forall e \in E$ , then  $(m, E, X)$  is called the *universal L-fuzzy soft set* and is denoted by  $\tilde{1}_E$ .

**4.10. Theorem.** [16] **[Lattice of fuzzy soft structures]**

Let  $\mathcal{M}$  be the set of all fuzzy soft structures on  $(E, X)$  equipped with the partial order  $\preceq$ .

Then  $(\mathcal{M}, \preceq)$  is a complete lattice where the supremum, the infimum and order-reversing involution of a family  $\{m_i \mid i \in I\}$  are defined respectively by  $\bigvee_{i \in I} m_i(e), \bigwedge_{i \in I} m_i(e)$  and  $m'_i(e) = (m_i(e))^\prime$ . In particular the top and the bottom elements in the lattice  $\mathcal{M}$  are given respectively by  $\mathcal{M}_\top(e) = 1_X \forall e \in E$  and  $\mathcal{M}_\perp(e) = 0_X \forall e \in E$ .

The definition of fuzzy soft mapping is similar to the definition of a soft mapping.

Let  $(m, E, X)$  and  $(n, F, Y)$  be two L-fuzzy soft sets and let  $(\psi, \varphi)$  be a fuzzy soft function from  $(m, E, X)$  to  $(n, F, Y)$ .

- (1) The *image* of  $(m, E, X)$  under the fuzzy soft function  $(\psi, \varphi)$  is the L-fuzzy soft set over  $Y$  defined by  $(\psi, \varphi)(m, E, X) = (\varphi(m), \psi(E), Y)$ , where  $\psi(E)$  is the image of  $E$  in the category **SET** and

$$\varphi(m)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\psi(e)=k} m_e(x), & \text{if } \varphi(x) = y; \\ 0, & \text{otherwise.} \end{cases} \quad \forall k \in \psi(E), \forall y \in Y.$$

- (2) The *pre-image* of  $(n, F, Y)$  under the fuzzy soft function  $(\psi, \varphi)$  is the L-fuzzy soft set over  $X$  defined by  $(\psi, \varphi)^{-1}(n, F, Y) = (\varphi^{-1}(n), \psi^{-1}(F), X)$ , where  $\psi^{-1}(F)$  is the pre-image of  $F$  in the category of sets and
- $$\varphi^{-1}(n)_f(x) = n_{\psi(f)}(\varphi(x)), \forall f \in \psi^{-1}(F), \forall x \in X.$$

[16] (for the case  $L = I$ , see [3])

If  $\varphi$  and  $\psi$  are injective (surjective), then  $(\psi, \varphi)$  is said to be injective (surjective).

If  $(\psi_1, \varphi_1)$  is a fuzzy soft mapping from  $X$  to  $Y$  and  $(\psi_2, \varphi_2)$  is a fuzzy soft mapping from  $Y$  to  $Z$  then the *composition* of  $(\psi_1, \varphi_1)$  and  $(\psi_2, \varphi_2)$  is denoted by  $[(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1)]$  and defined by

$$[(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1)] := (\psi_2 \circ \psi_1, \varphi_2 \circ \varphi_1).$$

**4.11. Definition.** An L-soft topology on a set  $X$  with respect to the parameters  $E$  is a mapping  $\mathcal{T} : E \rightarrow 2^{L^X}$  such that for all  $e \in E$ ,  $\mathcal{T}(e) \subset 2^{L^X}$ ,  $\mathcal{T}(e) : L^X \rightarrow 2$ , is an L-topology on  $X$ .

That is, for each  $e \in E$ ,  $\mathcal{T}(e)$  is an L-topology in the sense of Chang – Goguen ([7], [9]).

**4.12. Definition.** An L-fuzzifying soft topology on a set  $X$  with respect to the parameters  $E$  is a mapping  $\mathcal{T} : E \rightarrow L^{2^X}$  such that for all  $e \in E$ ,  $\mathcal{T}(e) \subset L^{2^X}$ ,  $\tau(e) : 2^X \rightarrow L$ , is an L-fuzzifying topology on  $X$ .

That is, for each  $e \in E$ ,  $\mathcal{T}(e)$  is an L-fuzzifying topology in Ying’s sense [23].

**4.13. Definition.** An L-fuzzy soft topology on a set  $X$  with respect to the parameters  $E$  is a mapping  $\mathcal{T} : E \rightarrow L^{L^X}$  such that for all  $e \in E$ ,  $\mathcal{T}(e) \subset L^{L^X}$ ,  $\mathcal{T}(e) : L^X \rightarrow L$ , is an L-fuzzy topology on  $X$ .

That is, for each  $e \in E$ ,  $\mathcal{T}(e) = \mathcal{T}_e$  is an L-fuzzy topology in Shostak’s sense [21].

The L-fuzzy soft topology is denoted by  $\mathcal{T}(X, E)$ . The triple  $(X, \mathcal{T}, E)$  is called an *L-fuzzy soft topological space*.

Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space and  $(m, E, X)$  an L-fuzzy soft set.  $\mathcal{T}_e(m(e))$  is called the *degree of openness* of the L-fuzzy set  $m(e)$ ,  $\forall e \in E$ .

**4.14. Example.** (1) Let  $(X, \mathcal{T}, E)$  be a soft topological space. If for each  $e \in E$ , we define  $\mathcal{T}(e) := \chi_{\mathcal{T}(e)} : 2^X \rightarrow 2$ , then we can consider  $(X, \mathcal{T}, E)$  as an L-fuzzy soft topological space.

(2) Let  $(X, \mathcal{T}, E)$  be an L-soft topological space. If for each  $e \in E$ , we define  $\mathcal{T}(e) := \chi_{\tau(e)} : L^X \rightarrow 2$ , then we can consider  $(X, \mathcal{T}, E)$  as an L-fuzzy soft topological space.

**4.15. Definition.** An L-fuzzy soft topology  $\mathcal{T}(X, E)$  is called *coarser* than an L-fuzzy soft topology  $\mathcal{T}^*(X, E)$  if for all  $e \in E$ ,  $\mathcal{T}^*(e) \geq \mathcal{T}(e)$ . (this means that for the L-fuzzy soft set  $(m, E, X)$ ,  $\mathcal{T}_e^*(m(e)) \geq \mathcal{T}_e(m(e))$ ).

**4.16. Definition.** Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space and  $(m, E, X)$  an L-fuzzy soft set.

- (1) The *closure*  $cl(m, E, X)$  of  $(m, E, X)$  is an L-fuzzy soft set with the same set of parameters, that is  $cl(m, E, X) = (clm, E, X)$ , where  $clm : E \rightarrow L^X$  and

$$clm(e) = cl(m(e)) = \bigwedge \{n \in L^X : \tau_e(n') > 0 \text{ and } m(e) \leq n\}, \forall e \in E.$$

- (2) The *interior* of  $(m, E, X)$  is  $int(m, E, X) = (intm, E, X)$  where  $intm : E \rightarrow L^X$  and

$$intm(e) = int(m(e)) = \bigvee \{n \in L^X : \tau_e(n) > 0 \text{ and } n \leq m(e)\}, \forall e \in E.$$

**4.17. Definition.** Let  $(X, \mathcal{T}, E)$  and  $(Y, \mathcal{T}^*, F)$  be two L-fuzzy soft topological spaces. Then  $(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  is called *fuzzy soft continuous* if for all  $e \in E$  and  $f = \psi(e) \in F$ ,  $\varphi : (X, \mathcal{T}(e)) \rightarrow (Y, \mathcal{T}^*(f))$  is fuzzy continuous.

Here,  $\mathcal{T}(e) = \mathcal{T}_e : L^X \rightarrow L$ ,  $\mathcal{T}^*(f) = \mathcal{T}_f^* : L^Y \rightarrow L$  and for all  $e \in E$  and  $f = \psi(e) \in F$ ,  $\varphi : (X, \mathcal{T}_e) \rightarrow (Y, \mathcal{T}_f^*)$  is fuzzy continuous if for all  $m \in L^Y$ ,  $\mathcal{T}_e(\varphi^{-1}(m)) \geq \mathcal{T}_f^*(m)$ .

**4.18. Proposition.** Let  $(X, \mathcal{T}_1, E_1)$ ,  $(Y, \mathcal{T}_2, E_2)$  and  $(Z, \mathcal{T}_3, E_3)$  be L-fuzzy soft topological spaces. If  $(\psi_1, \varphi_1) : (X, \mathcal{T}_1, E_1) \rightarrow (Y, \mathcal{T}_2, E_2)$  and  $(\psi_2, \varphi_2) : (Y, \mathcal{T}_2, E_2) \rightarrow (Z, \mathcal{T}_3, E_3)$  are fuzzy soft continuous, then their composition  $(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1) = (\psi_2 \circ \psi_1, \varphi_2 \circ \varphi_1)$  is also fuzzy soft continuous.

*Proof.* Since  $(\psi_2, \varphi_2)$  is fuzzy soft continuous, for each  $e_2 \in E_2$  and  $e_3 = \psi_2(e_2) \in E_3$ ,  $\varphi_2 : (Y, \mathcal{T}_2(e_2)) \rightarrow (Z, \mathcal{T}_3(e_3))$  is fuzzy continuous.

Since  $(\psi_1, \varphi_1)$  is fuzzy soft continuous, for each  $e_1 \in E_1$  and  $e_2 = \psi_1(e_1) \in E_2$ ,  $\varphi_1 : (X, \mathcal{T}_1(e_1)) \rightarrow (Y, \mathcal{T}_2(e_2))$  is fuzzy continuous.

The composition of two fuzzy continuous mappings is also a fuzzy continuous mapping. So,  $(\psi_2, \varphi_2) \circ (\psi_1, \varphi_1)$  is fuzzy soft continuous.  $\square$

## 4.2. L-fuzzy soft compactness.

**4.19. Definition.** Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space. If for all  $e \in E$ ,  $(X, \mathcal{T}(e))$  is fuzzy compact (fuzzy nearly compact, fuzzy almost compact), then  $(X, \mathcal{T}, E)$  is called *fuzzy soft compact* (fuzzy soft nearly compact, fuzzy soft almost compact).

That is, for each  $e \in E$ ,  $(X, \mathcal{T}(e))$  is fuzzy compact in Aygun's sense [5] and fuzzy nearly (almost) compact in Ramadan's sense [18].

Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space and  $(m, E)$  an L-fuzzy soft set on  $X$ . Then  $(m, E)$  is called a *fuzzy soft compact set* on  $X$  if for all  $e \in E$ ,  $m(e)$  is a fuzzy compact set.

That is, for each  $e \in E$ ,  $m(e)$  is a fuzzy compact set in Aygun's sense [5].

**4.20. Theorem.** Let  $(X, \mathcal{T}, E)$  and  $(Y, \mathcal{T}^*, F)$  be two L-fuzzy soft topological spaces. Let  $(\psi, \varphi) : (X, \mathcal{T}, E) \rightarrow (Y, \mathcal{T}^*, F)$  be a fuzzy soft continuous and onto mapping. If  $(X, \mathcal{T}, E)$  is a fuzzy soft compact (fuzzy soft nearly compact, fuzzy soft almost compact) space, then  $(Y, \mathcal{T}^*, F)$  is a fuzzy soft compact (fuzzy soft nearly compact, fuzzy soft almost compact) space.

*Proof.* Let  $(X, \mathcal{T}, E)$  be a fuzzy soft compact space. Then for all  $e \in E$ ,  $(X, \mathcal{T}(e))$  is fuzzy compact. Since,  $(\psi, \varphi)$  is fuzzy soft continuous for all  $\psi(e) = f$ , we have  $\varphi : (X, \mathcal{T}(e)) \rightarrow (Y, \mathcal{T}^*(f))$  is fuzzy continuous. Therefore,  $(Y, \mathcal{T}^*(f))$  is a fuzzy compact space.  $\square$

**4.21. Theorem.** Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space and let  $(m, E, X)$ ,  $(n, E, X)$  be two L-fuzzy soft sets. If  $(m, E, X)$  and  $(n, E, X)$  are fuzzy soft compact sets (fuzzy soft nearly compact, fuzzy soft almost compact), then  $(m, E, X) \sqcup (n, E, X)$  is fuzzy soft compact set (fuzzy soft nearly compact, fuzzy soft almost compact).

*Proof.* Let  $(m, E, X)$  and  $(n, E, X)$  be two fuzzy soft compact sets, i.e.,  $m, n : E \rightarrow L^X$  and  $\forall e \in E$ ,  $m(e), n(e) \in L^X$  are fuzzy compact sets. By the definition, we can write  $(m, E, X) \sqcup (n, E, X) = (k, E, X)$ , where  $k(e) = m(e) \vee n(e)$  for all  $e \in E$ . Since  $m(e)$  and  $n(e)$  are fuzzy compact sets, from [18] we know that  $k(e) = m(e) \vee n(e)$  is a fuzzy compact set. Hence,  $(m, E, X) \sqcup (n, E, X)$  is a fuzzy soft compact set.  $\square$

**4.22. Theorem.** Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space. If  $X$  is a finite set, then  $(X, \mathcal{T}, E)$  is fuzzy soft compact space.

*Proof.* Let  $(X, \mathcal{T}, E)$  be an L-fuzzy soft topological space, i.e.,  $\mathcal{T} : E \rightarrow L^{L^X}$  and for all  $e \in E$ ,  $(X, \mathcal{T}(e))$  is L-fuzzy topological space. Since  $X$  is finite,  $(X, \mathcal{T}(e))$  is a fuzzy compact space. Consequently,  $(X, \mathcal{T}, E)$  is a fuzzy soft compact space.  $\square$

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