ASYMPTOTIC EQUIVALENCE
OF DOUBLE SEQUENCES

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Received 01 : 01 : 2011 : Accepted 08 : 11 : 2011

Abstract

The goal of this paper is to present a four-dimensional matrix characterization of asymptotic equivalence of double sequences. This will be accomplished with the following notion of asymptotic equivalence of double sequences. Two double sequences are asymptotic equivalent if and only if

\[ P - \lim_{k,l \to \infty} \frac{x_{k,l}}{y_{k,l}} = 1, \]

where \( x \) and \( y \) are selected judicially.

Using this notion necessary and sufficient conditions on the entries of a four-dimensional matrix are given to ensure that the transformation will preserve asymptotic equivalence.

Keywords: Divergent double sequences, Subsequences of a double sequences, Pringsheim limit point, \( P \)-convergent, \( P \)-divergent, RH-regular.

2000 AMS Classification: Primary: 40 B 05. Secondary: 40 C 05.

1. Introduction

In 2002, Patterson [7] provided an answer to the following question: what type of four dimensional matrix transformation will preserve asymptotic equivalence under transformations \( \mu(x) \) ?

In this paper, we present a regularity type of characterization of asymptotic equivalence of double sequences by using four-dimensional matrix transformations. To accomplish this we established the following theorem:

Suppose \( A \) is a non-negative four-dimensional matrix. Then

\[ x \sim y \text{ implies } \mu(Ax) \sim \mu(Ay) \]

for any double sequences \( x, y \in P_\delta \) for some \( \delta > 0 \) if and only if \( A \) satisfies the following three conditions:

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