Hacettepe Journal of Mathematics and Statistics
Volume 41 (3) (2012), 365 – 374

PRODUCTS OF FP-SOFT SETS AND THEIR APPLICATIONS

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Received 05 : 01 : 2011 : Accepted 02 : 08 : 2012

Abstract

Soft set theory was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with fuzzy objects. In this work, t-norm and t-conorm products of fuzzy parameterized soft sets (FP-soft sets) are defined and their properties are investigated. By using these products, AND-FP-soft decision making and OR-FP-soft decision making methods are constructed. Finally, the methods are applied to solve a problems which contains uncertainties.

Keywords: Soft sets, Fuzzy sets, FP-soft sets, t-norm, t-conorm, t-norm product, t-conorm product, Decision making.

2000 AMS Classification: 62 C 86, 03 E 72.

1. Introduction

Many fields deal with fuzzy data that may not be successfully modeled by ordinary mathematics. Probability theory, fuzzy sets [24], rough sets [18], and other mathematical theories are well-known and are often useful approaches to describe uncertainty. However, all of these theories have their own difficulties which are pointed out in [17] by Molodtsov. Molodtsov proposed a completely new approach for modeling uncertainty, free from these difficulties, this so-called *soft set theory* has potential applications in many different fields. Maji *et al.* [14] worked on a detailed theoretical study of soft sets. Later, the properties and applications of soft set theory have been studied by many authors (e.g. [3, 4, 5, 11, 15, 20, 23]). The algebraic structure of soft set theory has also been studied (e.g. [1, 2, 9, 12]). Many applications of soft set theory have been expanded by using the ideas of fuzzy sets (e.g. [6, 10, 13, 16, 19, 21, 22]).

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Çağman *et al.* [7] give the definition of fuzzy parameterized soft sets (FP-soft sets) and studied their operations. In this paper, after giving FP-soft sets and their operations, t-norm and t-conorm products of FP-soft sets are defined and their properties are investigated. Unlike a soft set, the set of parameters of a FP-soft set is a fuzzy set which allows the construction of more efficient decision processes. Therefore, by using these products, AND-FP-soft decision making and OR-FP-soft decision making methods are constructed. Finally, these methods are applied to solve a problems which contains uncertainties. This approach may be more comprehensive in the future to solve related problems such as computer science, decision making, current life state, and so on.

The present expository paper is an abridgement of part of the dissertation [8].

2. Preliminary

In this section, we present the basic definitions of soft set theory [17] and fuzzy set theory [24] with norms that are useful for subsequent discussions. These and more detailed explanations related to soft sets and fuzzy sets can be found in [4, 13, 17] and [24, 25], respectively.

2.1. Definition. [17] Let U be an initial universe, $P(U)$ the power set of U, E a set of parameters and $X \subseteq E$. Then a soft set F_X over U is a set defined by a function representing a mapping

$$f_X : E \to P(U) \text{ such that } f_X(x) = \emptyset \text{ if } x \notin X.$$

Here, f_X is called the *approximate function* of the soft set F_X, and the value $f_X(x)$ is a set called the x-*element* of the soft set for all $x \in E$. It is worth noting that the sets $f_X(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set over U can be represented by the set of ordered pairs

$$F_X = \{(x, f_X(x)) : x \in E, f_X(x) \in P(U)\}$$

Note that the set of all soft sets over U will be denoted by S(U).

2.2. Example. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ a set of parameters. If $X = \{x_2, x_3, x_4, x_5\}$ and $f_X(x_2) = \{u_2, u_5\}$, $f_X(x_3) = \emptyset$, $f_X(x_4) = U$, $f_X(x_5) = \{u_1, u_2, u_5\}$ then the soft-set F_X is written as

$$F_X = \{(x_2, \{u_2, u_4\}), (x_4, U), (x_5, \{u_1, u_2, u_5\})\}$$

2.3. Definition. [24] Let U be a universe. A fuzzy set X over U is a set defined by a function μ_X representing a mapping

$$\mu_X : U \to [0, 1]$$

Here, μ_X is called the *membership function* of X, and the value $\mu_X(u)$ is called the *grade of membership* of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows,

$$X = \{(\mu_X(u)/u) : u \in U,\ \mu_X(x) \in [0, 1]\}.$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

2.4. Definition. [25] t-norms are associative, monotonic and commutative two valued functions t that map from $[0, 1] \times [0, 1]$ into $[0, 1]$. These properties are formulated with the following conditions:

(i) $t(0, 0) = 0$ and $t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x),\ x \in E$,

(ii) If $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $t(\mu_{X_1}(x), \mu_{X_2}(x)) \leq t(\mu_{X_3}x), \mu_{X_4}(x))$,

(iii) $t(\mu_{X_1}(x), \mu_{X_2}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x))$,

(iv) $t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$.

2.5. Definition. [25] t-conorms (s-norms) are associative, monotonic and commutative two placed functions s which map from $[0,1] \times [0,1]$ into $[0,1]$. These properties are formulated with the following conditions:

(i) $s(1,1) = 1$ and $s(\mu_{X_1}(x), 0) = s(0, \mu_{X_1}(x)) = \mu_{X_1}(x), x \in E$,
(ii) if $\mu_{X_1}(x) \le \mu_{X_3}(x)$ and $\mu_{X_2}(x) \le \mu_{X_4}(x)$, then $s(\mu_{X_1}(x), \mu_{X_2}(x)) \le s(\mu_{X_3}(x), \mu_{X_4}(x))$,
(iii) $s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x))$,
(iv) $s(\mu_{X_1}(x), s(\mu_{X_2}(x), \mu_{X_3}(x))) = s(s(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$.

t-norms and t-conorms are related in a sense of logical duality. Typical dual pairs of non parametrized t-norm and t-conorm are complied below:

(i) *Drastic product:*

$$t_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} \min\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & \max\{\mu_{X_1}(x), \mu_{X_2}(x)\} = 1, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) *Drastic sum:*

$$s_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} \max\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & \min\{\mu_{X_1}(x), \mu_{X_2}(x)\} = 0, \\ 1, & \text{otherwise.} \end{cases}$$

(iii) *Bounded product:*

$$t_1(\mu_{X_1}(x), \mu_{X_2}(x)) = \max\{0, \mu_{X_1}(x) + \mu_{X_2}(x) - 1\}$$

(iv) *Bounded sum:*

$$s_1(\mu_{X_1}(x), \mu_{X_2}(x)) = \min\{1, \mu_{X_1}(x) + \mu_{X_2}(x)\}$$

(v) *Einstein product:*

$$t_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) \cdot \mu_{X_2}(x)}{2 - [\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x) \cdot \mu_{X_2}(x)]}$$

(vi) *Einstein sum:*

$$s_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x)}{1 + \mu_{X_1}(x) \cdot \mu_{X_2}(x)}$$

(vii) *Algebraic product:*

$$t_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) \cdot \mu_{X_2}(x)$$

(viii) *Algebraic sum:*

$$s_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x) \cdot \mu_{X_2}(x)$$

(ix) *Hamacher product:*

$$t_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) \cdot \mu_{X_2}(x)}{\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x) \cdot \mu_{X_2}(x)}$$

(x) *Hamacher sum:*

$$s_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x) - 2 \cdot \mu_{X_1}(x).\mu_{X_2}(x)}{1 - \mu_{X_1}(x) \cdot \mu_{X_2}(x)}$$

(xi) *Minimum:*

$$t_3(\mu_{X_1}(x), \mu_{X_2}(x)) = \min\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

(xii) *Maximum:*

$$s_3(\mu_{X_1}(x), \mu_{X_2}(x)) = \max\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

3. FP-soft sets

In this section, we give FP-soft sets and their operations. The basic ideas of FP-soft set theory and its extensions, as well as its applications, can be found in [7]. In Section 2, the approximate function of a soft set is defined from a crisp set of parameters to crisp subsets of the universal set. But the approximate functions of FP-soft sets are defined from a fuzzy set of parameters to the crisp subsets of the universal set.

Throughout this section, fuzzy subsets of the set of parameters are denoted by the letters $\widetilde{X}, \widetilde{Y}, \widetilde{Z}, \ldots$ to avoid confusion and complexity of the symbols.

3.1. Definition. [7] Let U be an initial universe, $P(U)$ the power set of U, E the set of parameters and $\widetilde{X}$ a fuzzy set over E with membership function

$$f_{\widetilde{X}} : E \to P(U) \text{ such that } f_{\widetilde{X}}(x) = \emptyset \text{ if } \mu_{\widetilde{X}}(x) = 0.$$

Here, $f_{\widetilde{X}}$ is called the approximate function of the FP-soft set $F_{\widetilde{X}}$, and the value $f_{\widetilde{X}}(x)$ is a set called the x-element of the FP-soft set for all $x \in E$. Thus, an FP-soft set $F_{\widetilde{X}}$ over U can be represented by the set of ordered pairs

$$F_{\widetilde{X}} = \{(\mu_{\widetilde{X}}(x)/x, f_{\widetilde{X}}(x)) : x \in E,\ f_{\widetilde{X}}(x) \in P(U),\ \mu_{\widetilde{X}}(x) \in [0,1]\}.$$

The value $\mu_{\widetilde{X}}(x)$ is the degree of importance of the parameter x and depends on the decision-maker's requirements.

Note that the set of all FP-soft sets over U will be denoted by $\mu SS(U)$.

3.2. Definition. [7] Let $F_{\widetilde{X}} \in \mu SS(U)$. If $\mu_{\widetilde{X}}(x) = 0$ for all $x \in E$, then $F_{\widetilde{X}}$ is called the *empty FP-soft set*, denoted by $F_{\emptyset}$.

3.3. Definition. [7] Let $F_{\widetilde{X}} \in \mu SS(U)$. If $\mu_{\widetilde{X}}(x) = 1$ and $f_{\widetilde{X}} x) = U$ for all $x \in \widetilde{X}$, then $F_{\widetilde{X}}$ is called the $\widetilde{X}$*-universal FP-soft set*, denoted by $F_{\tilde{\widetilde{X}}}$.

If $\widetilde{X} = E$, then the $\widetilde{X}$-universal FP-soft set is called the *universal FP-soft set*, denoted by $F_{\tilde{E}}$.

3.4. Example. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ a set of parameters.

If $\widetilde{X} = \{0.2/x_2, 0.5/x_3, 1/x_5\}$ and $f_{\widetilde{X}}(x_2) = \{u_2, u_4\}$, $f_{\widetilde{X}}(x_3) = \emptyset$, $f_{\widetilde{X}}(x_5) = U$, then the FP-soft set $F_{\widetilde{X}}$ is written as

$$F_{\widetilde{X}} = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \emptyset), (1/x_5, U)\}.$$

If $\widetilde{Y} = \emptyset$, then the FP-soft set $F_{\widetilde{Y}}$ is the empty soft set. That is, $F_{\widetilde{Y}} = F_{\emptyset}$.

If $\widetilde{Z} = \{1/x_1, 1/x_2\}$ and $f_{\widetilde{Z}}(x_1) = U$, $f_{\widetilde{Z}}(x_2) = U$, then the FP-soft set $F_{\widetilde{Z}}$ is the $\widetilde{Z}$-universal FP-soft set. That is, $F_{\widetilde{Z}} = F_{\tilde{Z}}$.

If $\widetilde{X} = E$, and $f_{\widetilde{X}}(x_i) = U$ for all $x_i \in E$, $i = 1, 2, 3, 4, 5$ then the FP-soft set $F_{\widetilde{X}}$ is the universal FP-soft set. That is, $F_{\widetilde{X}} = F_{\tilde{E}}$.

3.5. Definition. [7] Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then $F_{\widetilde{X}}$ is a μ-soft-subset of $F_{\widetilde{Y}}$, denoted by $F_{\widetilde{X}} \widetilde{\subseteq} F_{\widetilde{Y}}$, if $\mu_{\widetilde{X}}(x) \le \mu_{\widetilde{Y}}(x)$ and $f_{\widetilde{X}}(x) \subseteq f_{\widetilde{Y}}(x)$ for all $x \in E$.

3.6. Remark. [7] $F_{\widetilde{X}} \widetilde{\subseteq} F_{\widetilde{Y}}$ does not imply that every element of $F_{\widetilde{X}}$ is an element of $F_{\widetilde{Y}}$, as in the definition of classical subset.

For example, assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ a set of parameters. If $\widetilde{X} = \{0.5/x_1\}$, $\widetilde{Y} = \{0.9/x_1, 0.1/x_3\}$, and $F_{\widetilde{X}} = \{(0.5/x_1, \{u_2, u_4\})\}$, $F_{\widetilde{Y}} = \{(0.9/x_1, \{u_2, u_3, u_4\}), (0.1/x_3, \{u_1, u_5\})\}$, then for all $x \in E$, $\mu_{\widetilde{X}}(x) \le \mu_{\widetilde{Y}}(x)$ and $f_{\widetilde{X}}(x) \subseteq f_{\widetilde{Y}}(x)$ is valid. Hence $F_{\widetilde{X}} \widetilde{\subseteq} F_{\widetilde{Y}}$.

It is clear that $(0.5/x_1, \{u_2, u_4\}) \in F_{\widetilde{X}}$ but $(0.5/x_1, \{u_2, u_4\}) \notin F_{\widetilde{Y}}$.

3.7. Definition. [7] Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$ are fps-equal, written as $F_{\widetilde{X}} = F_{\widetilde{Y}}$, if and only if $\mu_{\widetilde{X}}(x) = \mu_{\widetilde{Y}}(x)$ and $f_{\widetilde{X}}(x) = f_{\widetilde{Y}}(x)$ for all $x \in E$.

3.8. Definition. [7] Let $F_{\widetilde{X}} \in \mu SS(U)$. Then the complement $F_{\widetilde{X}}$, denoted by $F_{\widetilde{X^c}}$, is the FP-soft set defined by the approximate and membership functions

$$\mu_{\widetilde{X^c}}(x) = 1 - \mu_{\widetilde{X}}(x) \text{ and } f_{\widetilde{X^c}}(x) = U \setminus f_{\widetilde{X}}(x).$$

3.9. Definition. [7] Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then the *union* of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$, denoted by $F_{\widetilde{X}} \widetilde{\cup} F_{\widetilde{Y}}$, is defined by

$$\mu_{\widetilde{X} \widetilde{\cup} \widetilde{Y}}(x) = \max\{\mu_{\widetilde{X}}(x), \mu_{\widetilde{Y}}(x)\} \text{ and } f_{\widetilde{X} \widetilde{\cup} \widetilde{Y}}(x) = f_{\widetilde{X}}(x) \cup f_{\widetilde{Y}}(x), \text{for all } x \in E.$$

3.10. Definition. [7] Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then the *intersection* of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$, denoted by $F_{\widetilde{X}} \widetilde{\cap} F_{\widetilde{Y}}$, is a FP-soft sets defined by the approximate and membership functions

$$\mu_{\widetilde{X} \widetilde{\cap} \widetilde{Y}}(x) = \min\{\mu_{\widetilde{X}}(x), \mu_{\widetilde{Y}}(x)\} \text{ and } f_{\widetilde{X} \widetilde{\cap} \widetilde{Y}}(x) = f_{\widetilde{X}}(x) \cap f_{\widetilde{Y}}(x).$$

3.11. Remark. [7] Let $F_{\widetilde{X}} \in \mu SS(U)$. If $F_{\widetilde{X}} \neq F_{\emptyset}$ or $F_{\widetilde{X}} \neq F_{\tilde{E}}$, then $F_{\widetilde{X}} \widetilde{\cup} F_{\widetilde{X^c}} \neq F_{\tilde{E}}$ and $F_{\widetilde{X}} \widetilde{\cap} F_{\widetilde{X^c}} \neq F_{\emptyset}$.

4. t-norm and t-conorm products of FP-soft sets

In this section, we define the AND-t-norm, OR-t-norm, AND-t-conorm and OR-t-conorm products by using the t-norm and t-conorm on FP-soft sets. We then study some of their desired properties.

4.1. Definition. Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then the *$AND$-$t$-norm product* of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$, denoted by $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}$, is the FP-soft set defined as follows;

$$\mu_{\widetilde{X} \widehat{\otimes} Y} : E \times E \to [0,1], \;\; \mu_{\widetilde{X} \otimes \widetilde{Y}}(x) = \mu_{\widetilde{X}}(x) \mu_{\widetilde{Y}}(x)$$

and

$$f_{\widetilde{X} \widehat{\otimes} \widetilde{Y}} : E \times E \to P(U), \;\; f_{\widetilde{X} \widehat{\otimes} \widetilde{Y}}(x) = f_{\widetilde{X}}(x) \cap f_{\widetilde{Y}}(x)$$

Note that OR-t-norm product of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$, denoted by $(F_{\widetilde{X}} \otimes F_{\widetilde{X}})$, is defined in a similar way:

4.2. Definition. Let $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. Then the OR-t-conorm product of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$, denoted by $F_{\widetilde{X}} \oplus F_{\widetilde{Y}}$, is the FP-soft set defined as follows:

$$\mu_{\widetilde{X} \oplus \widetilde{Y}} : E \times E \to [0,1], \;\; \mu_{\widetilde{X} \oplus \widetilde{Y}}(x) = \mu_{\widetilde{X}}(x) + \mu_{\widetilde{Y}}(x) - \mu_{\widetilde{X}}(x) \mu_{\widetilde{Y}}(x)$$

and

$$f_{\widetilde{X} \oplus \widetilde{Y}} : E \times E \to P(U), \;\; f_{\widetilde{X} \oplus \widetilde{Y}}(x) = f_{\widetilde{X}}(x) \cup f_{\widetilde{Y}}(x).$$

Note that the AND-t-conorm product of $F_{\widetilde{X}}$ and $F_{\widetilde{X}}$, denoted by $(F_{\widetilde{X}} \widehat{\oplus} F_{X_2})$, is similarly defined.

We remark that the symbols $\oplus$, $\widehat{\oplus}$, $\otimes$ and $\widehat{\otimes}$ used in the subscripts of the approximate and membership functions are not operations of fuzzy sets. They indicate that $f_{\widetilde{X} \oplus \widetilde{Y}}$, $f_{X \widehat{\oplus} \widetilde{Y}}$, $f_{\widetilde{X} \otimes \widetilde{Y}}$ and $f_{\widetilde{X} \widehat{\otimes} \widetilde{Y}}$ are the approximate functions of $F_{\widetilde{X}} \oplus F_{\widetilde{Y}}$, $F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}$, $F_{\widetilde{X}} \otimes F_{\widetilde{Y}}$ and $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}$ and $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}$, respectively, and $\mu_{\widetilde{X} \oplus \widetilde{Y}}$, $\mu_{\widetilde{X} \widehat{\oplus} \widetilde{Y}}$, $\mu_{\widetilde{X} \otimes \widetilde{Y}}$ and $\mu_{\widetilde{X} \widehat{\otimes} \widetilde{Y}}$ are the membership functions of $F_{\widetilde{X}} \oplus F_{\widetilde{Y}}$, $F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}$, $F_{\widetilde{X}} \otimes F_{\widetilde{Y}}$ and $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}$, respectively.

4.3. Example. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ a set of parameters. If

$$\widetilde{X} = \{0.5/x_1, 0.7/x_2, 0.8/x_3, 1/x_4\, 0.5/x_5, 0.9/x_6, 0.2/x_8\}$$

and

$$\widetilde{Y} = \{0.9/x_1, 0.3/x_3, 0.4/x_4, 0.8/x_5, 0.7/x_6, 0.9/x_7, 0.8/x_8\}$$

are two fuzzy sets over E, then we can write the following FP-soft sets,

$$\begin{aligned}
F_{\widetilde{X}} = \big\{&(0.5/x_1, U), (0.7/x_2, \{u_1, u_2, u_4, u_5, u_6\}),\\
&(0.8/x_3, \{u_1, u_2, u_4, u_7, u_8, u_9\}), (1/x_4, \{u_1, u_6\}),\\
&(0.5/x_5, \{u_1, u_2, u_4, u_6, u_7, u_8, \}), (0.9/x_6, \{u_1, u_6, u_7, u_9\}),\\
&(0.2/x_8), \{u_3, u_4, u_7, u_8, u_9\})\big\}\\
F_{\widetilde{Y}} = \big\{&(0.9/x_1, \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}),\\
&(0.3/x_3, \{u_1, u_2, u_3, u_4, u_7, u_8, u_9\}), (0.4/x_4, \{u_2, u_5\}),\\
&(0.8/x_5, \{u_1, u_8, u_9\}), (0.7/x_6, U), (0.9/x_7, \{u_1, u_2, u_6\}),\\
&(0.8/x_8, \{u_2, u_3, u_4, u_5, u_7, u_8, u_9\})\big\}\\
F_{\widetilde{X}} \oplus F_{\widetilde{Y}} = \big\{&(0.95/x_1, U),\\
&(0.7/x_2, \{u_1, u_2, u_4, u_5, u_6\}), (0.86/x_3, \{u_1, u_2, u_3, u_4, u_7, u_8, u_9\}),\\
&(1/x_4, \{u_1, u_2, u_5, u_6\}), (0.9/x_5, \{u_1, u_2, u_4, u_6, u_7, u_8, u_9\}),\\
&(0.97/x_6, U), (0.7/x_7, \{u_1, u_2, u_6\}),\\
&(0.84/x_8, \{u_2, u_3, u_4, u_5, u_7, u_8, u_9\}), \big\}
\end{aligned}$$

The values of $F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}$, $F_{\widetilde{X}} \otimes F_{\widetilde{Y}}$ and $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}$ can be given similarly.

4.4. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. *Then,*

(i) $F_{\widetilde{X}} \oplus F_{\widetilde{Y}} = F_{\widetilde{Y}} \oplus F_{\widetilde{X}}$,
(ii) $F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}} = F_{\widetilde{Y}} \widehat{\oplus} F_{\widetilde{X}}$,
(iii) $F_{\widetilde{X}} \otimes F_{\widetilde{Y}} = F_{\widetilde{Y}} \oplus F_{\widetilde{X}}$,
(iv) $F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}} = F_{\widetilde{Y}} \widehat{\otimes} F_{\widetilde{X}}$. □

4.5. Proposition. *Let* $F_{\widetilde{X}}, F_E, F_\emptyset \in \mu SS(U)$. *Then,*

(i) $F_{\widetilde{X}} \oplus F_E = F_E$,
(ii) $F_{\widetilde{X}} \widehat{\otimes} F_E = F_{\widetilde{X}}$,
(iii) $F_{\widetilde{X}} \oplus F_\emptyset = F_{\widetilde{X}}$,
(iv) $F_{\widetilde{X}} \widehat{\otimes} F_\emptyset = F_\emptyset$. □

4.6. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}}, F_{\widetilde{Z}} \in \mu SS(U)$. *Then,*

(i) $F_{\widetilde{X}} \oplus (F_{\widetilde{Y}} \oplus F_{\widetilde{Z}}) = (F_{\widetilde{X}} \oplus F_{\widetilde{Y}}) \oplus F_{\widetilde{Z}}$,
(ii) $F_{\widetilde{X}} \widehat{\oplus} (F_{\widetilde{Y}} \widehat{\oplus} F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}) \widehat{\oplus} F_{\widetilde{Z}}$,
(iii) $F_{\widetilde{X}} \otimes (F_{\widetilde{Y}} \otimes F_{\widetilde{Z}}) = (F_{\widetilde{X}} \otimes F_{\widetilde{Y}}) \otimes F_{\widetilde{Z}}$,
(iv) $F_{\widetilde{X}} \widehat{\otimes} (F_{\widetilde{Y}} \widehat{\otimes} F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}) \widehat{\otimes} F_{\widetilde{Z}}$. □

4.7. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$. *Then, De Morgan type results are true.*

(i) $(F_{\widetilde{X}} \oplus F_{\widetilde{Y}})^c = F^c_{\widetilde{X}} \widehat{\otimes} F^c_{\widetilde{Y}}$,
(ii) $(F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}})^c = F^c_{\widetilde{X}} \oplus F^c_{\widetilde{Y}}$,
(iii) $(F_{\widetilde{X}} \otimes F_{\widetilde{Y}})^c = F^c_{\widetilde{X}} \widehat{\oplus} F^c_{\widetilde{Y}}$,
(iv) $(F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}})^c = F^c_{\widetilde{X}} \otimes F^c_{\widetilde{Y}}$.

Proof. (i) For all $x \in E$,

$$\begin{aligned}\mu_{(\widetilde{X}\oplus\widetilde{Y})^c}(x) &= 1-\mu_{(\widetilde{X}\oplus\widetilde{Y})}(x)\\ &= 1-(\mu_{\widetilde{X}}(x)+\mu_{\widetilde{Y}}(x)-\mu_{\widetilde{X}}(x)\mu_{\widetilde{Y}}(x))\\ &= 1-\mu_{\widetilde{X}}(x)-\mu_{\widetilde{Y}}(x)+\mu_{\widetilde{X}}(x)\mu_{\widetilde{Y}}(x)\\ &= (1-\mu_{\widetilde{X}}(x)(1-\mu_{\widetilde{Y}}(x))\\ &= \mu_{\widetilde{X^c}\otimes\widetilde{Y^c}}(x)\end{aligned}$$

and

$$\begin{aligned}f_{(\widetilde{X}\oplus\widetilde{Y})^c}(x,y) &= U\setminus(f_{\widetilde{X}}(x)\cup f_{\widetilde{Y}}(x))\\ &= (U\setminus f_{\widetilde{X}}(x))\cup(U\setminus f_{\widetilde{Y}}(x))\\ &= f_{\widetilde{X^c}}(x)\cap f_{\widetilde{Y^c}}(x)\end{aligned}$$

The proof of (ii), (iii) and (iv) can be made similarly. □

4.8. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}}, F_{\widetilde{Z}} \in \mu SS(U)$*. Then,*

(i) $F_{\widetilde{X}} \otimes (F_{\widetilde{Y}} \cap F_{\widetilde{Z}}) = (F_{\widetilde{X}} \otimes F_{\widetilde{Y}}) \cap (F_{\widetilde{X}} \otimes F_{\widetilde{Z}})$,
(ii) $F_{\widetilde{X}} \widehat{\otimes} (F_{\widetilde{Y}} \cap F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}) \cap (F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Z}})$,
(iii) $F_{\widetilde{X}} \otimes (F_{\widetilde{Y}} \cup F_{\widetilde{Z}}) = (F_{\widetilde{X}} \otimes F_{\widetilde{Y}}) \cup (F_{\widetilde{X}} \otimes F_{\widetilde{Z}})$,
(iv) $F_{\widetilde{X}} \widehat{\otimes} (F_{\widetilde{Y}} \cup F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}) \cup (F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Z}})$. □

4.9. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}}, F_{\widetilde{Z}} \in \mu SS(U)$*. Then,*

(i) $F_{\widetilde{X}} \oplus (F_{\widetilde{Y}} \cap F_{\widetilde{Z}}) = (F_{\widetilde{X}} \oplus F_{\widetilde{Y}}) \cap (F_{\widetilde{X}} \oplus F_{\widetilde{Z}})$,
(ii) $F_{\widetilde{X}} \widehat{\oplus} (F_{\widetilde{Y}} \cap F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}) \cap (F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Z}})$,
(iii) $F_{\widetilde{X}} \oplus (F_{\widetilde{Y}} \cup F_{\widetilde{Z}}) = (F_{\widetilde{X}} \oplus F_{\widetilde{Y}}) \cup (F_{\widetilde{X}} \oplus F_{\widetilde{Z}}$,
(iv) $F_{\widetilde{X}} \widehat{\oplus} (F_{\widetilde{Y}} \cup F_{\widetilde{Z}}) = (F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}) \cup (F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Z}}$. □

5. Soft fuzzification operators

In this section, we define soft fuzzification operators by using the AND-t-norm, OR-t-norm, AND-t-conorm and OR-t-conorm products. They convert two FP-soft sets to a fuzzy set.

5.1. Definition. In this definition, for the shake of brevity, we use T instead of $F_{\widetilde{X}} \oplus F_{\widetilde{Y}}$. Let f_T be the approximation function and μ_T the membership function of T. Then, an *OR-soft fuzzification operator*, denoted $s_\oplus$, is defined by

$$s_\oplus : F(E) \times FPS(U) \to F(U),$$

$$s_\oplus(T) = \{\mu_\oplus(u)/u : u \in f_{\mathrm{T}}(a), \mu_\oplus(u) \in [0,1]\},$$

where

$$\mu_\oplus(u) = \frac{1}{|U|}\sum_{i,j} \mu_{\mathrm{T}}(a_i)\chi_{\mathrm{T}(a_i)}(u_j)$$

and where

$$\chi_{\mathrm{T}(a_i)}(u_j) = \begin{cases} 1, & u_j \in \mathrm{T}(a_i), \\ 0, & u_j \notin \mathrm{T}(a_i). \end{cases}$$

The set $s_\oplus(\mathrm{T})$ is a fuzzy set called an *OR-decision fuzzy set over* U.

Note that $s_{\widehat{\oplus}}(F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}})$, $s_\otimes(F_{\widetilde{X}} \otimes F_{\widetilde{Y}})$ and $s_{\widehat{\otimes}}(F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}})$ are defined in a similar way.

5.2. Proposition. *Let* $F_{\widetilde{X}}, F_{\widetilde{Y}} \in \mu SS(U)$*. Then,*

(i) $s_\oplus(F_{\widetilde{X}} \oplus F_{\widetilde{Y}}) = s_\oplus(F_{\widetilde{Y}} \oplus F_{\widetilde{X}})$,
(ii) $s_{\widehat{\oplus}}(F_{\widetilde{X}} \widehat{\oplus} F_{\widetilde{Y}}) = s_{\widehat{\oplus}}(F_{\widetilde{Y}} \widehat{\oplus} F_{\widetilde{X}})$,
(iii) $s_\otimes(F_{\widetilde{X}} \otimes F_{\widetilde{Y}}) = s_\otimes(F_{\widetilde{Y}} \otimes F_{\widetilde{X}})$,
(iv) $s_{\widehat{\otimes}}(F_{\widetilde{X}} \widehat{\otimes} F_{\widetilde{Y}}) = s_{\widehat{\otimes}}(F_{\widetilde{Y}} \widehat{\otimes} F_{\widetilde{X}})$. □

6. FP-soft decision making methods

In this section, we construct FP-soft decision making methods. Assume that U be an initial universe that contains alternatives and E a set of parameters. We then construct an OR-FP-soft decision making method by the following algorithm to produce a decision fuzzy set from U.

Step 1: Choose feasible fuzzy subsets $\widetilde{X}$ and $\widetilde{Y}$ over E,
Step 2: Construct the FP-soft sets $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$ over U,
Step 3: Find the OR-t-conorm $F_{\widetilde{X}} \oplus F_{\widetilde{Y}}$,
Step 4: Compute the OR-decision fuzzy set $s_\oplus(F_{\widetilde{X}} \oplus F_{\widetilde{Y}})$.

Note that, in a similar way, we can construct an AND-FP-soft decision making method.

Now, we can give an example of the OR-FP-soft decision making method. Examples of the other method can also be given in a similar way.

6.1. Example. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ is a set of objects, $E = \{a_1, a_2, a_3, a_4, a_5\}$ is a set of parameters. Let us say that for $i = 1, 2, 3, 4, 5, 6, 7, 8$ the objects u_i stand for "Istanbul", "Denizli", "Ankara ", "Izmir", "Adana","Kilis", "Tokat" and "Bursa" and for $j = 1, 2, 3, 4, 5$ the parameters a_j stand for "transportation", "productive power", "marketing", "profit" and "sale" respectively.

There are two two members, called $\widetilde{X}$ and $\widetilde{Y}$, of the administrative body of a company who want to choose a city from the cities that are given above to construct a plant. If each member has to consider their own fuzzy set of parameters, then we select a city on the basis of the sets of members' parameters by using the Or-FP-soft decision making method as follows.

***Step* 1:** Assume that the members $\widetilde{X}$ and $\widetilde{Y}$ construct their own fuzzy sets as follows:

$$\widetilde{X} = \{0.5/x_1, 0.7/x_2, 0.8/x_3, 1/x_4\, 0.5/x_5, 0.9/x_6, 0.2/x_8\}$$

and

$$\widetilde{Y} = \{0.9/x_1, 0.3/x_3, 0.4/x_4, 0.8/x_5, 0.7/x_6, 0.9/x_7, 0.8/x_8\}$$

over E, respectively.

***Step* 2:** $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$ over U are written by

$$\begin{aligned} F_{\widetilde{X}} = \big\{&(0.5/x_1, U), (0.7/x_2, \{u_1, u_2, u_4, u_5, u_6\}),\\ &(0.8/x_3, \{u_1, u_2, u_4, u_7, u_8, u_9\}), (1/x_4, \{u_1, u_6\}),\\ &(0.5/x_5, \{u_1, u_2, u_4, u_6, u_7, u_8, \}), (0.9/x_6, \{u_1, u_6, u_7, u_9\}),\\ &(0.2/x_8, \{u_3, u_4, u_7, u_8, u_9\})\big\}\\ F_{\widetilde{Y}} = \big\{&(0.9/x_1, \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}),\\ &(0.3/x_3, \{u_1, u_2, u_3, u_4, u_7, u_8, u_9\}), (0.4/x_4, \{u_2, u_5\}),\\ &(0.8/x_5, \{u_1, u_8, u_9\}), (0.7/x_6, U), (0.9/x_7, \{u_1, u_2, u_6\}),\\ &(0.8/x_8, \{u_2, u_3, u_4, u_5, u_7, u_8, u_9\})\big\} \end{aligned}$$

***Step* 3:** The OR-t-conorm product of $F_{\widetilde{X}}$ and $F_{\widetilde{Y}}$ is formed as,

$$\begin{aligned}F_{\widetilde{X}} \oplus F_{\widetilde{Y}} = \big\{&(0.95/x_1, U), (0.7/x_2, \{u_1, u_2, u_4, u_5, u_6\}),\\ &(0.86/x_3, \{u_1, u_2, u_3, u_4, u_7, u_8, u_9\}), (1/x_4, \{u_1, u_2, u_5, u_6\}),\\ &(0.9/x_5, \{u_1, u_2, u_4, u_6, u_7, u_8, u_9\}), (0.97/x_6, U),\\ &(0.7/x_7, \{u_1, u_2, u_6\}), (0.84/x_8, \{u_2, u_3, u_4, u_5, u_7, u_8, u_9\})\big\}\end{aligned}$$

***Step* 4:** The OR-decision fuzzy set is computed as

$$\begin{aligned}s_{\oplus}(F_{\widetilde{X}} \oplus F_{\widetilde{Y}}) = \big\{&0.675/u_1, 0.768/u_2, 0.402/u_3, 0.580/u_4, 0.495/u_5,\\ &0.580/u_6, 0.502/u_7, 0.502/u_8, 0.502/u_9\big\}\end{aligned}$$

where the alternative u_2 is the decision-maker's best choice since it has the largest degree 0.768 among the others.

7. Conclusion

In this work, the t-norm and t-conorm products of FP-soft sets are defined and their properties are investigated. By using the products, AND-FP-soft decision making and OR-FP-soft decision making methods are constructed. This approach should be more comprehensive in the future to solve the related problems such as computer science, decision making, current life state, and so on.

Acknowledgement

The authors are grateful for financial support from the Research Fund of Gaziosmanpasa University under Grant No.: 2009/47.

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