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A NOVEL SEASONAL FUZZY TIME SERIES METHOD

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Abstract

Fuzzy time series forecasting methods, which have been widely studied in recent years, do not require constraints as found in conventional approaches. On the other hand, most of the time series encountered in real life should be considered as fuzzy time series due to the vagueness that they contain. Although numerous methods have been proposed for the analysis of time series in the literature, these methods fail to forecast seasonal fuzzy time series. The limited number of seasonal fuzzy time series methods consider only the fuzzy set having the highest membership value, rather than the membership value of observations belonging to each fuzzy set. This is contrary to fuzzy set theory and causes information loss, thus affecting forecasting performance negatively. In this study, a new seasonal fuzzy time series method which considers the membership value of the observations belonging to each set in both forecasting fuzzy relations and in the defuzzification step is proposed. The proposed method is applied to a real seasonal time series.

Keywords: Fuzzy time series, SARIMA, Fuzzy C-means, Feed forward artificial neural network.,

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1. Introduction

Fuzzy time series [20] which were first proposed by Song and Chissom are based on fuzzy set theory proposed by Zadeh [26]. Fuzzy time series methods do not require the assumptions that the conventional approaches do. In addition, environmental data such as air temperature and air quality, financial data such as stock index and exchange rate and data sets encountered in many areas of life should be considered as fuzzy time series due the vagueness that they contain. Due to the above mentioned reasons, interest in time series is increasing day by day.

Fuzzy time series methods consist of three steps, namely fuzzification, identification of the fuzzified relations and defuzzification, respectively. These three basic steps of fuzzy time series play an effective role on forecasting performance. Therefore, many studies on these three steps have been done in the literature.

First-order models for forecasting fuzzy time series were proposed in many studies such as those of Song and Chissom [20, 21] and Yolcu *et al.* [24] . Additionally, high-order forecasting models were proposed in the studies of Chen [5] and Hsu *et al.* [14]. Although these models were effectively used in forecasting fuzzy time series containing fuzzy relations, they fail to forecast fuzzy time series which contain vagueness in their observations, which can be frequently encountered in real life.

Song [19] proposed a method for the analysis of fuzzy time series but could not achieve a real fuzzy time series application. The model proposed by Song includes only lagged variable belonging to the period. In other words, $F(t-m)$ is the input and $F(t)$ is the output of the model whose period is m. This model coincides with the first order seasonal autoregressive (SAR (1)) model. However, numerous time series include more complex relations apart from this structure. In an effort to forecast these types of time series, Egrioglu *et al.* [10] proposed a partial high order fuzzy time series forecasting model based on the SARIMA model.

Although Egrioglu *et al.*'s approach in [10] has many advantages; it uses a universal set partition in the fuzzification step. The effect of the interval lengths, determined subjectively in the fuzzification step, on forecasting performance has been presented in many studies in the literature. In order to eliminate this problem, Uslu *et al.* developed the model proposed in Egrioglu *et al.* [10], and used the fuzzy C-means method (FCM) which does not require a universal set partition in the fuzzification step Uslu *et al.* [23].

All these studies aiming at forecasting seasonal fuzzy time series consider only the fuzzy set having the highest membership value for each fuzzy observation, and ignore the membership values of other fuzzy sets. This is contrary to fuzzy set theory and causes information loss, thus affecting forecasting performance negatively. In the literature there is only one method which considers memberships values in determining fuzzy relations, see Yu and Huarng [25]. This study includes a first order fuzzy time forecasting method but is not used for forecasting seasonal fuzzy time series. If high order models were used in this approach, which considers the membership values, a possible problem would be the excessive number of inputs of the artificial neural network used in determining the relation.

In this study, a new seasonal fuzzy time forecasting method for forecasting fuzzy time series is proposed in which a partial model order was determined via SARIMA and an artificial neural network (ANN) consisting of membership values of fuzzy observations regarded as fuzzy sets was used in determining the fuzzy relation. In this model, which considers membership values, the input number problem was eliminated by clustering the data set to form lagged variables and the unit number of the input layer of the feed forward artificial neural network was limited by the number of sets. The proposed method was applied to a real time series and was compared with some conventional time series approaches as well as fuzzy time series forecasting methods.

In the second chapter, SARIMA models which were used in determining the method order, FCM which was used in the fuzzification step and ANN which was used in the determination of the fuzzy relation are introduced. The third chapter deals with the basic fuzzy time series concept and definitions. In the fourth chapter, the proposed method and its algorithm are given. In the fifth chapter, the proposed method is applied to a real seasonal time series and the results obtained presented together with other results obtained from the other methods. In the last chapter, the results obtained are evaluated and discussed.

2. Review

A. SARIMA. For a time series with mean μ the model is expressed by equation (1):

$$\varphi(B)\Phi(B^s)(1-B)^d(1-B^s)^D(Z_t-\mu)=\theta(B)\Theta(B^s)a_t. \tag{1}$$

The model parameters can be given as follows;

$$\varphi(B)=(1-\varphi_1 B-\cdots-\varphi_p B^p), \tag{2}$$
$$\theta(B)=(1+\theta_1 B+\cdots+\theta_q B^q), \tag{3}$$
$$\Phi(B)=(1-\Phi_1 B^s-\cdots-\Phi_P B^{sP}), \tag{4}$$
$$\Theta(B)=(1+\Theta_1 B+\cdots+\Theta_Q B^{sQ}) \tag{5}$$

Detailed information on this model, which is called the *seasonal autoregressive integrated moving average* (SARIMA) model, and which is expressed as SARIMA$(p,d,q)(P,D,Q)_s$, can be obtained from the study of Box and Jenkins [3].

B. The Fuzzy C-Means (FCM) Clustering Method. The FCM clustering method was first introduced by Bezdek [2]. This is the most widely used clustering algorithm. In this approach fuzzy clustering is done by minimizing the least squared errors within groups. Let u_{ij} be the membership value, v_i the center of the cluster, n the number of variables and c the number of clusters. Then the objective function, which is minimized in fuzzy clustering, is

$$J_\beta(X,V,U)=\sum_{i=1}^{c}\sum_{j=1}^{n}u_{ij}^{\beta}d^2(x_j,v_i), \tag{6}$$

where β is a constant which satisfies $\beta>1$ and is called as the *fuzzy index.* $d(x_j,v_i)$ is a similarity measure between the observation and the center of the fuzzy cluster. J_β is minimized subject to

$$\begin{aligned} &0\le u_{ij}\le 1,\ \forall\, i,j,\\ &0<\sum_{j=1}^{n}u_{ij}\le n,\ \forall\, i,\\ &\sum_{i=1}^{c}u_{ij}=1,\ \forall\, j. \end{aligned} \tag{7}$$

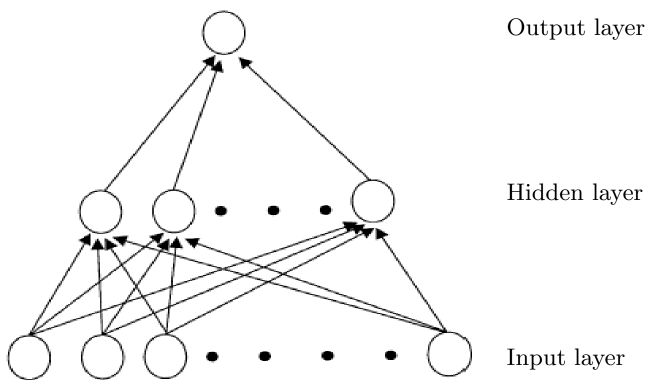
In this method, minimizing is done by an iterative algorithm. In each repetition the values of u_{ij} and v_i are updated by the formulas given in equation (8) and equation (9).

$$v_i = \frac{\sum_{j=1}^{n} u_{ij}^{\beta} x_j}{\sum_{j=1}^{n} u_{ij}^{\beta}}, \tag{8}$$

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d(x_j, v_i)}{d(x_j, v_k)} \right)^{\frac{2}{(\beta-1)}}}. \tag{9}$$

C. Feed Forward Neural Network. Artificial neural networks (ANN) can be defined as a mathematical algorithm that was inspired by biological neural networks. Artificial neural networks, defined as mathematical algorithms, are algorithms that can learn from examples and that can generalize what is learnt. The network presentation is a graphical expression of the mathematical algorithm Gunay *et al.* [11]. Artificial neural networks differ from biological ones in terms of their structure and ability Zurada [28]. Artificial neural networks are composed of a mathematical model Zhang *et al.* [27].

Figure 1. Architecture of a multilayer feed forward neural network



The three basic components that direct the operation of artificial neural networks are as follows;

Architectural Structure: The architecture structure of a multilayer feed forward artificial neural network consists of an input layer, hidden layer(s) and an output layer (see Figure 1). Each layer consists of neurons. The architecture structure is determined by deciding the number of neuron in each layer. These neurons are linked to each other by weights. There is no link among the neurons in the same layer.

Learning Algorithm: Although, there are many learning algorithms used in the determination of weights in artificial neural networks, the most widely used one is the *Back Propagation algorithm* which updates weights based on the difference between available data and the output of the network. The learning parameter, which is used in the back propagation algorithm and which can be taken as fixed or updated in the algorithm dynamically, plays an important role in reaching optimal results.

Activation Function: The proper selection of the activation function that enables curvilinear matching between input and output units, significantly affect the performance of the network. When the activation function, generally selected as unipolar, bipolar or linear, is not linear, the slope parameter should be determined. The slope parameter is another factor that plays an important role in reaching the appropriate conclusion.

3. Fuzzy time series

The definition of fuzzy time series was firstly introduced by Song and Chissom [20, 21]. Basic definitions of fuzzy time series not including constraints like the linear model, and the observation number can be given as follows;

3.1. Definition. —rm Fuzzy time series. Let $Y(t)$ $(t = \cdots, 0, 1, 2, \cdots)$, a subset of the real numbers, be the universe of discourse on which the fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection $f_1(t), f_1(t), \ldots$ then $F(t)$ is called a *fuzzy time series* defined on $Y(t)$.

3.2. Definition. First order seasonal fuzzy time series forecasting model. Let $F(t)$ be a fuzzy time series. Assume there exists seasonality in $\{F(t)\}$, a first order seasonal fuzzy time series forecasting model:

$$F(t-m) \to F(t), \tag{10}$$

where m denotes the period.

3.3. Definition. High order fuzzy time series forecasting model. Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2), \ldots, F(t-n)$, then this fuzzy logical relationship is represented by

$$F(t-n), \ldots, F(t-2), F(t-1) \to F(t) \tag{11}$$

and is called the n^{th} *order fuzzy time series forecasting model.*

3.4. Definition. First order bivariate fuzzy time series forecasting model. Let F and G be two fuzzy time series. Suppose that $F(t-1) = A_i$, $G(t-1) = A_k$ and $F(t) = A_j$. A bivariate fuzzy logical relationship is defined as $A_i, B_k \to A_j$, where A_i, B_k are referred to as the left hand side and A_j as the right hand side of the bivariate fuzzy logical relationship. Therefore, a first order bivariate fuzzy time series forecasting model is as follows:

$$F(t-1), G(t-1) \to F(t). \tag{12}$$

3.5. Definition. High order partial bivariate fuzzy time series forecasting model. Let F and G be two fuzzy time series. If $F(t)$ is caused by $F(t-m_1), \ldots, F(t-m_{k-1}), F(t-m_k), G(t-n_1), \ldots, G(t-n_{l-1}), G(t-n_l)$, where m_i $(i = 1, 2, \ldots, k)$ and n_j $(j = 1, 2, \ldots, l)$ are integers, $1 \leq m_1 < \cdots < m_k$, $1 \leq n_1 < \cdots < n_l$ then this FLR is represented by

$$\begin{aligned} &F(t-m_1), \ldots, F(t-m_{k-1}), F(t-m_k), \\ &G(t-n_1), \ldots, G(t-n_{l-1}), G(t-n_l) \to F(t) \end{aligned} \tag{13}$$

4. Proposed method

Most of the fuzzy time series encountered in real life include a seasonal component. The method proposed in Song [19] for the analysis of these types of time series includes lagged variable which belongs onlt to the period. Egrioglu *et al.* [10] presents a bivariate seasonal fuzzy time series forecasting model in which the model order was determined with SARIMA. The Egrioglu *et al.* [10] method requires universal set partition, but Uslu *et al.* [23] proposed a seasonal fuzzy time forecasting model which does not require universal set partition and uses the FCM algorithm. Although these studies have some advantages, due to the fact that these models ignore membership values which represent the order of observations belonging to fuzzy sets in determining the fuzzy relations and information loss, the forecasting performance of the model is affected negatively. The algorithm of the method proposed in this study which uses SARIMA in determining the model order, FCM in the fuzzification step and ANN in determining a fuzzy relation that takes into account membership values of the observations belonging to the fuzzy set is given below;

4.1. Algorithm. ***Step 1.*** *The model order is defined by SARIMA.*

The time series concerned is analyzed by the Box-Jenkins method after the model order is defined. Then we obtain the residuals series (a_t)*. As an illustration let us suppose we have defined the model as* $SARIMA(1,1,0)(0,1,1)_{12}$ *via Box-Jenkins method. This implies that* X_t *will be a linear combination of the corresponding lagged variables. That is,*

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-12}). \tag{14}$$

Therefore, (k,l)*, representing the order of the model, and the parameters* $m_1, \ldots, m_k$ *and* $n_1, \ldots, n_l$ *are determined based on the inputs of the SARIMA model. Accordingly* k *and* l *are defined as 5 and 1 respectively. Then the model will be a* $(5,1)^{th}$*-order partial bivariate fuzzy time series forecasting model and the fuzzy relationship can be given as follows;*

$$F(t-1), F(t-2), F(t-12), F(t-13), F(t-14), G(t-12)\} \rightarrow F(t). \tag{15}$$

This implies $m_1 = 1, m_2 = 2, m_3 = 12, m_4 = 13, m_5 = 14,$ $n_1 = 12,$ $F(t)$ *denotes the fuzzified time series* X_t *and* $G(t)$ *denotes the fuzzified residual series* a_t*.*

Step 2. *Data set of lagged variables is created.*

Depending on the model order defined in previous step, for each time series which should be included in the model X_t*, and residual series* a_t *for each lagged variable are lagged less than the order of lagged variables and data set is created. In other words, when a model given in equation (12) is considered, the lagged variables data set will include* $X_t, X_{t-1}, X_{t-11}, X_{t-12}, X_{t-13}, a_{t-11}$*.*

Step 3. *Data set of lagged variables is clustered via FCM.*

The number of fuzzy sets is determined by c, where $2 \leq c \leq n$ *and* n *is the number of observations. The data set which covers the delays in the times series is clustered via the FCM clustering method. Thus, the fuzzy set centers for each lagged variable constituting a data set and membership values showing the order of the observations belonging to the fuzzy sets for each observation are obtained. In this step, fuzzy sets are sorted according to set centers represented by* $v_r,$ $r = 1, 2, \ldots, c$ *and* $L_r,$ $r = 1, 2, \ldots, c$ *fuzzy sets are obtained.*

Step 4. *Fuzzy relations are determined via Feed Forward Artificial Neural Networks (ANN).*

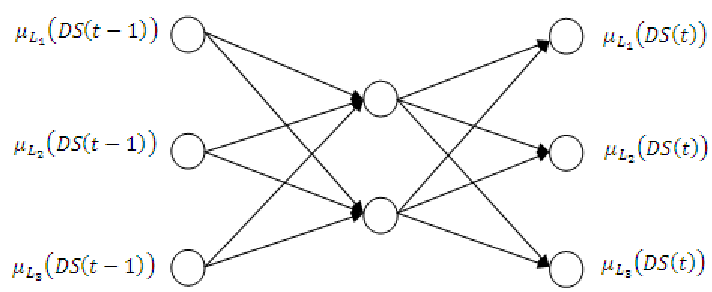
The number of neurons in the input and output layer of the feed forward artificial neural network used in determining the fuzzy relations equals the number of fuzzy set (c)*. The number of neurons in the hidden layer is determined by trial and error. Here, the point to take into consideration is that the hidden layer unit number should be selected in such a way that the ability of the feed forward artificial neural network to generalize is not lost. The architecture of a feed forward artificial neural network having two hidden layers for a model including three sets is presented in Figure 2. In Figure 2,* $\mu_{L_i}(DS(t))$ *represents the membership value of the lagged data set belonging to the* i^{th} *fuzzy set at time* t*. Moreover, while the membership value of the observation of the lagged data set belonging to the* c^{th} *fuzzy set at time* $t-1$ *constitutes the inputs of the ANN; the membership values of the observations of the lagged data set belonging to the* c^{th} *fuzzy set at time* t *constitutes the outputs of the ANN.*

In all layers of the feed forward artificial neural networks which is used in determining the fuzzy relation and whose architectural structure is exemplified above, the logistic activation function given in equation (16) was used.

$$f(x) = (1 + exp(-x))^{-1}. \tag{16}$$

Feed forward artificial neural networks are trained according to the Levenberg-Marquardt learning algorithm and optimal weights are obtained. The trained artificial neural network learns the relation between consecutive time series observations and membership values of sets.

Figure 2. Architecture of feed forward artificial neural network for three sets



Step 5. *Defuzzification of forecasts.*

In order to obtain fuzzy forecasts of the fuzzy time series at time t*, membership values of observations belonging to the fuzzy sets at time* $t-1$*, depending on the fuzzy set center* v_r*,* $r = 1, 2, \ldots, c$ *which was obtained by the FCM method, were determined and then these membership values were entered to the feed forward artificial neural networks as inputs and thus the outputs of the feed forward artificial neural networks were created. These outputs represent the membership values for a fuzzy forecast of the observation at time* t*. It must be noted that the sum of the membership values obtained for a fuzzy forecast value is not equal to 1, contrary to the FCM method. In the defuzzification step, membership values of fuzzy forecasts are converted to weights as in equation (17) and the defuzzified forecast is obtained as in equation (18).*

$$w_{it} = \frac{u_{it}}{u_{1t}, + u_{2t} + \ldots u_{ct}}, \tag{17}$$

$$X_t = \sum_{i=1}^{c} w_{it} v_i, \tag{18}$$

Here, $\widehat{u}_{it}$ *are the membership values of observation obtained from the outputs of the feed forward artificial neural network at time* t*, and* w_{it} *are the weights used in determining the fuzzified forecasts.*

5. Application

The proposed method was applied to the time series of the amount of sulfur dioxide in Ankara province between March 1994 and April 2006 (ANSO). In order to evaluate the performance of the proposed method, the last 10 observations were taken as a test set and the results obtained were compared with some conventional and alternative time series methods. In the application, in order to determine the order of the fuzzy time series forecasting model, the crisp time series was analyzed using the Box-Jenkins method, the optimal SARIMA model was determined and the residual time series (a_t) as well as the time series X_t obtained. In this step, the optimal model for the ANSO time series was SARIMA$(1,1,0)(0,1,1)_{12}$. As a linear function of X_t, this model can be expressed as;

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-12}). \tag{19}$$

Thus, the model will be a $(5,1)^{th}$-order partial high order fuzzy time series forecasting model where $k = 5$ and $l = 1$. This model can be expressed as;

$$F(t-1), F(t-2), F(t-12), F(t-13), F(t-14), G(t-12) \to F(t). \tag{20}$$

After determining the model order of the partial high order model, the lagged variables data set for each lagged variable that should be included in the model is created. The lagged variables data set for the $(5,1)^{th}$ order partial model was created using $X_t, X_{t-1}, X_{t-11}, X_{t-12}, X_{t-13}, a_{t-11}$ lagged variables. Here, it must be noted that the lagged variables data set consists of the one step leaded variable in the partial high order fuzzy time series forecasting model given in Song [19]. The data set created was clustered via FCM. Clustering was applied to all the lagged variable data sets together. In this step, the data set is clustered by shifting the number of sets 5 to 20. The Membership values of the observations belonging to each fuzzy set are also determined via the FCM method. The relationship between these membership values, in other words, the number of neurons in the hidden layer of the feed forward artificial neuron network which is used in determining the fuzzy relation were shifted between 1 and 15. In the light of this information, $16 \times 15 = 240$ different analyses were done and Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE) and Direction Accuracy were used as performance evaluation criteria.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} (X_t - \widehat{X}_t)^2}{n}},$$

$$\mathrm{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{X_t - \widehat{X}_t}{X_t} \right|.$$

$$\mathrm{DA} = \frac{1}{n-1} \sum_{t=1}^{n-1} \begin{cases} 1, & (X_{t+1} - X_t)(\widehat{X}_{t+1} - X_t) > 0 \\ 0, & o.w. \end{cases}$$

as a consequence of all the analyses, the best forecasting performance was obtained for the 14$^{\text{th}}$ set and the hidden layer unit number was 7. The results obtained from the proposed method and the results for some other methods are summarized in Table 1.

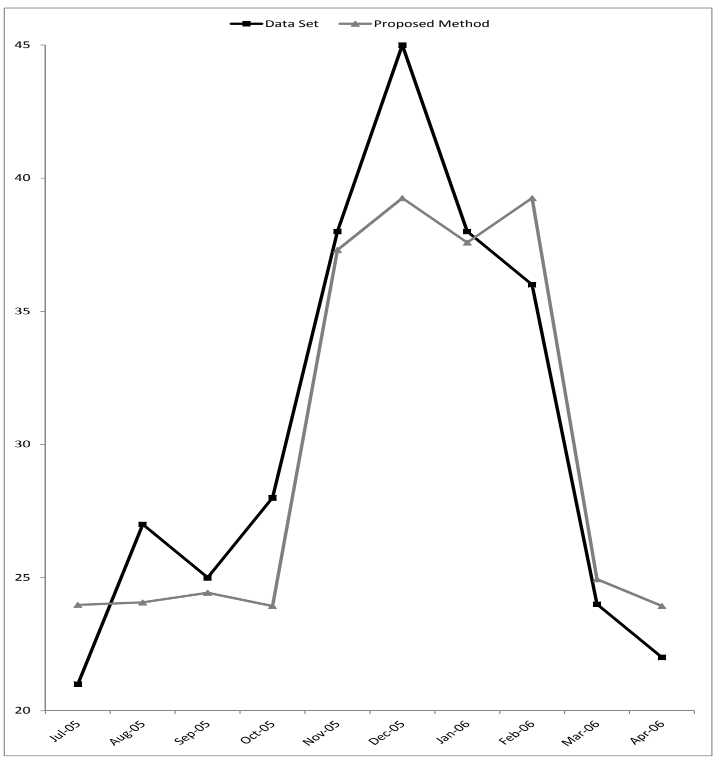
When Table 1 is analyzed, it is seen that the proposed method, which uses membership values in the determination of the fuzzy relations and in the defuzzification step, exhibits superior performance compared to both conventional time series approaches and fuzzy time series approaches. Moreover, the superior performance of the method can also be seen from Figure 3 in which the graph of the results obtained from the proposed method and real time series are presented.

Table 1. Results for the various methods

Data Set	SARIMA	WMES	[19]	[10]	[23]	Proposed Method
21	22.93	15.40	41.6667	20.00	22.7536	23.9734
27	22.35	16.11	27.5000	30.00	22.7536	24.0668
25	23.61	17.77	41.6667	20.00	22.7536	24.4291
28	28.81	25.12	41.6667	30.00	22.7536	23.9320
38	46.97	41.11	41.6667	30.00	42.0558	37.3027
45	54.62	46.12	46.7857	50.00	42.0558	39.2519
38	58.13	49.80	45.0000	40.00	42.0558	37.5819
36	46.99	44.24	46.7857	30.00	42.0558	39.2519
24	37.85	31.96	46.7857	30.00	22.7336	24.9392
22	24.76	18.39	27.5000	20.00	22.7536	23.9320
RMSE	9.62	7.11	12.74	4.56	3.66	2.88
MAPE	0.23	0.22	0.40	0.13	0.11	0.07
DA	55.56	66.67	44.44	100.00	77.78	77.78

WMES: Winters Multicaptive Exponential Smoothing

Figure 3. The graph of the results obtained from the proposed method and real time series



6. Discussion and conclusion

Although most of the methods proposed in the literature involve high order models, these methods fail to analyze seasonal fuzzy time series. Additionally, the few seasonal fuzzy time series forecasting methods proposed in the literature have some advantages as well as some substantial shortcomings. Probably, the most important one is ignoring the membership values representing the order of observation belonging to fuzzy sets in determining the fuzzy relations and the defuzzification steps. This leads to information loss and thus affects forecasting performance negatively. The seasonal fuzzy time series forecasting method proposed in this study eliminates this problem by considering the membership value and presents fuzzy relations more realistically. At this point, another issue worth-emphasizing is that the number of inputs of an artificial neural network used for relation determination in high order models is too high. In the proposed method, this problem was eliminated by clustering the data set which was composed of lagged variables and the input layer unit number of the feed forward artificial neural network was limited by the cluster number. It is evident that the partial high order seasonal fuzzy time series forecasting method which was proposed in this study and in which model order was determined via SARIMA and ANN was used in determining fuzzy relations has some advantages and exhibits superior forecasting performance.

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