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ON WEAKLY e-CONTINUOUS FUNCTIONS

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Abstract

The main goal of this paper is to introduce and look into some of the fundamental properties of weakly e-continuous functions defined via e-open sets introduced by E. Ekici (*On e-open sets, $\mathcal{DP}^*$-sets and $\mathcal{DPE}^*$-sets and decompositions of continuity*, Arab. J. Sci. Eng. **33** (2A), 269–281, 2008). Some characterizations and several properties concerning weakly e-continuous functions are obtained. The concept of weak e-continuity is weaker than both the weak continuity introduced by N. Levine (*A decomposition of continuity in topological spaces*, Amer. Math. Monthly **68**, 44–46, 1961) and the e-continuity introduced by Ekici, but stronger than weak β-continuity introduced by Popa and Noiri (*Weakly β-continuous functions*, An. Univ. Timis. Ser. Mat.-Inform. **32** (2), 83–92, 1994). In order to investigate some different properties we introduce the concept of e-strongly closed graphs and also investigate relationships between weak e-continuity and separation axioms, and e-strongly closed graphs and covering properties.

Keywords: Faint e-continuity, e-T_2 space, e-strongly closed graph, e-Lindelöf space, Weak e-continuity.

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1. Introduction

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represent nonempty topological spaces on which no separation axioms are assumed unless otherwise stated. Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{cl}(A)$ and $\mathrm{int}(A)$, respectively. $\mathcal{U}(x)$ denotes all open neighborhoods of the point $x \in X$.

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A subset A of a space X is called *regular open* [23] (resp. *regular closed* [23]) if $A = \text{int}(\text{cl}(A))$ (resp. $A = \text{cl}(\text{int}(A))$). A subset A of a space X is called *δ-semiopen* [19] (resp. *preopen* [12], *δ-preopen* [22], *α-open* [14], *semi-preopen* [3] or *β-open* [1], *b-open* [2] or *sp-open* [5] or *γ-open* [8], e-open [7]) if $A \subset \text{cl}(\text{int}_\delta(A))$ (resp. $A \subset \text{int}(\text{cl}(A))$, $A \subset \text{int}(\text{cl}_\delta(A))$, $A \subset \text{int}(\text{cl}(\text{int}(A)))$, $A \subset \text{cl}(\text{int}(\text{cl}(A)))$, $A \subset \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$, $A \subset \text{int}(\text{cl}_\delta(A)) \cup \text{cl}(\text{int}_\delta(A))$).

The complement of a δ-semiopen (resp. preopen, δ-preopen, α-open, β-open, b-open, e-open) set is said to be *δ-semiclosed* (resp. *preclosed*, *δ-preclosed*, *α-closed*, *β-closed*, *b-closed*, *e-closed*).

The family of all δ-semiopen (resp. preopen, δ-preopen, α-open, β-open, b-open, e-open) sets of X are denoted by $\delta SO(X)$ (resp. $PO(X)$, $\delta PO(X)$, $\alpha O(X)$, $\beta O(X)$, $BO(X)$, $eO(X)$). The family of all e-closed sets of X is denoted by $eC(X)$ and the family of all e-open sets of X containing a point $x \in X$ is denoted by $eO(X, x)$.

A set A is called *θ-open* [11] if every point of A has an open neighborhood whose closure is contained in A. The *θ-interior* [11] of A in X is the union of all θ-open subsets of A and is denoted by $\text{int}_\theta(A)$. Naturally, the complement of a θ-open set is called *θ-closed* [11]. Equivalently $\text{cl}_\theta(A) = \{x \mid U \in \mathcal{U}(x) \Rightarrow \text{cl}(U) \cap A \neq \emptyset\}$, and a set A is θ-closed if and only if $A = \text{cl}_\theta(A)$.

A set A is called *δ-open* [24] if every point of A has an open neighborhood whose interior of the closure is contained in A. The *δ-interior* [24] of A in X is the union of all δ-open subsets of A, and is denoted by $\text{int}_\delta(A)$. Naturally, the complement of a δ-open set is called *δ-closed* [24]. Equivalently $\text{cl}_\delta(A) = \{x \mid U \in \mathcal{U}(x) \Rightarrow \text{int}(\text{cl}(U)) \cap A \neq \emptyset\}$, and a set A is δ-closed if and only if $A = \text{cl}_\delta(A)$.

If A is a subset of a space X, then the *e-closure* of A, denoted by e-cl(A), is the smallest e-closed set containing A. The *e-interior* of A, denoted by e-int(A), is the largest e-open set contained in A.

1.1. Definition. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(a) *e-continuous* [7] (briefly, *e.c.*) if $f^{-1}(V)$ is e-open in (X, τ) for every open set V of (Y, σ);
(b) *Weakly continuous* [5] (briefly *w.c.*) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mathcal{U}(x)$ such that $f(U) \subset \text{cl}(V)$;
(c) *Weakly β-continuous* [21] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a β-open U of X containing x such that $f(U) \subset \text{cl}(V)$.

1.2. Lemma. [19,22] *The following properties hold for a set A in a space X:*

(a) δ-sint$(A) = A \cap \text{cl}(\text{int}_\delta(A))$;
(b) δ-pint$(A) = A \cap \text{int}(\text{cl}_\delta(A))$. □

1.3. Lemma. [7] *The following properties hold for a set A in a space X:*

(a) e-cl$(A) = A \cup (\text{int}(\text{cl}_\delta(A)) \cap \text{cl}(\text{int}_\delta(A)))$;
(b) e-int$(A) = A \cap (\text{int}(\text{cl}_\delta(A)) \cup \text{cl}(\text{int}_\delta(A)))$;
(c) e-cl$(X \backslash A) = X \backslash e$-int$(A)$;
(d) $x \in e$-cl(A) *if and only if* $A \cap U \neq \emptyset$ *for every* $U \in eO(X, x)$;
(e) $A \in eC(X)$ *if and only if* $A = e$-cl(A);
(f) e-int$(A) = \delta$-sint$(A) \cup \delta$-pint(A). □

We get the following lemma from the definition of e-continuity.

1.4. Lemma. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(a) *f is e-continuous.*

(b) *For each* $x \in X$ *and each* $V \in \mathcal{U}(f(x))$, *there exists* $U \in eO(X, x)$ *such that* $f(U) \subset V$.
(c) *The inverse image of each closed set in* Y *is* e-*closed in* X.
(d) $\mathrm{int}(\mathrm{cl}_\delta(f^{-1}(B))) \cap \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(B))) \subset f^{-1}(\mathrm{cl}(B))$ *for each* $B \subset Y$.
(e) $f(\mathrm{int}(\mathrm{cl}_\delta(A)) \cap \mathrm{cl}(\mathrm{int}_\delta(A))) \subset \mathrm{cl}(f(A))$ *for each* $A \subset X$.

Proof. (a) $\Longrightarrow$ (b) Let $x \in X$ and $V \in \mathcal{U}(f(x))$. Then $f^{-1}(V) \in eO(X, x)$. Set $U = f^{-1}(V)$ which contains x, then $f(U) \subset V$.

(b) $\Longrightarrow$ (a) Let $V \subset Y$ be open and $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists $U_x \in eO(X, x)$ such that $f(U_x) \subset V$. Then $x \in U_x \subset f^{-1}(V)$, and so $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Since the union of any family of e-open sets is an e-open set, we have $\bigcup_{x \in f^{-1}(V)} U_x \in eO(X)$. Then $f^{-1}(V) \in eO(X)$. Therefore, f is e-continuous.

(a) $\Longrightarrow$ (c) Clear.

(c) $\Longrightarrow$ (a) Clear.

(c) $\Longrightarrow$ (d) Let $B \subset Y$. Then $f^{-1}(\mathrm{cl}(B))$ is e-closed in X, i.e.

$$\begin{aligned}\mathrm{int}(\mathrm{cl}_\delta(f^{-1}(B))) \cap \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(B))) &\subset \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(B)))) \cap \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(B)))) \\ &\subset f^{-1}(\mathrm{cl}(B)).\end{aligned}$$

(d) $\Longrightarrow$ (e) Let $A \subset X$. Set $B = f(A)$ in (d), then

$$\mathrm{int}(\mathrm{cl}_\delta(f^{-1}(f(A)))) \cap \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(f(A)))) \subset f^{-1}(\mathrm{cl}(f(A))),$$

so that $\mathrm{int}(\mathrm{cl}_\delta(A)) \cap \mathrm{cl}(\mathrm{int}_\delta(A)) \subset f^{-1}(\mathrm{cl}(f(A)))$. This gives $f(\mathrm{int}(\mathrm{cl}_\delta(A)) \cap \mathrm{cl}(\mathrm{int}_\delta(A))) \subset \mathrm{cl}(f(A))$.

(e) $\Longrightarrow$ (a) Let $V \in \sigma$. Set $W = Y \backslash V$ and $A = f^{-1}(W)$. Then

$$\begin{aligned}f(\mathrm{int}(\mathrm{cl}_\delta(f^{-1}(Y \backslash V))) \cap \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(Y \backslash V)))) &\subset \mathrm{cl}(f(f^{-1}(Y \backslash V))) \\ &\subset \mathrm{cl}(Y \backslash V) = Y \backslash V,\end{aligned}$$

that is $f^{-1}(W)$ is e-closed in X, so f is e-continuous. □

2. Weakly e-continuous functions

In this section we obtain some characterizations and several properties concerning weakly e-continuous functions. Also, by defining faintly e-continuous functions we investigate relationships between faintly e-continuous functions and strongly θ-e-continuous functions and weakly e-continuous functions.

2.1. Definition. Let (X, τ) and (Y, σ) be topological spaces. $f : (X, \tau) \to (Y, \sigma)$ is a *weakly e-continuous* (briefly a *w.e.c.*) function at $x \in X$ if for each open set V of Y containing $f(x)$ there exists $U \in eO(X, x)$ such that $f(U) \subset \mathrm{cl}(V)$. The function f is w.e.c. iff f is *w.e.c.* for all $x \in X$.

2.2. Remark. From Definition 1.1 and Definition 2.1, we have the following diagram. The converses of these implications are not true in general, as shown in the following examples.

e-continuity
↓
weak continuity ⟶ weak e-continuity ⟶ weak β-continuity

2.3. Example. Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, X, \{b, c, d\}\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is weakly e-continuous but not e-continuous.

2.4. Example. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Consider the function $f : (X, \tau) \to (X, \sigma)$ defined as follows: $f(a) = a,\ f(b) = d,\ f(c) = c,\ f(d) = b$. Then f is weakly β-continuous but not weakly e-continuous.

2.5. Example. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\emptyset, X, \{c\}, \{a, b\}\}$. Then the identity $f : (X, \tau) \to (X, \sigma)$ is weakly e-continuous but not weakly continuous.

2.6. Lemma. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(a) *f is w.e.c. at $x \in X$;*
(b) *$x \in \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(V)))) \cup \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(V))))$ for each open neighborhood V of $f(x)$;*
(c) *$f^{-1}(V) \subset e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)))$ for each $V \in \sigma$.*

Proof. (a) $\Longrightarrow$ (b) Let $V \in \mathcal{U}(f(x))$. Since f is w.e.c. at x, there exists $U \in eO(X, x)$ such that $f(U) \subset \mathrm{cl}(V)$. Then $U \subset f^{-1}(\mathrm{cl}(V))$. Since U is e-open,

$$x \in U \subset \mathrm{cl}(\mathrm{int}_\delta(U)) \cup \mathrm{int}(\mathrm{cl}_\delta(U)) \subset \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(V)))) \cup \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(V)))).$$

(b) $\Longrightarrow$ (c) Let $x \in f^{-1}(V)$, so $f(x) \in V$. Then $x \in f^{-1}(\mathrm{cl}(V))$, and since $x \in \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(V)))) \cup \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(V))))$ we have

$$x \in f^{-1}(\mathrm{cl}(V)) \cap [\mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(V)))) \cup \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(V))))] = e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V))).$$

Hence $f^{-1}(V) \subset e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)))$.

(c) $\Longrightarrow$ (a) Let $V \in \mathcal{U}(f(x))$. Then $x \in f^{-1}(V) \subset e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)))$. Set $U = e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)))$. Then $U \in eO(X, x)$ and $f(U) \subset \mathrm{cl}(V)$. This shows that f is w.e.c. at $x \in X$. □

2.7. Theorem. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(a) *f is w.e.c.;*
(b) *$e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}(B)))) \subset f^{-1}(\mathrm{cl}(B))$ for every subset B of Y;*
(c) *$e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(F))) \subset f^{-1}(F)$ for every regular closed set F of Y;*
(d) *$e\text{-}\mathrm{cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{cl}(V))$ for every open set V of Y;*
(e) *$f^{-1}(V) \subset e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)))$ for every open set V of Y;*
(f) *$f^{-1}(V) \subset \mathrm{cl}(\mathrm{int}_\delta(f^{-1}(\mathrm{cl}(V)))) \cup \mathrm{int}(\mathrm{cl}_\delta(f^{-1}(\mathrm{cl}(V))))$ for every open set V of Y.*

Proof. (a) $\Longrightarrow$ (b) Let $B \subset Y$. Suppose that $x \in X \backslash f^{-1}(\mathrm{cl}(B))$. Then $f(x) \in Y \backslash \mathrm{cl}(B)$ and there exists an open set V containing $f(x)$ such that $V \cap B = \emptyset$; therefore $\mathrm{cl}(V) \cap \mathrm{int}(\mathrm{cl}(B)) = \emptyset$. Since f is w.e.c. there exists $U \in eO(X, x)$ such that $f(U) \subset \mathrm{cl}(V)$. Therefore, we have $U \cap f^{-1}(\mathrm{int}(\mathrm{cl}(B))) = \emptyset$, hence $x \in X \backslash e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}(B))))$. Thus we obtain $e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}(B)))) \subset f^{-1}(\mathrm{cl}(B))$.

(b) $\Longrightarrow$ (c) Let $F \in RC(Y)$. Then we have

$$e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(F))) = e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}(\mathrm{int}(F))))) \subset f^{-1}(\mathrm{cl}(\mathrm{int}(F))) = f^{-1}(F).$$

(c) $\Longrightarrow$ (d) For every $V \in \sigma$, $\mathrm{cl}(V)$ is regular closed in Y and we have $e\text{-}\mathrm{cl}(f^{-1}(V)) \subset e\text{-}\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}(V)))) \subset f^{-1}(\mathrm{cl}(V))$.

(d) $\Longrightarrow$ (e) Let $V \in \sigma$. Then $Y \backslash \mathrm{cl}(V)$ is open in Y, and using Lemma 1.3 (c) we have

$$X \backslash e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V)) = e\text{-}\mathrm{cl}(f^{-1}(Y \backslash \mathrm{cl}(V))) \subset f^{-1}(\mathrm{cl}(Y \backslash \mathrm{cl}(V))) \subset X \backslash f^{-1}(V).$$

Therefore we obtain $f^{-1}(V) \subset e\text{-}\mathrm{int}(f^{-1}(\mathrm{cl}(V))$.

(e) $\Longrightarrow$ (f) Let $V \in \sigma$. By Lemma 1.3 we have

$$f^{-1}(V) \subset e\text{-int}(f^{-1}(\text{cl}(V))) \subset \text{cl}(\text{int}_\delta(f^{-1}(\text{cl}(V)))) \cup \text{int}(\text{cl}_\delta(f^{-1}(\text{cl}(V)))).$$

(f) $\Longrightarrow$ (a) Let $x \in X$ and $V \in \mathcal{U}(f(x))$. Then

$$x \in f^{-1}(V) \subset \text{cl}(\text{int}_\delta(f^{-1}(\text{cl}(V)))) \cup \text{int}(\text{cl}_\delta(f^{-1}(\text{cl}(V)))).$$

It follows from Lemma 2.6 that f is w.e.c. □

2.8. Theorem. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(a) *f is w.e.c.;*
(b) *$e\text{-cl}(f^{-1}(\text{int}(\text{cl}(V)))) \subset f^{-1}(\text{cl}(V))$ for every e-open subset V of Y;*
(c) *$e\text{-cl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$ for every preopen subset V of Y;*
(d) *$f^{-1}(V) \subset e\text{-int}(f^{-1}(\text{cl}(V)))$ for every preopen subset V of Y;*

Proof. (a) $\Longrightarrow$ (b) Let $V \in eO(Y)$. Since f is w.e.c., from Theorem 2.7 (b) we have $e\text{-cl}(f^{-1}(\text{int}(\text{cl}(V)))) \subset f^{-1}(\text{cl}(V))$.

(b) $\Longrightarrow$ (c) Clear since $PO(Y) \subset eO(Y)$ and $V \subset \text{int}(\text{cl}(V))$.

(c) $\Longrightarrow$ (d) Similar to the proof of the implication (d) $\Longrightarrow$ (e) in Theorem 2.7.

(d) $\Longrightarrow$ (a) This follows from Theorem 2.7 since every open set is preopen. □

2.9. Theorem. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(a) *f is w.e.c.;*
(b) *$f(e\text{-cl}(A)) \subset \text{cl}_\theta(f(A))$ for every subset A of X;*
(c) *$e\text{-cl}(f^{-1}(B)) \subset f^{-1}(\text{cl}_\theta(B))$ for every subset B of Y;*
(d) *$e\text{-cl}(f^{-1}(\text{int}(\text{cl}_\theta(B)))) \subset f^{-1}(\text{cl}_\theta(B))$ for every subset B of Y.*

Proof. (a) $\Longrightarrow$ (b) Let $x \in e\text{-cl}(A)$, V be any open set of Y containing $f(x)$. Then there exists $U \in eO(X, x)$ such that $f(U) \subset \text{cl}(V)$. Then $U \cap A \neq \emptyset$ and $\emptyset \neq f(U) \cap f(A) \subset \text{cl}(V) \cap f(A)$, so that $f(x) \in \text{cl}_\theta(f(A))$.

(b) $\Longrightarrow$ (c) Let $B \subset Y$. Set $A = f^{-1}(B)$ in (b). Then we have $f(e\text{-cl}(f^{-1}(B))) \subset \text{cl}_\theta(B)$ and $e\text{-cl}(f^{-1}(B)) \subset f^{-1}(f(e\text{-cl}(f^{-1}(B)))) \subset f^{-1}(\text{cl}_\theta(B))$.

(c) $\Longrightarrow$ (d) Let B be a subset of Y. Since $\text{cl}_\theta(B)$ is closed in Y, we have

$$e\text{-cl}(f^{-1}(\text{int}(\text{cl}_\theta(B))) \subset f^{-1}(\text{cl}_\theta(\text{int}(\text{cl}_\theta(B)))) \subset f^{-1}(\text{cl}_\theta(B)).$$

(d) $\Longrightarrow$ (a) Let $V \in \sigma$. Then $V \subset \text{int}(\text{cl}(V)) = \text{int}(\text{cl}_\theta(V))$, and hence

$$e\text{-cl}(f^{-1}(V)) \subset e\text{-cl}(f^{-1}(\text{int}(\text{cl}_\theta(V)))) \subset f^{-1}(\text{cl}_\theta(V)) = f^{-1}(\text{cl}(V)).$$

It follows from Theorem 2.7 that f is w.e.c. □

2.10. Corollary. *If $f : (X, \tau) \to (Y, \sigma)$ is w.e.c., then $f^{-1}(V)$ is e-closed (resp. e-open) in X for every θ-closed (resp. θ-open) subset V of Y.*

Proof. If V is θ-closed, Theorem 2.9 (c) gives $e\text{-cl}(f^{-1}(V)) \subset f^{-1}(\text{cl}_\theta(V)) = f^{-1}(V)$, so $f^{-1}(V)$ is e-closed. If V is θ-open, $Y \backslash V$ is θ-closed and Theorem 2.9 gives

$$e\text{-cl}(f^{-1}(Y \backslash V)) \subset f^{-1}(\text{cl}_\theta(Y \backslash V)) = f^{-1}(Y \backslash V).$$

Now $e\text{-cl}(X \backslash f^{-1}(V)) \subset X \backslash f^{-1}(V)$, and then $X \backslash e\text{-int}(f^{-1}(V)) \subset X \backslash f^{-1}(V)$, so that $f^{-1}(V) \subset e\text{-int}(f^{-1}(V))$ and $f^{-1}(V)$ is e-open. □

2.11. Corollary. *If $f^{-1}(\text{cl}_\theta(B))$ is e-closed in X for every subset B of Y, then a function $f : (X, \tau) \to (Y, \sigma)$ is w.e.c.*

Proof. Since $f^{-1}(\mathrm{cl}_\theta(B))$ is e-closed in X, we have $e\text{-cl}(f^{-1}(B)) \subset e\text{-cl}(f^{-1}(\mathrm{cl}_\theta(B))) = f^{-1}(\mathrm{cl}_\theta(B))$, and by Theorem 2.9, f is w.e.c. □

2.12. Definition. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *faintly e-continuous* (briefly, *f.e.c.*) if for each $x \in X$ and each θ-open set V of Y containing $f(x)$, there exists an e-open subset U of X containing x such that $f(U) \subset V$.

2.13. Definition. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be strongly θ-e-continuous [17] (briefly, *st.θ.e.c.*) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an e-open set U of X containing x such that $f(e\text{-cl}(U)) \subset V$.

2.14. Lemma. *Let Y be a regular space. Then $f : X \to Y$ is st.θ.e.c. if and only if f is e-continuous.*

Proof. Let $x \in X$ and V an open subset of Y containing $f(x)$. Since Y is regular, there exists an open set W such that $f(x) \in W \subset \mathrm{cl}(W) \subset V$. If f is e-continuous, there exists $U \in eO(X, x)$ such that $f(U) \subset W$. We shall show that $f(e\text{-cl}(U)) \subset \mathrm{cl}(W)$. Suppose that $y \notin \mathrm{cl}(W)$. There exists an open set G containing y such that $G \cap W = \emptyset$. Since f is e-continuous, $f^{-1}(G) \in eO(X)$ and $f^{-1}(G) \cap U = \emptyset$, and hence $f^{-1}(G) \cap e\text{-cl}(U) = \emptyset$. Therefore, we obtain $G \cap f(e\text{-cl}(U)) = \emptyset$ and $y \notin f(e\text{-cl}(U))$. Consequently, we have $f(e\text{-cl}(U)) \subset \mathrm{cl}(W) \subset V$. The converse is obvious. □

2.15. Lemma. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:*

(a) *f is faintly e-continuous;*
(b) *The inverse image of every θ-open set in Y is e-open in X;*
(c) *The inverse image of every θ-closed set in Y is e-closed in X.*

Proof. (a) $\Longrightarrow$ (b) Let $V \subset Y$ be θ-open and $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists $U_x \in eO(X, x)$ such that $f(U_x) \subset V$. Then $x \in U_x \subset f^{-1}(V)$, and so $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Since the union of any family of e-open sets is an e-open set, we have $\bigcup_{x \in f^{-1}(V)} U_x \in eO(X)$. Then $f^{-1}(V) \in eO(X)$.

(b) $\Longrightarrow$ (a) Let $x \in X$ and $V \in \theta O(Y, f(x))$. Then $f^{-1}(V) \in eO(X, x)$. Set $U = f^{-1}(V)$ which contains x, then $f(U) \subset V$.

(a) $\Longrightarrow$ (c) Similar to (a) $\Longrightarrow$ (b) since the complement of every θ-closed set is θ-open.

(c) $\Longrightarrow$ (a) Similar to (b) $\Longrightarrow$ (a) since the complement of every θ-closed set is θ-open.

(b) $\Longrightarrow$ (c) Routine.

(c) $\Longrightarrow$ (b) Routine. □

2.16. Theorem. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and Y a regular space. Then the following are equivalent:*

(a) *f is st.θ.e.c.;*
(b) *f is e-continuous;*
(c) *$f^{-1}(\mathrm{cl}_\theta(B))$ is e-closed in X for every subset B of Y;*
(d) *f is w.e.c.;*
(e) *f is f.e.c.*

Proof. (a) $\Longrightarrow$ (b) Let $x \in X$ and let V be an open subset of Y containing $f(x)$. Then there exists $U \in eO(X, x)$ such that $f(e\text{-cl}(U)) \subset V$ but $f(U) \subset f(e\text{-cl}(U)) \subset V$, hence f is e.c.

(b) $\Longrightarrow$ (c) Since $\mathrm{cl}_\theta(B)$ is closed in Y for every subset B of Y, by Lemma 1.4 (c) $f^{-1}(\mathrm{cl}_\theta(B))$ is e-closed in X.

(c) $\Longrightarrow$ (d) Corollary 2.11.

(d) $\Longrightarrow$ (e) Corollary 2.10 and Lemma 2.15.

(e) $\Longrightarrow$ (a) Let V be any open subset of Y. Since Y is regular, V is θ-open in Y. By the faint e-continuity of f, $f^{-1}(V)$ is e-open in X. Therefore f is e-continuous, and then according to Lemma 2.14, f is st.θ.e.c. since Y is regular. □

2.17. Remark. Faint e-continuity does not imply strong θ-e-continuity.

2.18. Example. Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \emptyset\}$ and $\sigma = \{\emptyset, X, \{b, c, d\}\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is f.e.c. but not st.θ.e.c.

3. Some properties

3.1. Theorem. *If $f : (X, \tau) \to (Y, \sigma)$ is w.e.c. and $g : (Y, \sigma) \to (Z, \rho)$ is continuous, then the composition $g \circ f : (X, \tau) \to (Z, \rho)$ is w.e.c.*

Proof. Let $x \in X$ and $g(f(x)) \in W \in \rho$. Then $g^{-1}(W)$ is an open subset of Y containing $f(x)$, and there exists $U \in eO(X, x)$ such that $f(U) \subset \text{cl}(g^{-1}(W))$. Since g is continuous, we obtain $g(f(U)) \subset g(\text{cl}(g^{-1}(W))) \subset g(g^{-1}(\text{cl}(W))) \subset \text{cl}(W)$. □

3.2. Theorem. *If $g \circ f : (X, \tau) \to (Z, \rho)$ is w.e.c. and $f : (X, \tau) \to (Y, \sigma)$ is an open continuous surjection, then $g : (Y, \sigma) \to (Z, \rho)$ is w.e.c.*

Proof. Let $W \in \rho$. Since $g \circ f : (X, \tau) \to (Z, \rho)$ is w.e.c. and f is continuous we have $(g \circ f)^{-1}(W) \subset \text{cl}(\text{int}_\delta((g \circ f)^{-1}(\text{cl}(W)))) \cup \text{int}(\text{cl}_\delta((g \circ f)^{-1}(\text{cl}(W)))) = \text{cl}(\text{int}_\delta(f^{-1}(g^{-1}(\text{cl}(W))))) \cup \text{int}(\text{cl}_\delta(f^{-1}(g^{-1}(\text{cl}(W)))))$. Since f is an open continuous surjection, we have $g^{-1}(W) = f(f^{-1}(g^{-1}(W)))$ and

$$\begin{aligned} g^{-1}(W) &\subset f(\text{cl}(\text{int}_\delta(f^{-1}(g^{-1}(\text{cl}(W)))))) \cup f(\text{int}(\text{cl}_\delta(f^{-1}(g^{-1}(\text{cl}(W)))))) \\ &\subset \text{cl}(\text{int}_\delta(f(f^{-1}(g^{-1}(\text{cl}(W)))))) \cup \text{int}(\text{cl}_\delta(f(f^{-1}(g^{-1}(\text{cl}(W)))))) \\ &\subset \text{cl}(\text{int}_\delta(g^{-1}(\text{cl}(W)))) \cup \text{int}(\text{cl}_\delta(g^{-1}(\text{cl}(W)))), \end{aligned}$$

and by Theorem 2.7, g is w.e.c. □

Let $\{X_\alpha \mid \alpha \in I\}$ and $\{Y_\alpha \mid \alpha \in I\}$ be any two families of spaces with the same index set I. Let $f_\alpha : X_\alpha \to Y_\alpha$ be a function for each $\alpha \in I$. The product space $\Pi\{X_\alpha \mid \alpha \in I\}$ will be denoted by ΠX_α and $f : \Pi X_\alpha \to \Pi Y_\alpha$ will denote the product function defined by $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ for every $\{x_\alpha\} \in \Pi X_\alpha$. Moreover, let $p_\beta : \Pi X_\alpha \to X_\beta$ and $q_\beta : \Pi Y_\alpha \to Y_\beta$ be the natural projections. Then we have the following theorem.

3.3. Theorem. *If a function $f : \Pi X_\alpha \to \Pi Y_\alpha$ is w.e.c., then $f_\alpha : X_\alpha \to Y_\alpha$ is w.e.c. for each $\alpha \in I$.*

Proof. Suppose that f is w.e.c. Let $\beta \in I$. Since q_β is continuous, by Theorem 3.1, $q_\beta \circ f = f_\beta \circ p_\beta$ is w.e.c. Moreover, p_β is an open continuous surjection so by Theorem 3.2, f_β is w.e.c. □

4. Separation axioms and graph properties

In this section we define an e-strongly closed graph. We look into some relationships between weakly e-continuous functions and e-T_1 spaces and e-T_2 spaces.

4.1. Definition. A space X is called:

(a) Urysohn [27] if for each pair of distinct points x and y in X, there exist open sets U and V such that $x \in U$, $y \in V$ and $\mathrm{cl}(U) \cap \mathrm{cl}(V) = \emptyset$;
(b) e-T_1 [6] if for each pair of distinct points x and y in X, there exist e-open sets U and V of X containing x and y, respectively, such that $y \notin U$ and $x \notin V$;
(c) e-T_2 [6] if for each pair of distinct points x and y in X, there exist e-open sets U and V of X containing x and y, respectively, such that $U \cap V = \emptyset$.

4.2. Theorem. *Let $f : (X, \tau) \to (Y, \sigma)$ be a w.e.c. injective function. Then the following hold:*

(a) *If Y is Urysohn, then X is e-T_2.*
(b) *If Y is Hausdorff, then X is e-T_1.*

Proof. (a) Let x_1 and x_2 be any distinct points in X. Then $f(x_1) \neq f(x_2)$ and there exist open sets V_1 and V_2 of Y containing $f(x_1)$ and $f(x_2)$, respectively, such that $\mathrm{cl}(V_1) \cap \mathrm{cl}(V_2) = \emptyset$. Since f is w.e.c. there exists $U_i \in eO(X, x_i)$ such that $f(U_i) \subset \mathrm{cl}(V_i)$, for $i = 1, 2$. Since $f^{-1}(\mathrm{cl}(V_1))$ and $f^{-1}(\mathrm{cl}(V_2))$ are disjoint, we obtain $U_1 \cap U_2 = \emptyset$. Hence X is e-T_2.

(b) Let x_1 and x_2 be any distinct points in X. Then $f(x_1) \neq f(x_2)$ and there exist open sets V_1 and V_2 of Y such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$. Then we obtain $f(x_1) \notin \mathrm{cl}(V_2)$ and $f(x_2) \notin \mathrm{cl}(V_1)$. Since f is w.e.c., there exists $U_i \in eO(X, x_i)$ such that $f(U_i) \subset \mathrm{cl}(V_i)$, for $i = 1, 2$. Hence we obtain $x_2 \notin U_1$ and $x_1 \notin U_2$. This shows that X is e-T_1. □

4.3. Theorem. *If $g : (X, \tau) \to (Y, \sigma)$ is w.e.c. and A is a θ-closed subset of $X \times Y$ then $p_X(A \cap G(g))$ is e-closed in X, where p_X represents the projection of $X \times Y$ onto X and $G(g)$ denotes the graph of g.*

Proof. Let A be a θ-closed subset of $X \times Y$ and $x \in e\text{-}\mathrm{cl}(p_X(A \cap G(g)))$. Let U be any open subset of X containing x, and V any open set of Y containing $g(x)$. Since g is w.e.c., by Theorem 2.7, we have $x \in g^{-1}(V) \subset e\text{-}\mathrm{int}(g^{-1}(\mathrm{cl}(V)))$ and $U \cap e\text{-}\mathrm{int}(g^{-1}(\mathrm{cl}(V))) \in eO(X, x)$. Since $x \in e\text{-}\mathrm{cl}(p_X(A \cap G(g)))$ by Lemma 1.3,

$$[U \cap e\text{-}\mathrm{int}(g^{-1}(\mathrm{cl}(V)))] \cap p_X(A \cap G(g))$$

contains some point u of X. This implies that $(u, g(u)) \in A$ and $g(u) \in \mathrm{cl}(V)$. Thus we have $\emptyset \neq (U \times \mathrm{cl}(V)) \cap A \subset \mathrm{cl}(U \times V) \cap A$ and hence $(x, g(x)) \in \mathrm{cl}_\theta(A)$. Since A is θ-closed, $(x, g(x)) \in A \cap G(g)$ and $x \in p_X(A \cap G(g))$. Then $p_X(A \cap G(g))$ is e-closed by Lemma 1.3. □

4.4. Corollary. *If $f : (X, \tau) \to (Y, \sigma)$ has a θ-closed graph and $g : (X, \tau) \to (Y, \sigma)$ is w.e.c., then the set $\{x \in X \mid f(x) = g(x)\}$ is e-closed in X.*

Proof. Since $G(f)$ is θ-closed and $p_X(G(f) \cap G(g)) = \{x \in X \mid f(x) = g(x)\}$, it follows from Theorem 4.3 that $\{x \in X \mid f(x) = g(x)\}$ is e-closed. □

4.5. Definition. A function $f : (X, \tau) \to (Y, \sigma)$ is said to have an *e-strongly closed graph* if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist an e-open subset U of X and an open subset V of Y such that $(x, y) \in U \times V$ and $(U \times \mathrm{cl}(V)) \cap G(f) = \emptyset$. □

4.6. Theorem. *If Y is a Urysohn space and $f:(X,\tau)\to(Y,\sigma)$ is w.e.c., then $G(f)$ is e-strongly closed.*

Proof. Let $(x,y)\in(X\times Y)\backslash G(f)$. Then $y\neq f(x)$, and there exist open sets V and W of Y containing $f(x)$ and y, respectively, such that $\mathrm{cl}(V)\cap\mathrm{cl}(W)=\emptyset$. Since f is w.e.c., there exists an e-open subset U of X containing x such that $f(U)\subset\mathrm{cl}(V)$. Therefore, we obtain $f(U)\cap\mathrm{cl}(V)=\emptyset$. Since f is w.e.c., there exists a $W\in eO(X,x_2)$ such that $f(W)\subset\mathrm{cl}(V)$. Therefore, we have $f(U)\cap f(W)=\emptyset$, and hence $U\cap W=\emptyset$. This shows that X is e-T_2. □

4.7. Theorem. *Let $f:(X,\tau)\to(Y,\sigma)$ be a w.e.c. function having an e-strongly closed graph $G(f)$. If f is injective, then X is e-T_2.*

Proof. Let x_1 and x_2 be two distinct points of X. Since f is injective, $f(x_1)\neq f(x_2)$ and $(x_1,f(x_2))\notin G(f)$. Since $G(f)$ is e-strongly closed, there exist $U\in eO(X,x_1)$ and an open subset V of Y such that $(x_1,f(x_2))\in U\times V$ and $(U\times\mathrm{cl}(V))\cap G(f)=\emptyset$, and hence $f(U)\cap\mathrm{cl}(V)=\emptyset$. Since f is w.e.c., there exists a $W\in eO(X,x_2)$ such that $f(W)\subset\mathrm{cl}(V)$. Therefore, we have $f(U)\cap f(W)=\emptyset$ and hence $U\cap W=\emptyset$. This shows that X is e-T_2. □

5. Covering properties

Finally in this last section, by defining the notion of e-Lindelöf space we investigate some relationships between e-compact spaces and e-Lindelöf spaces and weakly e-continuous functions.

5.1. Definition. A Hausdorff space X is called *semicompact* [28] at a point x if every neighborhood U_x contains a V_x such that $B(V_x)$, the boundary of V_x, is compact. It is called semicompact if it has this property at every point.

5.2. Theorem. *If Y is a semicompact Hausdorff space and $f:(X,\tau)\to(Y,\sigma)$ is w.e.c., then f is e-continuous.*

Proof. Every semicompact Hausdorff space is regular, and it follows from Theorem 2.16 that f is e-continuous. □

5.3. Definition. A subset A of a space X is said to be an *H-set* [24] or to be *quasi H-closed relative to X* [20] if for every cover $\{U_\alpha \mid \alpha\in I\}$ of A by open sets of X, there exists a finite subset I_0 of I such that $A\subset\bigcup\{\mathrm{cl}(U_\alpha)\mid\alpha\in I_0\}$.

5.4. Definition. A topological space (X,τ) is said to be

(a) *e-compact* [6] (resp. *e-Lindelöf*) if every e-open cover of X has a finite (resp. countable) subcover;
(b) *Almost compact* [15] or *quasi H-closed* [20] if every cover of X by open sets has a finite subcover whose closures cover X;
(c) *Almost Lindelöf* [26] if every cover of X by open sets has a countable subcover whose closures cover X;
(d) *C-compact* [25] if for each closed subset $A\subset X$ and each open cover $\{U_\alpha:\alpha\in I\}$ of A, there exists a finite subset I_0 of I such that $A\subset\bigcup\{\mathrm{cl}(U_\alpha)\mid\alpha\in I_0\}$.

5.5. Theorem. *Let $f:(X,\tau)\to(Y,\sigma)$ be a w.e.c. surjection. Then the following hold:*

(a) *If X is e-compact, then Y is almost compact.*
(b) *If X is e-Lindelöf, then Y is almost Lindelöf.*

Proof. (a) Let $\{V_\alpha \mid \alpha \in I\}$ be a cover of Y by open subsets of Y. For each point $x \in X$ there exists $\alpha(x) \in I$ such that $f(x) \in V_{\alpha(x)}$. Since f is w.e.c., there exists an e-open set U_x of X containing x such that $f(U_x) \subset \mathrm{cl}(V_{\alpha(x)})$. The family $\{U_x \mid x \in X\}$ is a cover of X by e-open subsets of X, and hence there exists a finite subset X_0 of X such that $X = \bigcup_{x\in X_0} U_x$. Therefore, we obtain $Y = f(X) = \bigcup_{x\in X_0} \mathrm{cl}(V_{\alpha(x)})$. This shows that Y is almost compact.

(b) Analogous to (a). □

5.6. Theorem. *If a function $f : (X, \tau) \to (Y, \sigma)$ has an e-strongly closed graph $G(f)$, then $f(A)$ is θ-closed in Y for each subset A which is e-compact relative to X.*

Proof. Let A be e-compact relative to X and $y \in Y\backslash f(A)$. Then for each $x \in A$ we have $(x, y) \notin G(f)$, and there exist $U_x \in eO(X, x)$ and an open V_x of Y containing y such that $f(U_x) \cap \mathrm{cl}(V_x) = \emptyset$. The family $\{U_x \mid x \in A\}$ is a cover of A by e-open subsets of X. Since A is e-compact relative to X, there exists a finite subset A_0 of A such that $A \subset \bigcup\{U_x \mid x \in A_0\}$. Put $V = \bigcap_{x\in A_0} V_x$. Then V is an open set in Y, $y \in V$ and

$$f(A) \cap \mathrm{cl}(V) \subset [\cup_{x\in A_0} f(U_x)] \cap \mathrm{cl}(V) \subset [\bigcup_{x\in A_0} f(U_x) \cap \mathrm{cl}(V)] = \emptyset.$$

Therefore $y \notin \mathrm{cl}_\theta(f(A))$, and hence $f(A)$ is θ-closed in Y. □

We recall that a space X is said to be *submaximal* [4] if every dense subset of X is open in X. A space X is said to be *extremally disconnected* [4] if the closure of each open set of X is open in X.

5.7. Theorem. *Let X be a submaximal extremally disconnected space. If a function $f : X \to Y$ has an e-strongly closed graph then $f^{-1}(A)$ is closed in X for each subset A which is an H-set in Y.*

Proof. Let A be an H-set of Y and $x \notin f^{-1}(A)$. For each $y \in A$ we have $(x, y) \in X \times Y\backslash G(f)$, and there exist an e-open set U_y of X containing x and an open set V_y of Y containing y such that $f(U_y) \cap \mathrm{cl}(V_y) = \emptyset$, hence $U_y \cap f^{-1}(\mathrm{cl}(V_y)) = \emptyset$. The family $\{V_y \mid y \in A\}$ is a cover of A by open sets of Y. Since A is an H-set in Y, there exists a finite subset A_0 of A such that $A \subset \bigcup\{\mathrm{cl}(V_y) \mid y \in A_0\}$. Since X is submaximal extremally disconnected, each U_y is open in X. Set $U = \bigcap_{y\in A_0} U_y$, then U is an open set containing x and

$$f(U) \cap A \subset \cup_{y\in A_0}[f(U) \cap \mathrm{cl}(V_y)] \subset \cup_{y\in A_0}[f(U_y) \cap \mathrm{cl}(V_y)] = \emptyset.$$

Therefore we have $U \cap f^{-1}(A) = \emptyset$. Hence $f^{-1}(A)$ is closed in X. □

5.8. Corollary. *Let $f : X \to Y$ be a function with an e-strongly closed graph, from a submaximal extremally disconnected space X into a C-compact space Y. Then f is continuous.*

Proof. Let A be a closed subset in the C-compact space Y. Then A is an H-set and $f^{-1}(A)$ is closed in X according to Theorem 5.7. Therefore f is continuous. □

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