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A NOTE ON GENERALIZED LEFT (θ, ϕ)-DERIVATIONS IN PRIME RINGS

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Abstract

In this paper we describe generalized left (θ, ϕ)-derivations in prime rings, and prove that an additive mapping in a ring R acting as a homomorphism or anti-homomorphism on an additive subgroup S of R must be either a mapping acting as a homomorphism on S or a mapping acting as an anti-homomorphism on S, through which some related results are improved.

Keywords: Prime rings, Generalized left (θ, ϕ)-derivations, Mappings acting as homomorphisms, Mappings acting as anti-homomorphisms, Mappings acting as homomorphisms or anti-homomorphisms.

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1. Introduction

Let R be an associative ring. Recall that an additive mapping $\mu : R \to R$ is called a derivation if $\mu(xy) = x\mu(y) + \mu(x)y$ holds for all $x, y \in R$. An additive mapping $\delta : R \to R$ is called a generalized derivation if there exists a derivation μ of R such that $\delta(xy) = x\delta(y) + \mu(x)y$ holds for all $x, y \in R$. An additive mapping $\mu : R \to R$ is called a (θ, ϕ)-derivation if $\mu(xy) = \theta(x)\mu(y) + \mu(x)\phi(y)$ holds for all $x, y \in R$, where θ, ϕ are endomorphisms of R. An additive mapping $\delta : R \to R$ is called a generalized (θ, ϕ)-derivation if there exists a (θ, ϕ)-derivation μ such that $\delta(xy) = \theta(x)\delta(y) + \mu(x)\phi(y)$

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holds for all $x, y \in R$, where θ, ϕ are endomorphisms of R. Obviously the relations among these concepts can be described as the following diagram

$$\begin{array}{ccccc} & & \text{generalized derivations} & & \\ & \nearrow & & \searrow & \\ \text{derivations} & & & & \text{generalized } (\theta,\phi)\text{-derivations,} \\ & \searrow & & \nearrow & \\ & & (\theta,\phi)\text{-derivations} & & \end{array}$$

where $\rightarrow$ denotes that the right covers the left as concepts. Generalized derivations, (θ, ϕ)-derivations and generalized (θ, ϕ)-derivations are studied mainly in (semi)-prime rings. For example one can search them in Brešar [6], Havala [10], Lee [11], Chang *et al.* [8], Cheng *et al.* [9], Ashraf *et al.* [3], and so on.

The concept of left derivations was given by Brešar and Vukman in [7]. Recall that an additive mapping $\mu : R \to R$ is called a *left derivation* if $\mu(xy) = x\mu(y) + y\mu(x)$ holds for all $x, y \in R$. They proved that a prime ring R having a nonzero left derivation must be commutative. In fact, they stated the results in a more general form (see [7, Proposition 1.6]).

Similar to the development of the concept of derivations, the development of the concept of left derivations should have an analogue

$$\begin{array}{ccccc} & & \text{generalized left derivations} & & \\ & \nearrow & & \searrow & \\ \text{left derivations} & & & & \text{generalized left } (\theta,\phi)\text{-derivations.} \\ & \searrow & & \nearrow & \\ & & \text{left } (\theta,\phi)\text{-derivations} & & \end{array}$$

In fact, Ashraf and copartners have given the concept of left (θ, ϕ)-derivations in [12, 2], and generalized left derivations in [4]. According to Ashraf *et al.*, an additive mapping $\delta : R \to R$ is called a *generalized left derivation* if there exists a left derivation $\mu : R \to R$ such that $\delta(xy) = x\delta(y) + y\mu(x)$ holds for all $x, y \in R$. An additive mapping $\mu : R \to R$ is called a *left* (θ, ϕ)*-derivation* if $\mu(xy) = \theta(x)\mu(y) + \phi(y)\mu(x)$ holds for all $x, y \in R$, where θ, ϕ are endomorphisms of R. And so it is natural to give the concept of generalized left (θ, ϕ)-derivations as that an additive mapping $\delta : R \to R$ is called a *generalized left* (θ, ϕ)*-derivation* if there exists a left (θ, ϕ)-derivation μ such that $\delta(xy) = \theta(x)\delta(y) + \phi(y)\mu(x)$ holds for all $x, y \in R$, where θ, ϕ are endomorphisms of R.

Particularly in a prime ring, for a generalized left (θ, ϕ)-derivation δ of R, the left (θ, ϕ)-derivation μ such that $\delta(xy) = \theta(x)\delta(y) + \phi(y)\mu(x)$ holds for all $x, y \in R$ in the definition is unique. Hence generally in a prime ring, the unique μ decided by δ is called the *associated left* (θ, ϕ)*-derivation of* δ.

Obviously in commutative rings, derivations (resp. generalized derivations, (θ, ϕ)-derivations, generalized (θ, ϕ)-derivations) act in accord with left derivations (resp. generalized left derivations, left (θ, ϕ)-derivations, generalized left (θ, ϕ)-derivations). However in a noncommutative ring, the case is quite different in general.

In this paper, firstly, we will give a note which describes the form of generalized left (θ, ϕ)-derivations in prime rings under the assumption that θ, ϕ are automorphisms of R (see Theorem 2.1).

At the other hand, Bell and Kappe [5] discussed derivations acting as homomorphisms or anti-homomorphisms on a nonzero right ideal of a prime ring. Recall that an additive mapping f from a ring R into itself is said to act as a homomorphism or as an anti-homomorphism on S, an additive subgroup of R, if for each pair $x, y \in S$, either $f(xy) = f(x)f(y)$ or $f(xy) = f(y)f(x)$ holds. Certainly the concept of mappings acting as homomorphisms on S, and the concept of mappings acting as anti-homomorphisms on S can be defined in a similar way.

Particularly mappings acting as homomorphisms on S and mappings acting as anti-homomorphisms on S are all mappings acting as homomorphisms or anti-homomorphisms on S. But one will ask whether or not these two kinds of mappings are the unique mappings acting as homomorphisms or anti-homomorphisms on an additive subgroup S of R. In this paper, secondly, we will give another note which gives a firm answer on this problem (see Lemma 2.3).

Finally in this paper, using the two results above, we will generalize the following results on left (θ, ϕ)-derivations to those on generalized left (θ, ϕ)-derivations (see Corollary 2.5 and Proposition 2.8).

1.1. Theorem. [2, Theorem 4.2] *Let R be a prime ring and K a nonzero ideal of R, and let θ, ϕ be automorphisms of R. Suppose $d : R \to R$ is a left (θ, ϕ)-derivation of R.*

(1) *If d acts as a homomorphism on K, then $d = 0$ on R.*
(2) *If d acts as an anti-homomorphism on K, then $d = 0$ on R.*

1.2. Theorem. [1, Theorem 2.7] *Let R be a 2-torsion free prime ring and J a nonzero Jordan ideal and a subring of R. Suppose that θ, ϕ are automorphisms of R, and that $d : R \to R$ is a left (θ, ϕ)-derivation of R.*

(1) *If d acts as a homomorphism on J, then $d = 0$ on R.*
(2) *If d acts as an anti-homomorphism on J, then $d = 0$ on R.*

2. Results

Now we describe the generalized left (θ, ϕ)-derivation of a prime ring R under the assumption that θ, ϕ are automorphisms of R.

2.1. Theorem. *Let R be a prime ring with automorphisms θ, ϕ. Then a generalized left (θ, ϕ)-derivation δ must take one of the following forms:*

(1) *There exists a left R-homomorphism $d :_R R \to_R R$ such that $\delta = \theta \circ d$.*
(2) *R is a commutative domain with δ as its a generalized (θ, ϕ)-derivation.*

Proof. Let μ be the associated left (θ, ϕ)-derivation of δ.

Firstly, we consider the case that $\mu = 0$. Then $\delta(xy) = \theta(x)\delta(y)$ holds for all $x, y \in R$. Let $d = \theta^{-1} \circ \delta$, we can obtain $\delta = \theta \circ d$ with d a left R-homomorphism from ${}_R R$ into itself, which is the first case.

Finally, we consider the case that $\mu \neq 0$. Then for all $x, y, z \in R$, we have

$$\begin{aligned} \delta(xyz) &= \delta((xy)z) = \theta(xy)\delta(z) + \phi(z)\mu(xy) \\ &= \theta(x)\theta(y)\delta(z) + \phi(z)\theta(x)\mu(y) + \phi(z)\phi(y)\mu(x). \end{aligned}$$

At the other hand, for all $x, y, z \in R$, we have

$$\begin{aligned} \delta(xyz) &= \delta(x(yz)) = \theta(x)\delta(yz) + \phi(yz)\mu(x) \\ &= \theta(x)\theta(y)\delta(z) + \theta(x)\phi(z)\mu(y) + \phi(y)\phi(z)\mu(x). \end{aligned}$$

So for all $x, y, z \in R$, we have

$$[\theta(x), \phi(z)]\mu(y) + [\phi(y), \phi(z)]\mu(x) = 0. \tag{2.1}$$

Setting $z = y$ in (2.1), we have that

$$[\theta(x), \phi(y)]\mu(y) = 0 \tag{2.2}$$

holds for all $x, y \in R$. Setting $x = xz$ in (2.2), we have

$$\theta(x)[\theta(z), \phi(y)]\mu(y) + [\theta(x), \phi(y)]\theta(z)\mu(y) = 0$$

holds for all $x, y, z \in R$. By (2.2), for all $x, y, w \in R$, we have $[\theta(x), \phi(y)]w\mu(y) = 0$. Hence for each $y \in R$, either $\mu(y) = 0$ or $\phi(y) \in Z(R)$ since R is a prime ring. That is

$$\{y \in R \mid \phi(y) \in Z(R)\} \cup \{y \in R \mid \mu(y) = 0\} = R.$$

Hence either $\{y \in R \mid \phi(y) \in Z(R)\} = R$ or $\{y \in R \mid \mu(y) = 0\} = R$. Since $\mu \neq 0$, we have $\{y \in R \mid \phi(y) \in Z(R)\} = R$. Then R is a commutative domain which completes the proof. □

By Theorem 2.1, we give the form of the left (θ, ϕ)-derivation of a prime ring R under the assumption that θ, ϕ are automorphisms of R.

2.2. Corollary. *Let R be a prime ring with automorphisms θ, ϕ. Then μ is a nonzero left (θ, ϕ)-derivation of R if and only if R is a commutative domain with μ as its a nonzero (θ, ϕ)-derivation.* □

Now we give another note on mappings acting as homomorphisms or anti-homomorphisms on an additive subgroup of a ring.

2.3. Lemma. *Let R be a ring with S its an additive subgroup. Let $f : R \to R$ be an additive mapping. Then f acts as a homomorphism or an anti-homomorphism on S if and only if either f acts as a homomorphism on S or f acts an anti-homomorphism on S.*

Proof. We will deal with the only if part, for the other part is obvious. For each $s \in S$, let $H_s = \{x \in S \mid f(sx) = f(s)f(x)\}$ and $H'_s = \{x \in S \mid f(sx) = f(x)f(s)\}$. Obviously H_s and H'_s are all subgroups of S, and $H_s \cup H'_s = S$. So either $H_s = S$ or $H'_s = S$. Let $H = \{s \in S \mid H_s = S\}$ and $H' = \{s \in S \mid H'_s = S\}$. Obviously H, H' are all subgroups of S and $H \cup H' = S$. So either $H = S$ or $H' = S$ which completes the proof. □

Note that Theorem 1.1 and 1.2 can be stated in a new form by an application of Lemma 2.3, on which we will not say more. Now we give an equivalent condition under which a generalized left (θ, ϕ)-derivation δ of a prime ring R with the associated (θ, ϕ)-derivation μ has the property that $\mu \neq 0$ acts as a homomorphism or an anti-homomorphism on a nonzero subring of R.

2.4. Theorem. *Let R be a prime ring. Let δ be a generalized left (θ, ϕ)-derivation of R with associated left (θ, ϕ)-derivation μ such that $\mu \neq 0$, where θ, ϕ are automorphisms of R. Then δ acts as a homomorphism or an anti-homomorphism on S, a nonzero subring of R, if and only if one of the following holds:*

(1) *$\delta = 0$ on S.*
(2) *$\delta = \theta$ on S and $\mu = 0$ on S.*
(3) *$\delta = \phi$ on S and $\mu = \phi - \theta$ on S.*

Proof. By Theorem 2.1, R is a commutative domain with δ as its a generalized (θ, ϕ)-derivation since $\mu \neq 0$. We will only prove the only if part, the proof for the other part is obvious. Assume that $\delta \neq 0$ on S. Then for all $x, y \in S$, we have

$$\delta(xy) = \theta(x)\delta(y) + \phi(y)\mu(x) = \delta(x)\delta(y)$$

since R is commutative. Then

$$(\delta - \theta)(x)\delta(y) = \mu(x)\phi(y) \tag{2.3}$$

holds for all $x, y \in S$. Setting $x = xz$ in (2.3), then for all $x, y, z \in S$, we have

$$(\delta - \theta)(xz)\delta(y) = \mu(xz)\phi(y).$$

That is

$$(\theta(x)\delta(z) + \phi(z)\mu(x) - \theta(x)\theta(z))\delta(y) = \theta(x)\mu(z)\phi(y) + \phi(z)\mu(x)\phi(y)$$

holds for all $x, y, z \in S$. Then by (2.3) for all $x, y, z \in S$, we have $\phi(z)\mu(x)(\delta - \phi)(y) = 0$. Since $S \neq 0$, $\mu(S)(\delta - \phi)(S) = 0$. Hence either $\mu = 0$ on S or $\delta = \phi$ on S. When $\mu = 0$ on S, we obtain $\delta = \theta$ on S from (2.3). When $\mu \neq 0$ on S, we have $\delta = \phi$ on S. Then we obtain $\mu = \phi - \theta$ on S from (2.3). □

Particularly when S is either a nonzero ideal of a prime ring or a nonzero Jordan ideal and subring of a 2-torsionfree prime ring R in Theorem 2.4, we have

2.5. Corollary. *Let S be either a nonzero ideal of a prime ring R or a nonzero Jordan ideal and subring of a 2-torsionfree prime ring R. Let δ be a generalized left (θ, ϕ)-derivation of R with the associated left (θ, ϕ)-derivation μ such that $\mu \neq 0$, where θ, ϕ are automorphisms of R. Then δ acts as a homomorphism or an anti-homomorphism on S if and only if $\delta = \phi$ and $\mu = \phi - \theta$.*

Proof. When S is a nonzero ideal of a prime ring R, we consider the three cases in Theorem 2.4 separately. Firstly, if $\delta = 0$ on S, then for all $s \in S$ and for all $r \in R$, we have that

$$0 = \delta(rs) = \theta(r)\delta(s) + \phi(s)\mu(r) = \phi(s)\mu(r).$$

Then since $S \neq 0$, we have $\mu = 0$, which contradicts $\mu \neq 0$. Secondly, if $\delta = \theta$ and $\mu = 0$ on S, then for all $s \in S$ and for all $r \in R$, we have that

$$\theta(r)\theta(s) = \theta(rs) = \delta(rs) = \theta(r)\delta(s) + \phi(s)\mu(r) = \theta(r)\theta(s) + \phi(s)\mu(r).$$

Then $\phi(s)\mu(r) = 0$ holds for all $s \in S$ and for all $r \in R$, which shows that $\mu = 0$, a contradiction. Thirdly, if $\delta = \phi$ and $\mu = \phi - \theta$ on S, then for all $s \in S$ and for all $r \in R$, we have that

$$\phi(r)\phi(s) = \phi(rs) = \delta(rs) = \theta(r)\delta(s) + \phi(s)\mu(r) = \phi(s)(\mu(r) + \theta(r)),$$

which shows that $\mu(r) = (\phi - \theta)(r)$ holds for all $r \in R$ since $S \neq 0$. On the other hand, for all $s \in S$ and for all $r \in R$, we have that

$$\phi(s)\phi(r) = \phi(sr) = \delta(sr) = \theta(s)\delta(r) + \phi(r)\mu(s) = \theta(s)\delta(r) + \phi(r)(\phi(s) - \theta(s)).$$

Then for all $s \in S$ and for all $r \in R$, we have that $\theta(s)(\delta(r) - \phi(r)) = 0$, which shows that $\delta = \phi$ since $S \neq 0$.

When S is a nonzero Jordan ideal and subring of 2-torsionfree prime ring R, noting that $(2r)s = sr + rs \in S$ for all $r \in R$ and for all $s \in S$ since R is commutative, in a similar way to the ideal case, we have that either $2\mu(x) = 0$ holds for all $x \in R$ or $2(\delta - \phi)(x) = 2(\phi - \theta - \mu)(x) = 0$ holds for all $x \in R$. Hence the conclusion is obtained since R is 2-torsionfree. □

The left (θ, ϕ)-derivation version of Theorem 2.4 and Corollary 2.5 can be obtained immediately.

2.6. Corollary. *Let R be a prime ring . Let μ be a left (θ, ϕ)-derivation of R, where θ, ϕ are automorphisms of R. Then μ acts as a homomorphism or an anti-homomorphism on S, a nonzero subring of R, if and only if $\mu = 0$ on S.* □

2.7. Corollary. [Theorem 1.1 and 1.2] *Let S be either a nonzero ideal of a prime ring R or a nonzero Jordan ideal and subring of 2-torsionfree prime ring R. Let μ be a left (θ, ϕ)-derivation of R, where θ, ϕ are automorphisms of R. Then μ acts as a homomorphism or an anti-homomorphism on S if and only if $\mu = 0$.* □

For completeness, we discuss Corollary 2.5 further when $\mu = 0$.

2.8. Proposition. *Let S be either a nonzero ideal of a prime ring R or a nonzero Jordan ideal and subring of a 2-torsionfree prime ring R. Let δ be a generalized left (θ, ϕ)-derivation of R with the associated left (θ, ϕ)-derivation μ such that $\mu = 0$, where θ, ϕ are automorphisms of R. Then δ acts as a homomorphism or an anti-homomorphism on S if and only if either $\delta = \theta$ or $\delta = 0$.*

Proof. By Theorem 2.1, there exists a left R-homomorphism $d :_R R \to_R R$ such that $\delta = \theta \circ d$. And it is easy to see that d also acts as a homomorphism or an anti-homomorphism on S.

Firstly, We consider the case that S is a nonzero ideal of a prime ring R. When d acts as a homomorphism on S, then for all $s, t \in S$ and for all $x, y, z \in R$, we have

$$d(sxytz) = sxytd(z) = d(sx)d(ytz) = sd(x)ytd(z).$$

Then $s(d(x) - x)ytd(z) = 0$ holds for all $s, t \in S$ and for all $x, y, z \in R$. Hence either $S(d(x) - x) = 0$ holds for all $x \in R$ or $Sd(z) = 0$ holds for all $z \in R$. And so either $d = 1_R$ or $d = 0$. Thus either $\delta = \theta$ or $\delta = 0$.

When d acts as an anti-homomorphism on S, then for all $s, t \in S$, we have $d(st) = sd(t) = d(t)d(s)$. For all $x \in R$, set $s = xs$ in the above formula, we have that

$$d(t)xd(s) = d(t)d(xs) = d((xs)t) = xsd(t) = xd(t)d(s)$$

holds for all $s, t \in S$ and for all $x \in R$. That is $[d(t), x]d(s) = 0$ holds for all $s, t \in S$ and for all $x \in R$. Then for all $y \in R$, replacing x by xy in $[d(t), x]d(s) = 0$, we have that $[d(t), x]yd(s) = 0$ holds for all $s, t \in S$ and for all $x, y \in R$, which shows that $d(S) \subseteq Z(R)$. Hence d acts as an anti-homomorphism on S which has been dealt with.

Secondly, we consider the case that S is a nonzero Jordan ideal and subring of a 2-torsionfree prime ring R. Note the following two facts:

(1) For all $s, t \in S$ and for all $x \in R$, $2sxt \in S$,
(2) For any $a \in R$, either $Sa = 0$ or $aS = 0$ implies $a = 0$.

For all $s, t \in S$ and for all $x \in R$, we have

$$2sxt + (st)x + x(st) = s(tx + xt) + (sx + xs)t \in S.$$

By $(st)x + x(st) \in S$, the first fact is proved. If $Sa = 0$, then $(sx + xs)a = 0$ for all $s \in S$ and all $x \in R$. Then $SRa = 0$ implies $a = 0$ since $S \neq 0$, which proves the second fact.

If d acts as a homomorphism on S, then for all $r, s, t \in S$ and for all $x \in R$, we have $d(r(2sxt)) = d(r)d(2sxt) = rd(2sxt)$. Then $2(d(r) - r)SRd(t) = 0$ holds for all $r, t \in S$. Since R is 2-torsionfree, either $d(r) = r$ holds for all $r \in S$ or $d = 0$ on S. Then for all $x \in R$ and for all $r \in S$, we have either that

$$xs + sx = d(xs + xs) = xd(s) + sd(x) = xs + sd(x)$$

or that $0 = d(xs+xs) = xd(s)+sd(x) = sd(x)$. Then either $S(d(x)-x) = 0$ holds for all $x \in R$ or $Sd(x) = 0$ holds for all $x \in R$, which proves the conclusion. If d acts as an anti-homomorphism on S, then for all $s, t \in S$, we have $d(st) = sd(t) = d(t)d(s)$. For all $r \in S$ and for all $x \in R$, setting $s = 2rxs$ in $sd(t) = d(t)d(s)$, we have $2rxsd(t) = d(t)(2rx)d(s)$. At the other hand, multiplying $sd(t) = d(t)d(s)$ by $2rx$ from the left hand side, we have $2rxsd(t) = 2rxd(t)d(s)$ for all $r, s, t \in S$ and for all $x \in R$. Hence $2[rx, d(t)]d(s) = 0$

holds for all $r, s, t \in S$ and for all $x \in R$. And so for all $r, r', s, t \in S$ and for all $x, x' \in R$, we have $2[rx, d(t)]r'x'd(s) = 0$. Then $[rx, d(S)]SRd(S) = 0$ holds for all $r \in S$ and for all $x \in R$ since R is 2-torsionfree. Hence $[rx, d(S)] = 0$ holds for all $r \in S$ and for all $x \in R$. For all $y \in R$, setting $x = xy$, we have that $SR[R, d(S)] = 0$. So $d(S) \subseteq Z(R)$. This shows that d acts as a homomorphism on S which we have dealt with. $\square$

Now we give two examples in order to show that for the Jordan ideal case the condition that R is 2-torsionfree is necessary in Corollary 2.5, Corollary 2.7 and Proposition 2.8.

2.9. Example. Let $R = \mathbb{Z}_2[x, y]$ and

$$S = \{f(x, y) \in R \mid f(x, y) \text{ is a symmetrical polynomial}\}.$$

It is easy to see that S is a nonzero Jordan ideal and subring of 2-torsion prime ring R. Let $\theta = 1_R$ and $\phi : R \to R$ such that $\phi(f(x, y)) = f(y, x)$ for all $f(x, y) \in R$. It can be checked that ϕ is an automorphism of R. Set $\mu = \phi - \theta$, then μ is a nonzero left (θ, ϕ)-derivation of R and $\mu \neq \phi$. However $\mu(S) = 0$ shows that μ acts as a homomorphism or an anti-homomorphism on S. This shows that for the Jordan ideal case the condition that R is 2-torsionfree is necessary in Corollary 2.5 and 2.7.

2.10. Example. Let $R = \mathbb{M}_2(\mathbb{Z}_2)$ and $S = \{0, I_2\} \subseteq R$. Then S is a nonzero Jordan ideal and subring of a 2-torsion prime ring R. Let $\theta = 1_R$ and $f : R \to R$ such that $f(x) = xe_{11}$ for all $x \in R$. It is easy to see that f acts as a homomorphism on S. However $f \neq 1_R = \theta$ and $f \neq 0$. This shows that for the Jordan ideal case the condition that R is 2-torsionfree is necessary in Proposition 2.8.

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