ON GENERALIZED DERIVATIONS AND COMMUTATIVITY OF PRIME AND SEMIPRIME RINGS

Asma Ali∗†, Deepak Kumar∗ and Phool Miyan∗

Received 07 : 01 : 2010 : Accepted 31 : 05 : 2010

Abstract

Let \( R \) be a prime ring and \( \theta, \phi \) endomorphisms of \( R \). An additive mapping \( F : R \rightarrow R \) is called a generalized \((\theta, \phi)\)-derivation on \( R \) if there exists a \((\theta, \phi)\)-derivation \( d : R \rightarrow R \) such that \( F(xy) = F(x) \theta(y) + \phi(x)d(y) \) for all \( x, y \in R \). Let \( S \) be a non-empty subset of \( R \). In the present paper for various choices of \( S \) we study the commutativity of a semiprime (prime) ring \( R \) admitting a generalized \((\theta, \phi)\)-derivation \( F \) satisfying any one of the properties: (i) \( F(x)F(y) − xy \in Z(R) \), (ii) \( F(x)F(y) + xy \in Z(R) \), (iii) \( F(x)F(y) − yx \in Z(R) \), (iv) \( F(x)F(y) + yx \in Z(R) \), (v) \( F[x, y] − [x, y] \in Z(R) \), (vi) \( F[x, y] + [x, y] \in Z(R) \), (vii) \( F(x \circ y) − x \circ y \in Z(R) \), and (viii) \( F(x \circ y) + x \circ y \in Z(R) \), for all \( x, y \in S \).

Keywords: Lie ideals, Torsion free rings, Derivations, \((\theta, \phi)\)-derivations, Generalized derivations, Generalized \((\theta, \phi)\)-derivations.

2000 AMS Classification: 16 W 25, 16 N 60, 16 U 80.

Introduction

Let \( R \) be an associative ring with centre \( Z(R) \). A ring \( R \) is said to be prime (resp. semiprime) if \( aRa = \{0\} \) implies that either \( a = 0 \) or \( b = 0 \) (resp. \( aRa = \{0\} \) implies that \( a = 0 \)). For any \( x, y \in R \) we shall write \([x, y] = xy − yx\) and \( x \circ y = xy + yx \). An additive subgroup \( U \) of \( R \) is said to be a Lie ideal of \( R \) if \([x, u] \in U \) for all \( x \in R \) and \( u \in U \). An additive mapping \( d : R \rightarrow R \) is called a derivation if \( d(xy) = d(x)y + xd(y) \) for all \( x, y \in R \). Let \( \theta, \phi \) be endomorphisms of \( R \). An additive mapping \( d : R \rightarrow R \) is called

∗Department of Mathematics, Aligarh Muslim University, Aligarh - 202002, India.
E-mail: (A. Ali) asma.ali2@rediffmail.com (D. Kumar) deep_math1@yahoo.com (P. Miyan) phoolmiyan83@gmail.com
†Corresponding Author.

∗Supported by the University Grants Commission, India Grant F. No. 33-106/2007 (SR)