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EQUIPRIME N-IDEALS OF MONOGENIC N-GROUPS

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Abstract

In this paper we introduce the notion of equiprime N-ideals where N is a near-ring. We consider the interconnections of equiprime, 3-prime and completely prime N-ideals of a monogenic N-group Γ. We show that if P is an equiprime N-ideal of Γ, then $(P : \Gamma)_N$ is an equiprime ideal of N, and that the converse holds when N is a right permutable near-ring and Γ is a monogenic N-group.

Keywords: Prime near-ring, Prime ideal, Prime N-group, Equiprime N-ideal.

2000 AMS Classification: 16 Y 30.

1. Introduction

There are various ways to generalize prime ideals of rings to near-rings. Prime ideals in near-rings have been extensively studied by several authors. In 1970, Holcombe [9] studied three different concepts of primeness, which he called 0-prime (or prime), 1-prime and 2-prime. Ramakotaiah and Rao [13], defined the concepts of prime ideal of type 1 and prime ideal of type 2. Groenewald [8] used the phrase "3-prime ideal" for "prime ideal of type 1". In the literature the phrase "completely prime (or c-prime) ideal" has been used for "prime ideal of type 2". Booth, Groenewald and Veldsman [5] presented another generalization of prime rings, called equiprime or e-prime. These notions of primeness above are in general distinct for near-rings.

Prime rings and their extensions to ring modules have been studied by various authors [7, 11, 15]. What about the extensions of prime near-rings to prime N-groups? Juglal, Groenewald and Lee [10] generalized the various notions of primeness that were defined in N to the N-group Γ. They also provided characterizations of prime N-groups and showed equivalences between these characterizations.

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In this study, we define the notion of equiprime N-ideals and investigate the interconnections of equiprime, 3-prime and completely prime N-ideals of monogenic N-groups. Also, we obtain some relationships between an equiprime N-ideal P of an N-group Γ and the ideal $(P : \Gamma)$ of the near-ring N.

2. Preliminary definitions and results

N will always denote a right near-ring. It is assumed that the reader is familiar with the basic definitions of right near-ring, zero-symmetric near-ring and ideal (cf.[12]).

2.1. Definition. We recall from Holcombe [9], Groenewald [8] and Booth [5]; an ideal $P \lhd N$ is *v-prime*, if for $A, B \subseteq N$ with

A, B ideals of N if $v = 0$

A, B left ideals of N if $v = 1$

A, B N-subgroups of N if $v = 2$,

the inclusion $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

An ideal P of a near-ring N is called a *3-prime ideal* if for all $a, b \in N$, $aNb \subseteq P$ implies $a \in P$ or $b \in P$.

If for all $a, b \in N$, $ab \in P$ implies $a \in P$ or $b \in P$, then $P \lhd N$ is called a *completely prime ideal* [13].

$P \lhd N$ is called an *equiprime ideal*, if $a \in N \backslash P$ and $x, y \in N$ are such that $anx - any \in P$ for all $n \in N$, then $x - y \in P$ [5, Proposition 2.2.].

If the zero ideal of N is v-prime ($v = 0, 1, 2, 3, c, e$), then N is called a *v-prime near-ring*.

It is already known that if $P \lhd N$, then P is completely prime $\Longrightarrow$ P is 3-prime $\Longrightarrow$ P is 2-prime, P is 1-prime $\Longrightarrow$ P is 0-prime and P is 2-prime $\Longrightarrow$ P is 1-prime when N is zero-symmetric. Furthermore, any equiprime ideal is 3-prime [14].

Also playing a role in this paper are the identities:

If for all $a, b, c, d \in N$, $abc = acb$ (resp. $abc = bac$, $abcd = acbd$), then N is called a *right permutable* (resp. *left permutable, medial*) *near-ring* [3].

If $abc = abac$ (resp. $abc = acbc$), then N is called a *left self distributive* -LSD- (resp. *right self distributive* -RSD-) *near-ring* [4].

Birkenmeier and Heatherly [4] showed that 3-prime ideals in an LSD or RSD zero-symmetric near-ring are also completely prime. Furthermore, Atagün [1] proved that if P is an IFP ideal (for all $a, b \in N$, $ab \in P$ implies $anb \in P$ for all $n \in N$) and a 3-(semi)prime ideal of N, then P is a completely (semi)prime ideal.

The following proposition will be used in Section 4.

2.2. Proposition. *Let P be a 3-prime ideal of a near-ring N.*

a) *If N is right permutable, then P is a completely prime ideal of N* [2, Proposition 2.2].
b) *If N is medial, then P is a completely prime ideal of N* [3, Proposition 2.7].

If Γ is an additive group, then it is called an *N-group* if for all $x, y \in N$, $\gamma \in \Gamma$,

a) $x\gamma \in \Gamma$,
b) $(x + y)\gamma = x\gamma + y\gamma$,
c) $(xy)\gamma = x(y\gamma)$.

A subgroup Δ of Γ with $N\Delta \subseteq \Delta$ is said to be an *N-subgroup* of Γ ($\Delta \leq_N \Gamma$). A normal subgroup Δ of Γ is called an *N-ideal* of Γ ($\Delta \lhd_N \Gamma$) if $\forall\, \gamma \in \Gamma$, $\forall\, \delta \in \Delta$, $\forall\, n \in N$: $n(\gamma + \delta) - n\gamma \in \Delta$ [12].

Γ is called *monogenic* if there exists $\gamma \in \Gamma$ such that $N\gamma = \{x\gamma : x \in N\} = \Gamma$. Let A, B be subsets of Γ. Then the *Noetherian quotient* $(A : B)_N$ is defined as the set $\{n \in N : nB \subseteq A\}$.

2.3. Lemma. *Let N be a zero-symmetric near-ring and Γ an N-group. If $P \lhd_N \Gamma$, then $NP \subseteq P$.*

Proof. It is clear that $n0_\Gamma = 0_\Gamma$ for all $n \in N$, since N is zero-symmetric. Now, for all $n \in N$ and $p \in P$, $np = n(0_\Gamma + p) - n0_\Gamma \in P$ since $P \lhd_N \Gamma$. Hence $NP \subseteq P$. □

An N-group Γ is said to be *equiprime* [6] if

a) $N\Gamma \neq 0_\Gamma$,
b) If $a \in N$ with $a \notin (0_\Gamma : \Gamma)_N = \{n \in N : n\Gamma = 0_\Gamma\}$ and $\gamma_1, \gamma_2 \in \Gamma$, then $an\gamma_1 = an\gamma_2$ for all $n \in N$, implies $\gamma_1 = \gamma_2$,
c) $N0_\Gamma = 0_\Gamma$.

3. Prime N-ideals

Throughout this paper, N will always denote a zero-symmetric near-ring, Γ a left N-group and P a subset of N.

3.1. Definition. (cf. [10]) Let $P \lhd_N \Gamma$ be such that $N\Gamma \nsubseteq P$. Then P is called:

a) *0-prime* if for an ideal A of N and an N-ideal B of Γ, $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
b) *1-prime* if for a left ideal A of N and an N-ideal B of Γ, $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
c) *2-prime* if for a left N-subgroup A of N and an N-subgroup B of Γ, $AB \subseteq P$ implies $A\Gamma \subseteq P$ or $B \subseteq P$.
d) *3-prime* if $nN\gamma \subseteq P$ implies that $n\Gamma \subseteq P$ or $\gamma \in P$ for $n \in N$ and $\gamma \in \Gamma$.
e) *completely prime* (*c-prime*) if $n\gamma \in P$ implies that $n\Gamma \subseteq P$ or $\gamma \in P$ for $n \in N$ and $\gamma \in \Gamma$.

Γ is said to be a *v-prime N-group* ($v = 0, 1, 2, 3, c$) if $N\Gamma \neq 0$ and 0 is a v-prime N-ideal of Γ.

We state the following results from [10] which will be used in this paper.

3.2. Proposition. [10, Proposition 1.7] *Let $P \lhd_N \Gamma$. Then P is completely prime $\Longrightarrow$ P is 3-prime $\Longrightarrow$ P is 2-prime $\Longrightarrow$ P is 0-prime.* □

3.3. Corollary. [10, Corollary 1.8] *If Γ is an N-group, then Γ is completely prime $\Longrightarrow$ Γ is 3-prime $\Longrightarrow$ Γ is 2-prime $\Longrightarrow$ Γ is 0-prime.* □

In general, a 3-prime N-ideal need not to be a completely prime N-ideal. However, in [10] the authors gave the following proposition.

3.4. Proposition. [10, Proposition 1.10] *Let $P \lhd_N \Gamma$. Then the following are equivalent:*

a) *P is 3-prime and $(P : \gamma) \lhd N$ for every $\gamma \in \Gamma \backslash P$.*
b) *$N\Gamma \nsubseteq P$ and $(P : \gamma) = (P : \Gamma)$ for every $\gamma \in \Gamma \backslash P$.*
c) *P is a completely prime N-ideal.* □

In section 4, we investigate conditions under which a 3-prime N-ideal is completely prime.

If $P \lhd_N \Gamma$, we recall that $\widetilde{P} = (P : \Gamma)$ is an ideal of N. If P is a v-prime N-ideal ($v = 0, 1, 2, 3, c$), then does this imply that $\widetilde{P}$ is also a v-prime ideal of N? Juglal, Groenewald and Lee investigate this in the proposition that follows:

3.5. Proposition. [10, Proposition 1.16] *Let $P \lhd_N \Gamma$.*

a) *If P is a 2-prime N-ideal of Γ, then $\widetilde{P}$ is a 2-prime ideal of N.*
b) *If P is a 3-prime N-ideal of Γ, then $\widetilde{P}$ is a 3-prime ideal of N.*
c) *If P is a c-prime N-ideal of Γ, then $\widetilde{P}$ is a c-prime ideal of N.* □

4. Equiprime N-ideals

4.1. Definition. Let $P \lhd_N \Gamma$ be such that $N\Gamma \not\subseteq P$. If $a \in N$ and $\gamma_1, \gamma_2 \in \Gamma$, then $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$, implies $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$, then P is called an *equiprime N-ideal* of Γ.

It is known that if the ideal P of N is an equiprime ideal, then P is a 3-prime ideal. We show a similar relationship holds among the N-ideals of Γ.

4.2. Proposition. *Let $P \lhd_N \Gamma$. If P is an equiprime N-ideal, then P is a 3-prime N-ideal.*

Proof. Suppose that P is an equiprime N-ideal and $aN\gamma \subseteq P$ for $a \in N$ and $\gamma \in \Gamma$. Since for every $n \in N$, $an\gamma \in P$, then $an\gamma = an\gamma - an0_\Gamma \in P$. Since P is an equiprime N ideal, we have $a\Gamma \subseteq P$ or $\gamma = \gamma - 0_\Gamma \in P$. So P is a 3-prime N-ideal. □

4.3. Proposition. *Let N be a right permutable near-ring and $P \lhd_N \Gamma$. Then P is a completely prime N-ideal if and only if P is a 3-prime N-ideal.*

Proof. Assume P is a 3-prime N-ideal and $n\gamma \in P$ for $n \in N$ and $\gamma \in \Gamma$. By Lemma2.3, $nNn\gamma \subseteq NP \subseteq P$. Then $nNn\gamma = nnN\gamma = n^2N\gamma \subseteq P$, since N is right permutable. This implies that $n^2\Gamma \subseteq P$ or $\gamma \in P$, since P is 3-prime. So we have $n^2 \in (P : \Gamma)$ or $\gamma \in P$. If $\gamma \in P$, then we are done. Otherwise, since P is a 3-prime N-ideal of Γ, $(P : \Gamma)$ is a 3-prime ideal of N by Proposition 3.5 (b), and then $(P : \Gamma)$ is a completely prime ideal of N by Proposition 2.2 (a). Hence, $n^2 \in (P : \Gamma)$ implies that $n \in (P : \Gamma)$; whence $n\Gamma \subseteq P$. Therefore P is a completely prime N-ideal. Conversely, if P is completely prime, it is also 3-prime by Proposition 3.2. □

4.4. Example. Let $(N, +)$ be the Klein four group with multiplication defined as per Pilz [12, Scheme 1, p.408];

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Then $(N, +, \cdot)$ is a zero-symmetric right permutable near-ring. Consider the N-group $\Gamma = N$. It can be easily seen that $P = 0$ is both a 3-prime and a completely prime N-ideal of Γ.

From now on, in this section all N-groups will be monogenic N-groups.

4.5. Proposition. *Let N be a left permutable near-ring and $P \lhd_N \Gamma$. Then P is a completely prime N-ideal if and only if P is a 3-prime N-ideal.*

Proof. By Proposition 3.2, if $P \lhd_N \Gamma$ is completely prime then it is 3-prime.

Assume N is left permutable and P is 3-prime. Let $n \in N$ and $\gamma \in \Gamma$ be such that $n\gamma \in P$. Then $Nn\gamma \subseteq NP \subseteq P$ by Lemma 2.3. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence $\gamma = x\gamma_0$ for some $x \in N$. Since N is left permutable, $Nn\gamma = Nnx\gamma_0 = nNx\gamma_0 = nN\gamma \subseteq P$. This implies that $n\Gamma \subseteq P$ or $\gamma \in P$, because P is 3-prime. Hence, P is completely prime. □

4.6. Proposition. *Let N be a medial near-ring and $P \lhd_N \Gamma$. Then P is a completely prime N-ideal if and only if P is a 3-prime N-ideal.*

Proof. Assume P is a 3-prime N-ideal and $n\gamma \in P$ for $n \in N$ and $\gamma \in \Gamma$. Then $Nn\gamma \subseteq NP \subseteq P$ by Lemma 2.3. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence, $\gamma = x\gamma_0$ for some $x \in N$. Thus, $nNnx\gamma_0 = nNn\gamma \subseteq NP \subseteq P$. Then $nNnx\gamma_0 = nnNx\gamma_0 = n^2Nx\gamma_0 = n^2N\gamma \subseteq P$, since N is medial. Since P is 3-prime, it follows that $n^2\Gamma \subseteq P$ or $\gamma \in P$. If $\gamma \in P$, then we are done. If $n^2\Gamma \subseteq P$, then $n^2 \in (P : \Gamma)$. As P is a 3-prime N-ideal, $(P : \Gamma)$ is a 3-prime ideal of N by Proposition 3.5 (b). This implies that $(P : \Gamma)$ is completely prime by Proposition 2.2 (b). Hence, $n \in (P : \Gamma)$. Therefore $n\Gamma \subseteq P$; whence P is a completely prime N-ideal.

Conversely, if P is completely prime, it is 3-prime by Proposition 3.2. □

4.7. Proposition. *Let N be an LSD near-ring and $P \lhd_N \Gamma$. Then P is a completely prime N-ideal if and only if P is a 3-prime N-ideal.*

Proof. Suppose that P is a 3-prime N-ideal. Let $n \in N$ and $\gamma \in \Gamma$ be such that $n\gamma \in P$. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence, $\gamma = x\gamma_0$ for some $x \in N$. Since N is LSD, $nn'\gamma = nn'x\gamma_0 = nn'nx\gamma_0 = nn'n\gamma$ for each $n' \in N$. Then, $nn'\gamma = nn'n\gamma \in NP \subseteq P$, so $nn'\gamma \in P$ for each $n' \in N$. Hence, $nN\gamma \subseteq P$. Thus $n\Gamma \subseteq P$ or $\gamma \in P$ because P is 3-prime. Therefore P is a completely prime N-ideal.

If P is completely prime, it is also 3-prime by Proposition 3.2. □

4.8. Corollary. *Let P be an equiprime N-ideal.*

a) *If N is right permutable, then P is a completely prime N-ideal.*
b) *If N is left permutable, then P is a completely prime N-ideal.*
c) *If N is medial, then P is a completely prime N-ideal.*
d) *If N is LSD, then P is a completely prime N-ideal.*

Proof. The result follows from Proposition 4.2 and Propositions 4.3, 4.5, 4.6, 4.7. □

The remaining results in this section are about relationships between equiprime N-ideals and completely prime N-ideals.

4.9. Proposition. *Let N be a right permutable near-ring and $P \lhd_N \Gamma$ be such that $N_d \backslash (P : \Gamma) \neq \emptyset$. Then P is an equiprime N-ideal if and only if P is a completely prime N-ideal.*

Proof. Under the given assumptions, if P is equiprime it is completely prime by Corollary 4.8 (a).

Conversely, suppose P is completely prime and $a \in N$, $\gamma_1, \gamma_2 \in \Gamma$, $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$. We need to show that $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence, $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$ for some $x, y \in N$. Since N is right permutable, $an\gamma_1 - an\gamma_2 = anx\gamma_0 - any\gamma_0 = axn\gamma_0 - ayn\gamma_0 = (ax - ay)n\gamma_0 \in P$ for all $n \in N$. Hence $(ax - ay)N\gamma_0 \subseteq P$.

By Proposition 3.2, a completely prime N-ideal is also 3-prime, so $(ax - ay)N\gamma_0 \subseteq P$ implies $(ax - ay)\Gamma \subseteq P$ or $\gamma_0 \in P$. If $\gamma_0 \in P$, then $\Gamma = N\gamma_0 \subseteq NP \subseteq P$ by Lemma 2.3.

This implies that $\Gamma = P$, which contradicts $N\Gamma \not\subseteq P$. Hence, we have $(ax - ay)\Gamma \subseteq P$, whence $(ax - ay) \in (P : \Gamma)$. Then $n_d(ax - ay) \in (P : \Gamma)$ for an $n_d \in N_d\backslash(P : \Gamma)$ because $(P : \Gamma)$ is an ideal of N. Since $n_d \in N_d\backslash(P : \Gamma)$ and N is right permutable, we get $n_d(ax - ay) = n_d ax - n_d ay = n_d xa - n_d ya = n_d(x - y)a \in (P : \Gamma)$. Since P is a completely prime N-ideal, $(P : \Gamma)$ is a completely prime ideal of N by Proposition 3.5 (c). Now since $(P : \Gamma)$ is a completely prime ideal of N and $n_d(x - y)a \in (P : \Gamma)$, we have $n_d \in (P : \Gamma)$ or $(x - y)a \in (P : \Gamma)$.

Since $n_d \in N_d\backslash(P : \Gamma)$ and $(P : \Gamma)$ is a completely prime ideal, we get $(x-y) \in (P : \Gamma)$ or $a \in (P : \Gamma)$. If $a \in (P : \Gamma)$, then $a\Gamma \subseteq P$ and we are done. If $(x - y) \in (P : \Gamma)$, we get $(x-y)\Gamma \subseteq P$. In particular, for $\gamma_0 \in \Gamma$, it follows that $(x-y)\gamma_0 = x\gamma_0 - y\gamma_0 = \gamma_1 - \gamma_2 \in P$. Therefore, P is an equiprime N-ideal. □

The condition $N_d\backslash(P : \Gamma) \neq \emptyset$ cannot be removed from Proposition 4.9. We have the following example:

4.10. Example. Consider the near-ring $(N, +, \cdot)$ in Example 4.4. Let the N-group $\Gamma = N$ and let $P = 0$. We know that N is a zero-symmetric right permutable near-ring and $P = 0$ is a completely prime N-ideal. It is seen that $N_d\backslash(P : \Gamma) = \emptyset$ and P is not an equiprime N-ideal of Γ.

4.11. Proposition. *Let N be a left permutable near-ring and $P \lhd_N \Gamma$. Then P is an equiprime N-ideal if and only if P is a completely prime N-ideal.*

Proof. By Corollary 4.8 (b), if P is equiprime, then it is completely prime.

Suppose P is completely prime. Let $a \in N$, $\gamma_1, \gamma_2 \in \Gamma$, $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$. We need to show that $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. So there exist $x, y \in N$ such that $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$. Then

$$an\gamma_1 - an\gamma_2 = anx\gamma_0 - any\gamma_0 = nax\gamma_0 - nay\gamma_0 = (nax - nay)\gamma_0 \in P,$$

since N is left permutable. Then, $(nax - nay)\Gamma \subseteq P$ or $\gamma_0 \in P$, since P is a completely prime N-ideal. If $\gamma_0 \in P$, then $\Gamma = N\gamma_0 \subseteq NP \subseteq P$ by Lemma 2.3. This implies that $\Gamma = P$, which contradicts that $N\Gamma \not\subseteq P$. Hence, we have $(nax - nay)\Gamma \subseteq P$, whence $(nax - nay) \in (P : \Gamma)$. Furthermore, since $(P : \Gamma) \neq N$, there exists a $q \in N\backslash(P : \Gamma)$. Then, $(nax - nay)q \in (P : \Gamma)$, since $(P : \Gamma)$ is an ideal of N. Hence,

$$(nax - nay)q = naxq - nayq = nxaq - nyaq = (nx - ny)aq \in (P : \Gamma)$$

because N is left permutable. By Proposition 3.5 (c), $(P : \Gamma)$ is a completely prime ideal of N since P is a completely prime N-ideal of Γ, which means that either $(nx - ny)a \in (P : \Gamma)$ or $q \in (P : \Gamma)$. Since $q \in N\backslash(P : \Gamma)$ and $(P : \Gamma)$ is a completely prime ideal, we get $(nx - ny)a \in (P : \Gamma)$ and therefore $(nx - ny) \in (P : \Gamma)$ or $a \in (P : \Gamma)$. If $a \in (P : \Gamma)$, then $a\Gamma \subseteq P$ and we are done.

If $(nx - ny) \in (P : \Gamma)$, then $(nx - ny)q \in (P : \Gamma)$ because $(P : \Gamma)$ is an ideal of N. Since N is left permutable, $(nx - ny)q = nxq - nyq = xnq - ynq = (x - y)nq$. Since $(P : \Gamma)$ is completely prime and $q \notin (P : \Gamma)$, $(x - y)n \in (P : \Gamma)$, it follows that $(x - y) \in (P : \Gamma)$ or $n \in (P : \Gamma)$ for all $n \in N$. If $n \in (P : \Gamma)$ for all $n \in N$, this contradicts that $N\Gamma \not\subseteq P$. Hence, $(x-y) \in (P : \Gamma)$. In particular, since $\gamma_0 \in \Gamma$, it follows that $(x - y)\gamma_0 = x\gamma_0 - y\gamma_0 = \gamma_1 - \gamma_2 \in P$. Thus, P is an equiprime N-ideal. □

4.12. Proposition. *Let N be a medial near-ring and P an N-ideal of Γ such that $N_d\backslash(P : \Gamma) \neq \emptyset$. Then P is an equiprime N-ideal if and only if P is a completely prime N-ideal.*

Proof. Suppose P is a completely prime N-ideal. Let $a \in N\backslash(P:\Gamma)$, $\gamma_1, \gamma_2 \in \Gamma$, $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$. We need to show that $\gamma_1 - \gamma_2 \in P$. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence, $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$ for some $x, y \in N$. Since $an\gamma_1 - an\gamma_2 = anx\gamma_0 - any\gamma_0 = (anx - any)\gamma_0 \in P$ and P is a completely prime N-ideal, $(anx - any)\Gamma \subseteq P$ or $\gamma_0 \in P$.

If $\gamma_0 \in P$, then $\Gamma = N\gamma_0 \subseteq NP \subseteq P$. Since $P \lhd_N \Gamma$, this contradicts that $N\Gamma \nsubseteq P$. Hence we have $(anx - any)\Gamma \subseteq P$, whence $anx - any \in (P:\Gamma)$. Let $n_d \in N_d\backslash(P:\Gamma)$. Then,

$$n_d(anx - any)n_d = n_danxn_d - n_danyn_d = n_dnaxn_d - n_dnayn_d \in (P:\Gamma)$$

since N is medial and $(P:\Gamma) \lhd N$, and again since N is medial, we get

$$\begin{aligned} n_dnaxn_d - n_dnayn_d &= n_da(nx)n_d - n_da(ny)n_d = n_d(nx)an_d - n_d(ny)an_d \\ &= n_d(nx - ny)an_d \in (P:\Gamma). \end{aligned}$$

In view of Proposition 3.5 (c) and the fact that $a, n_d \notin (P:\Gamma)$, $(nx - ny) \in (P:\Gamma)$. Now for $n_d \in N_d\backslash(P:\Gamma)$, we have $n_d(nx - ny)n_d \in (P:\Gamma)$. Since N is medial, $n_d(nx - ny)n_d = n_dnxn_d - n_dnyn_d = n_dxnn_d - n_dynn_d = n_d(x - y)nn_d$. Hence, $n_d(x - y)nn_d \in (P:\Gamma)$ for all $n \in N$. Since $(P:\Gamma)$ is a completely prime ideal and $n_d \in N_d\backslash(P:\Gamma)$ we have $(x - y)n \in (P:\Gamma)$ which implies that $(x - y) \in (P:\Gamma)$ or $n \in (P:\Gamma)$ for all $n \in N$.

If $n \in (P:\Gamma)$ for all $n \in N$, this contradicts that $N\Gamma \nsubseteq P$. Hence, $(x - y) \in (P:\Gamma)$. Therefore, for $\gamma_0 \in \Gamma$, $(x - y)\gamma_0 = x\gamma_0 - y\gamma_0 = \gamma_1 - \gamma_2 \in P$, which implies that P is an equiprime N-ideal. The converse comes from Corollary 4.8 (c). □

5. Equiprime N-ideals and equiprime ideals

5.1. Proposition. *If P is an equiprime N-ideal of Γ, then $(P:\Gamma)$ is an equiprime ideal of N.*

Proof. Assume P is equiprime and $a, x, y \in N$ are such that $anx - any \in (P:\Gamma)$ for all $n \in N$. Then for every $\gamma \in \Gamma$, we have $(anx - any)\gamma \in P$. Since Γ is an N-group, we get

$$(anx - any)\gamma = anx\gamma - any\gamma = an\gamma_1 - an\gamma_2 \in P,$$

where $\gamma_1 = x\gamma$ and $\gamma_2 = y\gamma$. Then $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$, since P is an equiprime N-ideal. If $a\Gamma \subseteq P$, then $a \in (P:\Gamma)$ and we are done. If $\gamma_1 - \gamma_2 \in P$, then $\gamma_1 - \gamma_2 = x\gamma - y\gamma = (x - y)\gamma \in P$ for all $\gamma \in \Gamma$. Hence, $(x - y) \in (P:\gamma)$ for all $\gamma \in \Gamma$. Therefore $(x - y) \in (P:\Gamma)$, which implies that $(P:\Gamma)$ is an equiprime ideal of N. □

5.2. Proposition. *Let N be a right permutable near-ring, Γ a monogenic N-group and $P \lhd_N \Gamma$ be such that $N\Gamma \nsubseteq P$. If $(P:\Gamma)$ is an equiprime ideal of N, then P is an equiprime N-ideal of Γ.*

Proof. Suppose that $(P:\Gamma)$ is an equiprime ideal of N and $a \in N$, $\gamma_1, \gamma_2 \in \Gamma$, $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$. We need to show that $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$.

Suppose $a\Gamma \nsubseteq P$ and $\gamma_1 - \gamma_2 \notin P$. If $a\Gamma \nsubseteq P$, then $a \notin (P:\Gamma)$. On the other hand, since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Hence, $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$ for some $x, y \in N$.

If $\gamma_1 - \gamma_2 \notin P$, then $\gamma_1 - \gamma_2 = x\gamma_0 - y\gamma_0 = (x - y)\gamma_0 \notin P$, which implies $(x - y) \notin (P:\gamma_0) \supseteq (P:\Gamma)$. Since $(P:\Gamma)$ is equiprime and $a, (x - y) \notin (P:\Gamma)$, there exists an $m \in N$ such that $amx - amy \notin (P:\Gamma)$. Hence, $amx - amy \notin (P:N\gamma_0)$. Then, there exists $n' \in N$ such that $(amx - amy)n'\gamma_0 \notin P$. So,

$$(amx - amy)n'\gamma_0 = amxn'\gamma_0 - amyn'\gamma_0 = amn'x\gamma_0 - amn'y\gamma_0 = amn'\gamma_1 - amn'\gamma_2,$$

since N is right permutable. Hence, there exists an $mn' \in N$ such that $amn'\gamma_1 - amn'\gamma_2 \notin P$. But this is a contradiction with the assumption. Hence, $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$, and therefore it follows that P is an equiprime N-ideal. □

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