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A NEW VIEW ON SOFT RINGS

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Abstract

In this paper we study soft rings and ideals. Firstly, we define a new binary relation on soft sets using binary relations on the universe and parameter sets. Then, we introduce the notion of soft ring and soft ideal over a ring, and some examples are given. Also, we obtain some new properties of soft rings and soft ideals. Lastly, we define extended sum, restricted sum, extended product, and restricted product of soft sets, and derive their basic properties.

Keywords: Soft sets, Soft rings, Soft subrings, Soft ring homomorphism, Soft ideals.

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1. Introduction

Many complicated problems in economics, engineering, the environment, social science, medical science and many other fields involve uncertain data. These problems which one come face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the notion of the exact solution of this model is determined. Because of that the mathematical model is too complex, the exact solution cannot be found. There are several well-known theories to describe uncertainty. For instance fuzzy set theory [31], rough set theory [26] and other mathematical tools. But all of these theories have their inherit difficulties as pointed out by Molodtsov [25]. To overcome these difficulties, Molodtsov introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties affecting existing methods.

The theory of soft sets has rich potential for applications in several directions, few of which had been demonstrated by Molodtsov in his pioneer work [25]. At present, works on soft set theory are making progress rapidly. Maji *et al.* [22] described an application

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of soft set theory to solve a decision making problem and studied several operations on the theory of soft sets. Pei and Miao [27] discussed the relationship between soft sets and information systems. Maji *et al.* [21] studied several operations on soft sets. Ali *et al.* [7] gave some new notions, such as restricted intersection, restricted union, restricted difference and extended intersection of two soft sets. Majumdar and Samanta [23] gave a notion of soft mappings, and some of their properties. Also they studied images and inverse images of crisp sets and soft sets under soft mappings.

Some researches have studied algebraic properties of soft sets. Initially, Aktaş and Çağman [2] introduced the basic concepts of soft set theory, and compared soft sets to the related concepts of fuzzy sets and rough sets. They also discussed the notion of soft groups and derived their basic properties using Molodtsov's definition of the soft sets. Chen *et al.* [5] defined the parameterization reduction of soft sets, and improved an application of a soft set in a decision making problem. Jun and Park [16] discussed the applications of soft sets in the ideal theory of BCK/BCI-algebras. Jun [12] applied the notion of soft sets to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Feng *et al.* [8] introduced the notions of soft semirings, soft ideals and idealistic soft semirings, and then investigated several related properties. Jin-liang *et al.* [11] defined operations on fuzzy soft groups, and proved some results about them. Sun *et al.* [28] presented the definition of soft modules, and constructed some basic properties using modules. Yang *et al.* [30] combined interval-valued fuzzy sets and soft sets, and then introduced the concept of the interval-valued fuzzy soft set. Jun *et al.* [15] introduced the notions of soft p-ideals and p-idealistic soft BCI-algebras, and then gave characterizations of p-ideals in BCI-algebras. Aygünoğlu and Aygün [3] gave the concept of fuzzy soft group and defined fuzzy soft functions and fuzzy soft homomorphisms. Jun *et al.* [13] defined the notions of soft d-algebras, soft d^*-algebras, soft d-ideals, soft $d^\sharp$-ideals, soft d^* ideals, and d-idealistic soft d algebras, and surveyed their properties. Jun and Song [17] introduced the notion of $\in$-soft set and q-soft set, based on a fuzzy set, and investigated conditions for $\in$-soft sets and q-soft sets to be idealistic soft BCK/BCI-algebras. Çağman and Enginoğlu [6] defined soft matrices and their operations. They also constructed a soft max-min decision making method. Jun *et al.* [14] derived the notion of fuzzy soft BCK/BCI algebras and investigated its properties. Feng *et al.* [9] provided a framework to combine fuzzy sets, rough sets and soft sets all together. Babitha and Sunil [4] introduced concepts of soft set relations, and discussed many related concepts such as equivalent soft set relationss, partitions, composition, function etc. Liu *et al.* [20] described some classes of soft rings and give the first, second and third fuzzy isomorphism theorems for soft rings. Kazancı *et al.* [19] defined soft BCH-algebras and gave the theorems of homomorphic image and homomorphic pre-image of soft sets. Majumdar and Samanta [24] defined generalized fuzzy soft sets and studied some of their properties. They also showed applications of generalized fuzzy soft sets. Zhan and Jun [32] investigated soft BL-algebras based on fuzzy sets. Jun *et al.* [18] investigated $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy p-ideals and fuzzy p-ideals with thresholds related to soft set theory. Lastly, Acar *et al.* [1] defined soft rings and introduced basic notions of soft rings.

From the beginning the majority of studies on soft sets for algebraic structures such as, groups, rings, semirings, modules and BCK/BCI-algebras have concentrated on usual binary operations [1, 2, 7, 8, 12, 16, 19, 28, 29]. However, this seems to restrict the application of algebraic sets. To solve this problem, we define new binary relations and soft functions on soft sets. From this point of view, in Section 2, we summarize some basic concepts of soft sets which will be used throughout the paper. In Section 3, we recall the concept of soft rings and soft ideals as introduced by Acar *et al.* [1], and its further properties are discussed. Also, we introduce the concept of soft ring homomorphism for rings, and we examine some of its properties. Finally some new binary relations, named extended sum, restricted sum, extended product, and restricted product are introduced and some important results obtained for them.

2. Basic definitions on soft set theory

In this section, we will give some known and useful definitions and notations. The definitions may be found in references [7, 8, 21, 25, 27].

Molodtsow [25] defined the notion of a soft set in the following way: Let U be an initial universe set and E a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E. A pair (F, A) is called a *soft set* over U, where F is a mapping $F : A \to \mathcal{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-approximate elements of the soft set (F, A).

2.1. Definition. [8] For a soft set (F, A), the set $\mathrm{Supp}(F, A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the *support* of the soft set (F, A). If $\mathrm{Supp}(F, A) \neq \emptyset$, then the soft set (F, A) is called non-null.

2.2. Definition. [25] Let (F, A) and (G, B) be two soft sets over U. Then,

(i) (F, A) is said to be a *soft subset* of (G, B), denoted by $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$,
(ii) (F, A) and (G, B) are said to be *soft equal*, denoted by $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

2.3. Definition. [7, 8] Let (F, A) and (G, B) be two soft sets over U. Then,

(1) The *extended intersection* (H, C) of soft sets (F, A) and (G, B), denoted by $(F, A) \cap_\varepsilon (G, B)$, is defined as $C = A \cup B$, and for all $c \in C$

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

(2) The *restricted intersection* (H, C) of soft sets (F, A) and (G, B), denoted by $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and for all $c \in C$, $H(c) = F(c) \cap G(c)$.
(3) The *extended union* (H, C) of soft sets (F, A) and (G, B), denoted by $(F, A) \cup (G, B)$, is defined as $C = A \cup B$, and for all $c \in C$

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

(4) The *restricted union* (H, C) of soft sets (F, A) and (G, B), denoted by $(F, A) \cup_\Re (G, B)$, is defined as $C = A \cap B$, and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Note that restricted intersection was also known as bi-intersection in Feng *et al.* [8], and extended union was first introduced and called union by Maji *et al.* [21].

Now, we define a binary operation on soft sets in the following way: Suppose that $\oplus$ is a binary operation on $\mathcal{P}(E)$, and $\otimes$ is a binary operation on $\mathcal{P}(U)$. Then for any two soft sets (F, A) and (G, B) over U, $(F, A) \oplus_\otimes (G, B)$ is defined as the soft set (H, C), where $C = A \oplus B$ and

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) \otimes G(c) & \text{if } \in A \cap B \\ \emptyset & \text{if otherwise.} \end{cases}$$

for all $c \in C$. Here we describe a general binary operation. However, many researchers gave more specific binary relations [7, 21, 25]. As an example, the following binary operations given in Definition 2.3 can be obtained a special case of above binary relation in the following way:

(i) If $\oplus = \cup$, and $\otimes = \cap$, then $(F, A) \oplus_\otimes (G, B)$ is the extended intersection of (F, A) and (G, B),
(ii) If $\oplus = \cap$, and $\otimes = \cap$, then $(F, A) \oplus_\otimes (G, B)$ is the restricted intersection of (F, A) and (G, B),
(iii) If $\oplus = \cup$, and $\otimes = \cup$, then $(F, A) \oplus_\otimes (G, B)$ is the extended union of (F, A) and (G, B),
(iv) If $\oplus = \cap$, and $\otimes = \cup$, then $(F, A) \oplus_\otimes (G, B)$ is the restricted union of (F, A) and (G, B).

Molodtsov [25] also proposed a general way to define binary operations over soft sets. Assume that we have a binary operation on $\mathcal{P}(U)$, which is denoted by $\oplus$. Let (F, A) and (G, B) be soft sets over U. Then, the operation $\oplus$ for soft sets in defined by $(F, A) \oplus (G, B) = (H, A \times B)$, where $H(a, b) = F(a) \oplus G(b)$, $a \in A$, $b \in B$. Maji et al. [21] introduced the following operations over soft sets, which can be seen as the implementations of Molodtsovs idea above.

2.4. Definition. [8, 21] Let (F, A) and (G, B) be two soft sets over U. Then,

(1) The $\wedge$-*intersection* of two soft sets (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \wedge (G, B)$ over U, where $C = A \times B$, and $H(a, b) = F(a) \cap G(b)$ for all $(a, b) \in A \times B$;
(2) The $\vee$-*union* of two soft sets (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \vee (G, B)$ over U, where $C = A \times B$, and $H(a, b) = F(a) \cup G(b)$ for all $(a, b) \in A \times B$.
(3) Let (F, A) and (H, B) be two soft sets over G and K, respectively. The *cartesian product* of the soft sets (F, A) and (H, B), denoted by $(F, A) \times (H, B)$, is defined as $(F, A) \times (H, B) = (U, A \times B)$, where $U(x, y) = F(x) \times H(y)$ for all $(x, y) \in A \times B$.

The following definitions are generalizations of the above.

2.4. Definition. [7, 8, 19] Let $\{(H_i, A_i) \mid i \in \Lambda\}$ be a family of soft sets over U. Then,

(1) The *extended intersection* of the family (H_i, A_i), denoted by $(\bigcap_\varepsilon)_{i \in \Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \bigcup_{i \in \Lambda} A_i, \quad H(a) = \bigcap_{i \in \Lambda(a)} H_i(a) \ \forall\, a \in A,$$

(2) The *restricted intersection* of the family (H_i, A_i), denoted by $\bigcap_{i \in \Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \bigcap_{i \in \Lambda} A_i, \quad H(a) = \bigcap_{i \in \Lambda} H_i(a) \ \forall\, a \in A,$$

(3) The *extended union* of the family (H_i, A_i), denoted by $\bigcup_{i\in\Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \bigcup_{i\in\Lambda} A_i, \quad H(a) = \bigcup_{i\in\Lambda(a)} H_i(a)\ \forall\, a \in A,$$

(4) The *restricted union* of the family (H_i, A_i), denoted by $(\bigcup_{\Re})_{i\in\Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \bigcap_{i\in\Lambda} A_i, \quad H(a) = \bigcup_{i\in\Lambda} H_i(a)\ \forall\, a \in A.$$

2.5. Definition. [8, 19, 21] Let $\{(H_i, A_i) \mid i \in \Lambda\}$ be a family of soft sets over U. Then,

(1) The $\wedge$-*intersection* of the family (H_i, A_i), denoted by $\bigwedge_{i\in\Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \prod_{i\in\Lambda} A_i, \quad H((a_i)_{i\in\Lambda}) = \bigcap_{i\in\Lambda} H_i(a_i)\ \forall\,(a_i)_{i\in\Lambda} \in A,$$

(2) The $\vee$-*union* of the family (H_i, A_i), denoted by $\bigvee_{i\in\Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \prod_{i\in\Lambda} A_i, \quad H((a_i)_{i\in\Lambda}) = \bigcup_{i\in\Lambda} H_i(a_i)\ \forall\,(a_i)_{i\in\Lambda} \in A),$$

(3) The *cartesian product* of the family (H_i, A_i), denoted by $\prod_{i\in\Lambda}(H_i, A_i)$, is the soft set (H, A) defined as:

$$A = \prod_{i\in\Lambda} A_i, \quad H((a_i)_{i\in\Lambda}) = \prod_{i\in\Lambda} H_i(a_i)\ \forall\,(a_i)_{i\in\Lambda} \in A.$$

2.6. Definition. [7] Let (F, A) be soft set over U. Then,

(i) (F, A) is said to be a *relative null soft set*, denoted by $\mathcal{N}_A$, if $F(e) = \emptyset$ for all $e \in A$,

(ii) (F, A) is said to be a *relative whole soft set*, denoted by $\mathcal{W}_A$, if $F(e) = U$ for all $e \in A$.

2.7. Definition. [29] Let (F, A) and (G, B) be two soft sets over U and U' respectively, $f : U \to U'$, $g : A \to B$ be two functions. Then we say that the pair (f, g) is a *soft function* from (F, A) to (G, B), denoted by $(f, g) : (F, A) \to (G, B)$, if the following condition is satisfied: $f(F(x)) = G(g(x))$ for all $x \in A$. If f and g are injective (resp. surjective, bijective), then (f, g) is said to be injective (resp. surjective, bijective).

The concept of soft homomorphism on groups was at first introduced by Aktaş *et al.* [2].

2.8. Lemma. [29] *Let (F, A), (G, B) and (H, C) be soft sets over U, U' and U'', respectively. Let $(f, g) : (F, A) \to (G, B)$ and $(f', g') : (G, B) \to (H, C)$ be two soft functions. Then $(f' \circ f, g' \circ g) : (F, A) \to (H, C)$ is a soft function.* □

2.9. Definition. [29] Let (F, A) and (G, B) be two soft sets over U and U' respectively, (f, g) a soft function from (F, A) to (G, B).

The *image* of (F, A) under the soft function (f, g), denoted by $(f, g)(F, A) = (f(F), B)$, is the soft set over U' defined by

$$f(F)(y) = \begin{cases} \bigcup_{g(x)=y} f(F(x)) & \text{if } y \in \text{Img}, \\ \emptyset & \text{otherwise.} \end{cases}$$

for all $y \in B$.

The *pre-image* of (G, B) under the soft function (f, g), denoted by $(f, g)^{-1}(G, B) = (f^{-1}(G), A)$, is the soft set over U defined by $f^{-1}(G)(x) = f^{-1}(G(g(x)))$ for all $x \in A$.

It is clear that $(f, g)(F, A)$ is a soft subset of (G, B) and (F, A) is a soft subset of $(f, g)^{-1}(G, B)$. In particular, if g is the identity function on A, the soft sets $(f(F), A)$ and $(f^{-1}(G), A)$ are as given in [2, 19].

3. Soft rings and soft ideals

We begin by giving some known and useful definitions and notations of the theory of rings from [10]. A *ring* R is a structure consisting of a non-empty set R together with two binary operations on R, called addition and multiplication, denoted in the usual manner, such that

- R together with addition is a commutative group,
- R together with multiplication is a semigroup, and
- $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in R$.

If R contains an element 1_R such that $1_R a = a 1_R$ for all $a \in R$, then R is said to be a *ring with identity*. A zero element of a ring R is an element 0 (necessarily unique) such that $0 + x = x + 0 = x$ for all $x \in R$. A non-empty subset I of a ring R is called a *subring* if and only if $a - b \in I$ and $ab \in I$ for all $a, b \in I$. A non-empty subset I of R is called an *ideal*, denoted by $I \triangleleft R$, if and only if $a - b \in I$ and $ra, ar \in I$ for all $a, b \in I$ and $r \in R$. The subrings $\{0\}$ and R are called *trivial subrings* of R.

Let R and S be rings. A mapping $f : R \to S$ is called a ring *homomorphism* if it satisfies

$$f(a + b) = f(a) + f(b) \text{ and } f(ab) = f(a)f(b)$$

for all $a, b \in R$. That is, the mapping f preserves the ring operations. A ring homomorphism $f : R \to S$ is called a *monomorphism* [resp. *epimorphism*, *isomorphism*] if it is an injective [resp. surjective, bijective] mapping. The *kernel* of a ring homomorphism $f : R \to S$ is its kernel as a map of additive groups; that is, $\mathrm{Ker} f = \{r \in R \mid f(r) = 0\}$. Similarly the *image* of f, denoted $\mathrm{Im}\ f$, is $\{s \in S \mid f(r) = s \text{ for some } r \in R\}$.

If A and B are non-empty subsets of R, $A + B$ denote the set of all sums $\{a + b \mid a \in A, b \in B\}$ and $A \cdot B$ denote the set of all finite sums of products $\{a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \mid n \in \mathbb{N},\ a_i \in A,\ b_i \in B\}$.

Let $\{S_i \mid i \in I\}$ be a family of subrings (ideals) of R, then their intersection $\bigcap_{i \in I} S_i$ is a subring (ideal) of R.

Let S_i be a subring (ideal) of R_i $(i \in I)$, then their cartesian product $\prod_{i \in I} S_i$ is subring (ideal) of $\prod_{i \in I} R_i$.

3.1. Definition. Let (F, A) be a non-null soft set over a ring R. Then (F, A) is called a *soft ring over* R if $F(x)$ is a subring of R for all $x \in \mathrm{Supp}(F, A)$.

In Definition 3.1., $F(x)$ is always a non-empty set for all $x \in \mathrm{Supp}(F, A)$. If F is chosen such that $F : A \to \mathcal{P}(R) \setminus \{\emptyset\}$ or $\mathrm{Supp}(F, A) = A$, then Definition 3.1. coincides with [1, Definition 3.1].

3.2. Definition. Let (F, A) be a non-null soft set over a ring R. Then (F, A) is called a *soft ideal* over R if $F(x)$ is an ideal of R for all $x \in \mathrm{Supp}(F, A)$. This definition is similar to that of *idealistic soft ring* [1, Definition 5.1].

It is clear that every soft ideal is a soft ring.

Let us illustrate these definitions using the following examples.

3.3. Example. Let R be a ring and I a subring (ideal) of R. Let $F : A \to \mathcal{P}(R)$ be a mapping such that $F(x) = I$ for all $x \in A$. Then (F, A) is a soft ring (ideal) over R.

In Example 3.3., If $I = \{0_R\}$, then (F, A) is said to be a *relative null soft ring* (*ideal*) over R. If $I = R$, then (F, A) is said to be a *relative whole soft ring (ideal)* over R.

3.4. Example. Let R be a ring and $F : R \to \mathcal{P}(R)$ a mapping such that $F(r) =< r >$, where $< r >$ is the ideal generated by $r \in R$ for all $r \in R$. Then (F, R) is a soft ideal over R.

3.5. Example. Let (F, A) be a soft set over $\mathbb{Z}_6$, where $A = \{0, 1, 2\}$ and $F : A \to \mathcal{P}(\mathbb{Z}_6)$ is a set-valued function defined by

$$F(x) = \{y \in \mathbb{Z}_6 \mid x \cdot y = 0\}$$

for all $x \in A$. Since $F(x)$ is an ideal of $\mathbb{Z}_6$ for all $x \in \mathrm{Supp}(F, A)$, then (F, A) is a soft ideal over $\mathbb{Z}_6$.

3.6. Example. Let (F, A) be a soft set over $\mathbb{Z}$, where $A = \mathbb{Z}$ and $F : A \to \mathcal{P}(\mathbb{Z})$ is the function defined by

$$F(x) = \begin{cases} x\mathbb{Z} & \text{if } 2 \mid x, \\ \emptyset & \text{otherwise,} \end{cases}$$

for all $x \in A$. The support set of the soft set (F, A) is $\mathrm{Supp}(F, A) = \{2n \mid n \in \mathbb{Z}\}$. Since $F(x)$ is an ideal of $\mathbb{Z}$ for all $x \in \mathrm{Supp}(F, A)$, then (F, A) is a soft ideal over $\mathbb{Z}$.

3.7. Example. Let R be a ring with identity element and $F : R \to \mathcal{P}(R)$ a mapping such that $F(x) = \{r \mid rx = xr\}$ for all $x \in R$. Then (F, R) is a soft ring over R. On the other hand, (F, R) is a soft ideal if and only if R is a commutative ring.

3.8. Example. Let $M_{n\times n}(\mathbb{R})$ be the set of all $n \times n$ matrices over $\mathbb{R}$ and (F, A) a soft set over R, where $R = A = M_{n\times n}(\mathbb{R})$ and $F : A \to \mathcal{P}(R)$ is the function defined by $F(B) = \{C \cdot B \mid C \in M_{n\times n}(\mathbb{R})\}$ for all $B \in A$. Then (F, A) is a soft ring over R, but (F, A) is not a soft ideal over R.

3.9. Theorem. [1] *Let $\{(F_i, A_i) \mid i \in \Lambda\}$ be a non-empty family of soft rings over R. Then,*

(1) *The restricted intersection of the family $\{(F_i, A_i) \mid i \in \Lambda\}$ is a soft ring over R if it is non-null.*
(2) *If $A_i \cap A_j = \emptyset$ for all $i, j \in \Lambda$, $i \neq j$, then the extended union of the family $\{(F_i, A_i) \mid i \in \Lambda\}$ is a soft ring over R.* □

3.10. Theorem. *Let $\{(F_i, A_i) \mid i \in \Lambda\}$ be a non-empty family of soft ideals over R. Then,*

(1) *The restricted intersection of the family $\{(F_i, A_i) \mid i \in \Lambda\}$ is a soft ideal over R if it is non-null.*
(2) *If $A_i \cap A_j = \emptyset$ for all $i, j \in \Lambda$, $i \neq j$, then the extended union of the family $\{(F_i, A_i) \mid i \in \Lambda\}$ is a soft ideal over R.*

Proof. (1) The result is obvious since the intersection of an arbitrary non-empty family of ideals of a ring is an ideal.

(2) Similar to the proof of Theorem 3.9 (2). □

3.11. Theorem. *Let $\{(F_i, A_i) \mid i \in \Lambda\}$ be a non-empty family of soft rings (ideals) over R. Then,*

(1) *If* $F_i(x) \subseteq F_j(x)$ *or* $F_j(x) \subseteq F_i(x)$ *for all* $i, j \in \Lambda(x)$, $x \in \bigcup_{i\in\Lambda} A_i$, *then the extended union of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$ *is a soft ring (ideal) over* R.
(2) *The extended intersection of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$ *is a soft ring (ideal) over* R *if it is non-null.*
(3) *If* $F_i(x) \subseteq F_j(x)$ *or* $F_j(x) \subseteq F_i(x)$ *for all* $i, j \in \Lambda$, $x \in \bigcap_{i\in\Lambda} A_i$, *then the restricted union of the family* $\{(F_i, A_i) \mid i \in \Lambda\}$ *is a soft ring (ideal) over* R.

Proof. (1) We can write $\bigcup_{i\in\Lambda}(F_i, A_i) = (F, B)$. Then $B = \bigcup_{i\in\Lambda} A_i$ and for all $x \in B$, $F(x) = \bigcup_{i\in\Lambda(x)} F_i(x)$. Note first that (F, B) is non-null since

$$\mathrm{Supp}(F, B) = \bigcup_{i\in\Lambda} \mathrm{Supp}(F_i, A_i) \neq \emptyset.$$

Let $x \in \mathrm{Supp}(F, B)$. Then $F(x) = \bigcup_{i\in\Lambda(x)} F_i(x) \neq \emptyset$, and so we have $F_{i_0}(x) \neq \emptyset$ for some $i_0 \in \Lambda(x)$. By the hypothesis, we know that $F_i(x) \subseteq F_j(x)$ or $F_j(x) \subseteq F_i(x)$ for all $i, j \in \Lambda(x)$, $x \in \bigcup_{i\in\Lambda} A_i$. Hence $\bigcup_{i\in\Lambda} F_i(x)$ is a subring (ideal) of R. Moreover the extended union $\bigcup_{i\in\Lambda}(F_i, A_i)$ is a soft ring (ideal) over R.

(2) We can write $(\bigcap_{\varepsilon})_{i\in\Lambda}(F_i, A_i) = (F, B)$. Then $B = \bigcup_{i\in\Lambda} A_i$, and for all $x \in B$, $F(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$. Note first that (F, B) is non-null since

$$\mathrm{Supp}(F, B) = \bigcup_{i\in\Lambda} \mathrm{Supp}(F_i, A_i) \neq \emptyset.$$

Let $x \in \mathrm{Supp}(F, B)$. Then $F(x) = \bigcap_{i\in\Lambda(x)} F_i(x) \neq \emptyset$, and so we have $F_i(x) \neq \emptyset$ for all $i \in \Lambda(x)$. By the hypothesis, we know that $F_i(x)$ is subring (ideal) of R for all $i \in \Lambda(x)$, $x \in \bigcup_{i\in\Lambda} A_i$. Hence $\bigcap_{i\in\Lambda} F_i(x)$ is a subring (ideal) of R. Moreover the extended intersection $(\bigcap_{\varepsilon})_{i\in\Lambda}(F_i, A_i)$ is a soft ring (ideal) over R.

(3) Similar to (1). □

We obtain following corollaries from Theorem 3.10 and Theorem 3.11.

3.12. Corollary. *Let* (F, A) *and* (G, B) *be two soft ideals over* R. *Then the restricted intersection of* (F, A) *and* (G, B) *is a soft ideal over* R *if it is non-null.* □

3.13. Corollary. *Let* (F, A) *and* (G, B) *be two soft rings (ideals) over* R. *Then,*

(1) *If* $F(x) \subseteq G(x)$ *or* $G(x) \subseteq F(x)$ *for all* $x \in A \cap B$, *then the extended union of* (F, A) *and* (G, B) *is a soft ring (ideal) over* R.
(2) *The extended intersection of* (F, A) *and* (G, B) *is a soft ring (ideal) over* R *if it is non-null.*
(3) *If* $F(x) \subseteq G(x)$ *or* $G(x) \subseteq F(x)$ *for all* $x \in A \cap B$, *then the restricted union of* (F, A) *and* (G, B) *is a soft ring (ideal) over* R. □

3.14. Theorem. [1] *Let* $\{(F_i, A_i) \mid i \in \Lambda\}$ *be a non-empty family of soft rings over* R. *Then the* $\wedge$*-intersection* $\bigwedge_{i\in\Lambda}(F_i, A_i)$ *is a soft ring over* R *if it is non-null.* □

3.15. Theorem. *Let* $\{(F_i, A_i) \mid i \in \Lambda\}$ *be a non-empty family of soft ideals over* R. *Then the* $\wedge$*-intersection* $\bigwedge_{i\in\Lambda}(F_i, A_i)$ *is a soft ideal over* R *if it is non-null.*

Proof. By Definition 2.6. (1), we can write $\bigwedge_{i\in\Lambda}(F_i, A_i) = (F, B)$. Let $(x_i)_{i\in\Lambda} \in \mathrm{Supp}(F, B)$. Then $F((x_i)_{i\in\Lambda}) = \bigcap_{i\in\Lambda} F_i(x_i) \neq \emptyset$. By the hypothesis, we know that $F_i(x_i)$ ideal of R, we obtain $\bigcap_{i\in\Lambda} F_i(x_i)$ is a ideal of R. Hence the $\wedge$-intersection $\bigwedge_{i\in\Lambda}(F_i, A_i)$ is a soft ideal over R. □

3.16. Theorem. *Let* $\{(F_i, A_i) \mid i \in \Lambda\}$ *be a non-empty family of soft rings (ideals) over* R. *If* $F_i(a_i) \subseteq F_j(a_j)$ *or* $F_j(a_j) \subseteq F_i(a_i)$ *for all* $i, j \in \Lambda$, $a_i \in A_i$, *then the* $\vee$*-union* $\bigvee_{i\in\Lambda}(F_i, A_i)$ *is a soft ring (ideal) over* R.

Proof. We can write $\bigvee_{i\in\Lambda}(F_i, A_i) = (F, B)$. Let $(x_i)_{i\in\Lambda} \in \mathrm{Supp}(F, B)$. Then $F((x_i)_{i\in\Lambda}) = \bigcup_{i\in\Lambda} F_i(x_i) \neq \emptyset$, and so we have $F_{i_0}(x_{i_0}) \neq \emptyset$ for some $i_0 \in \Lambda$. By the hypothesis, we know that $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$ for all $i, j \in \Lambda$, $x_i \in A_i$, and $F_i(x_i)$ is a subring (ideal) of R, we obtain $\bigcup_{i\in\Lambda} F_i(x_i)$ is a subring (ideal) of R. Hence the $\vee$-union $\bigvee_{i\in\Lambda}(F_i, A_i)$ is a soft ring (ideal) over R. □

We obtain following corollaries from Theorem 3.15 and Theorem 3.16.

3.17. Corollary. *Let (F, A) and (G, B) be two soft ideals over R. Then the $\wedge$-intersection of the two soft set (F, A) and (G, B) is a soft ideal over R if it is non-null.* □

3.18. Corollary. *Let (F, A) and (G, B) be two soft rings (ideals) over R. If $F(x) \subseteq G(y)$ or $G(y) \subseteq F(x)$ for all $(x, y) \in A \times B$, then the $\vee$-union of the two soft set (F, A) and (G, B) is a soft ring (ideal) over R.* □

3.19. Theorem. *Let (F_i, A_i) be soft ring (ideal) over R_i, $(i \in \Lambda)$. Then the cartesian product of the family $\{(F_i, A_i) \mid i \in \Lambda\}$ is a soft ring (ideal) over $\prod_{i\in\Lambda} R_i$.*

Proof. We can write $\prod_{i\in\Lambda}(F_i, A_i) = (F, B)$. Let $(x_i)_{i\in\Lambda} \in \mathrm{Supp}(F, B)$. Then $F((x_i)_{i\in\Lambda}) = \prod_{i\in\Lambda} R_i(x_i) \neq \emptyset$, and so we have $F_i(x_i) \neq \emptyset$ for all $i \in \Lambda$. Since $F_i(x_i)$ subring (ideal) of R_i, we obtain $\prod_{i\in\Lambda} R_i(x_i)$ is a subring (ideal) of $\prod_{i\in\Lambda} R_i$. Hence $\prod_{i\in\Lambda}(R_i, A_i)$ is a soft ring (ideal) over $\prod_{i\in\Lambda} R_i$. □

3.20. Corollary. *Let (F, A) and (G, B) be two soft rings (ideals) over R_1 and R_2, respectively. Then the cartesian product of the soft sets (F, A) and (G, B) is a soft ring (ideal) over $R_1 \times R_2$.* □

3.21. Definition. Let (F, A) and (G, B) be two soft rings over R_1 and R_2, respectively. Let (f, g) be a soft function from (F, A) to (G, B). If f is a ring homomorphism from R_1 to R_2, then (f, g) is said to be a *soft ring homomorphism*, and that (F, A) is *soft homomorphic* to (G, B). The latter is denoted by $(F, A) \sim (G, B)$.

If f is an epimorphism and g is surjective, then this definition is coincides with [1, Definition 5.13].

In this definition, if f is an isomorphism from R_1 to R_2 and g is a bijective mapping from A onto B, then we say that (f, g) is a *soft ring isomorphism* and that (F, A) is *soft isomorphic* to (G, B). The latter is denoted by $(F, A) \simeq (G, B)$.

3.22. Theorem. *Let (F, A) and (G, B) be soft sets over R_1 and R_2. Let (f, g) be a soft ring homomorphism from (F, A) to (G, B).*

(i) *If g is bijective mapping and (F, A) is a soft ring over R_1, then $(f(F), B)$ is a soft ring over R_2.*
(ii) *If (G, B) is a soft ring over R_2, then $(f^{-1}(G), A)$ is a soft ring over R_1 if it is non-null.*

Proof. (i) Since (F, A) is a soft ring over R_1, it is clear that $(f(F), B)$ is a non-null soft set over R_2. Let $y \in \mathrm{Supp}(f(F), B)$. Then $\bigcup_{g(x)=y} f(F(x)) \neq \emptyset$. Since g is bijective, then there exist a unit $x \in A$ such that $g(x) = y$ and $f(F(x)) \neq \emptyset$. Because $F(x)$ is a subring of R_1, and f is ring homomorphism, then $f(F(x))$ is a subring of R_2. Since g is injective mapping, then $f(F)(y) = f(F(x))$ is a subring over R_2. Consequently, $(f(F), B)$ is a soft ring over R_2.

(ii) Let $x \in \mathrm{Supp}(f^{-1}(G), A)$. Then $f^{-1}(G)(x) \neq \emptyset$. We know that $f^{-1}(G)(x) = f^{-1}(G(g(x)))$. Since $g(x) \in B$ and (G, B) is a soft ring over R_2, then $G(g(x))$ is subring of R_2. Moreover the non-empty set $G(g(x))$ is a subring of R_2 and its homomorphic inverse image $f^{-1}(G(g(x)))$ is also a subring of R_1. Hence $(f^{-1}(G), A)$ is a soft ring over R_1. □

3.23. Theorem. *Let (F, A) and (G, B) be soft sets over R_1 and R_2. Let (f, g) be a soft ring homomorphism from (F, A) to (G, B).*

(i) *If g is bijective, f is a surjective mapping and (F, A) is a soft ideal over R_1, then $(f(F), B)$ is a soft ideal over R_2.*

(ii) *If (G, B) is a soft ideal over R_2, then $(f^{-1}(G), A)$ is a soft ideal over R_1 if it is non-null.*

Proof. Similar to the proof of Theorem 3.22. □

In Theorem 3.22., If $g = I_A$, then the following corollary is obtained.

3.24. Corollary.

(i) *Let (F, A) be a soft ring over R_1 and the function $f : R_1 \to R_2$ be a ring homomorphism. If $F(a) = \mathrm{Ker} f$ for all $a \in A$, then $(f(F), A)$ is a relative null soft ring over R_2.*

(ii) *Let (F, A) be a relative whole soft ring over R_1, and the function $f : R_1 \to R_2$ be an epimorphism of rings. Then $(f(F), A)$ is a relative whole soft ring over R_2.* □

3.25. Theorem. *Let (F, A), (G, B) and (H, C) be soft sets over R_1, R_2 and R_3, respectively. Let $(f, g) : (F, A) \to (G, B)$ and $(f', g') : (G, B) \to (H, C)$ be two soft ring homomorphisms. Then $(f' \circ f, g' \circ g) : (F, A) \to (H, C)$ is a soft ring homomorphism.*

Proof. Straightforward. □

We introduce the following new definitions for soft sets on a ring R.

3.26. Definition. Let (F, A) and (G, B) be two soft sets over a ring R. The *extended sum* of (F, A) and (G, B) is denoted by $(F, A) \oplus_\cup (G, B)$, and is defined as $(F, A) \oplus_\cup (G, B) = (H, C)$, where $C = A \cup B$ and

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) + G(c) & \text{if } c \in A \cap B, \end{cases}$$

for all $c \in C$.

3.27. Definition. Let (F, A) and (G, B) be two soft sets over a ring R such that $A \cap B \neq \emptyset$. The *restricted sum* of (F, A) and (G, B) is denoted by $(F, A) \oplus_\cap (G, B)$, and is defined as $(F, A) \oplus_\cap (G, B) = (H, C)$, where $C = A \cap B$ and $H(c) = F(c) + G(c)$ for all $c \in C$.

3.28. Definition. Let (F, A) and (G, B) be two soft sets over a ring R. The *extended product* of (F, A) and (G, B) is denoted by $(F, A) \odot_\cup (G, B)$, and is defined as $(F, A) \odot_\cup (G, B) = (H, C)$, where $C = A \cup B$ and

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B, \\ G(c) & \text{if } c \in B \setminus A, \\ F(c) \cdot G(c) & \text{if } c \in A \cap B, \end{cases}$$

for all $c \in C$.

3.29. Definition. Let (F, A) and (G, B) be two soft sets over a ring R such that $A \cap B \neq \emptyset$. The *restricted product* of (F, A) and (G, B) is denoted by $(F, A) \odot_\cap (G, B)$, and is defined as $(F, A) \odot_\cap (G, B) = (H, C)$, where $C = A \cap B$ and $H(c) = F(c) \cdot G(c)$ for all $c \in C$.

3.30. Theorem. *Let (F, A) and (G, B) be two soft ideals over R. Then the extended sum of two soft set (F, A) and (G, B) is a soft ideal over R.*

Proof. From Definition 3.26 we can write $(F, A) \oplus_{\cup} (G, B) = (H, A \cup B)$. If $c \in A \setminus B$ or $c \in B \setminus A$ for all $c \in A \cup B$, then it is obvious $H(c)$ is an ideal of R. If $c \in A \cap B$, then $H(c) = F(c) + G(c)$ is an ideal of R since the sum of two ideal is an ideal. Hence, we find that the extended sum of two soft ideals (F, A) and (G, B) is a soft ideal over R. □

3.31. Theorem. *Let (F, A) and (G, B) be two soft ideals over R. Then the restricted sum of two soft set (F, A) and (G, B) is a soft ideal over R.*

Proof. Similar to the proof of Theorem 3.30. □

3.32. Theorem. *Let (F, A) and (G, B) be two soft ideals over R. Then the extended product of two soft set (F, A) and (G, B) is a soft ideal over R.*

Proof. From Definition 3.28, we can write $(F, A) \odot_{\cup} (G, B) = (H, A \cup B)$. If $c \in A \setminus B$ or $c \in B \setminus A$ for all $c \in A \cup B$, then it is obvious that $H(c)$ is an ideal of R. If $c \in A \cap B$, then $H(c) = F(c) \cdot G(c)$ is an ideal of R since the product of two ideal is an ideal. Hence, we find that the extended product of two soft ideals (F, A) and (G, B) is a soft ideal over R. □

3.33. Theorem. *Let (F, A) and (G, B) be two soft ideals over R. Then the restricted product of two soft set (F, A) and (G, B) is a soft ideal over R.*

Proof. Similar to the proof of Theorem 3.32. □

3.34. Example. Let $R = \mathbb{Z}$, $A = \{2n \mid n \in \mathbb{Z}\}$, $B = \{3n \mid n \in \mathbb{Z}\}$. Consider the functions $F : A \to \mathcal{P}(\mathbb{Z})$ and $G : B \to \mathcal{P}(\mathbb{Z})$ defined by

$$F(x) = \begin{cases} \{0\} & \text{if } x = 0, \\ k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z}, \end{cases}$$

and

$$G(x) = \begin{cases} \{0\} & \text{if } x = 0, \\ 2^k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z}, \end{cases}$$

which are ideals of R. Thus (F, A) and (G, B) are soft ideals over R. Let $(F, A) \oplus_{\cup} (G, B) = (H, C)$, where $C = A \cup B$. Then, for all $x \in C$, we have

$$H(x) = \begin{cases} F(x) & \text{if } 2 \mid x,\ 3 \nmid x, \\ G(x) & \text{if } 3 \mid x,\ 2 \nmid x, \\ F(x) + G(x) & \text{if } \quad 6 \mid x, \end{cases}$$

$$= \begin{cases} k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \nmid x, \\ \mathbb{Z} & \text{if } 2 \nmid x,\ 3 \mid x, \\ k\mathbb{Z} + 2^k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \mid x, \\ \{0\} & \text{if } x = 0. \end{cases}$$

Let $(F, A) \oplus_{\cap} (G, B) = (K, D)$, where $D = A \cap B = \{6n \mid n \in \mathbb{Z}\}$. Then, for all $x \in D$, we have

$$K(x) = F(x) + G(x) = \begin{cases} k\mathbb{Z} + 2^k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \mid x, \\ \{0\} & \text{if } x = 0. \end{cases}$$

It is clear that the extended sum and restricted sum of two soft ideal (F, A) and (G, B) is a soft ideal over R.

Let $(F,A) \odot_{\cup} (G,B) = (H,C)$, where $C = A \cup B$. Then, for all $x \in C$, we have

$$H(x) = \begin{cases} F(x) & \text{if } 2 \mid x,\ 3 \nmid x, \\ G(x) & \text{if } 3 \mid x,\ 2 \nmid x, \\ F(x) \cdot G(x) & \text{if } 6 \mid x, \end{cases}$$

$$= \begin{cases} k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \nmid x, \\ \mathbb{Z} & \text{if } 2 \nmid x, 3 \mid x, \\ k.2^k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \mid x, \\ \{0\} & \text{if } x = 0. \end{cases}$$

Let $(F,A) \odot_{\cap} (G,B) = (K,D)$, where $D = A \cap B = \{6n \mid n \in \mathbb{Z}\}$. Then, for all $x \in D$, we have

$$K(x) = F(x) \cdot G(x) = \begin{cases} k.2^k\mathbb{Z} & \text{if } x \in 2^k\mathbb{Z} \setminus 2^{k+1}\mathbb{Z},\ k \geq 1,\ 3 \mid x, \\ \{0\} & \text{if } x = 0. \end{cases}$$

It is clear that the extended product and restricted product of two soft ideal (F,A) and (G,B) is a soft ideal over R.

3.35. Definition. [1] Let (F,A) and (G,B) be soft rings over R. Then the soft ring (F,A) is called a soft subring of (G,B), denoted by $(F,A) \leq_{\Re} (G,B)$, if it satisfies the following conditions:

(i) $A \subseteq B$,

(ii) $F(x)$ is a subring of $G(x)$ for all $x \in \mathrm{Supp}(F,A)$.

3.36. Theorem. *Let (F,A) be a soft ring over R and $\{(F_i,A_i) \mid i \in \Lambda\}$ a non-empty family of soft subrings of (F,A). Then,*

(1) *The restricted intersection of the family $\{(F_i,A_i) \mid i \in \Lambda\}$ is a soft subring of (F,A) if it is non-null.*

(2) *If $F_i(x) \subseteq F_j(x)$ or $F_j(x) \subseteq F_i(x)$ for all $i,j \in \Lambda(x)$, $x \in \bigcup_{i\in\Lambda} A_i$, then the extended union of the family $\{(F_i,A_i) \mid i \in \Lambda\}$ is a soft subring of (F,A).*

(3) *The extended intersection of the family $\{(F_i,A_i) \mid i \in \Lambda\}$is a soft subring of (F,A) if it is non-null.*

(4) *If $F_i(x) \subseteq F_j(x)$ or $F_j(x) \subseteq F_i(x)$ for all $i,j \in \Lambda$, $x \in \bigcap_{i\in\Lambda} A_i$, then the restricted union of the family $\{(F_i,A_i) \mid i \in \Lambda\}$ is a soft subring of (F,A).*

Proof. Similar to the proofs of Theorem 3.10. and Theorem 3.11. □

3.37. Theorem. *Let $\{(F_i,A_i) \mid i \in \Lambda\}$ be a non-empty family of soft rings over R. Let (G_i,B_i) be a soft subring of (F_i,A_i), $(i \in \Lambda)$. Then:*

(1) *The $\wedge$-intersection $\bigwedge_{i\in\Lambda}(G_i,B_i)$ is a soft subring of $\bigwedge_{i\in\Lambda}(F_i,A_i)$ if it is non-null.*

(2) *If $G_i(a_i) \subseteq G_j(a_j)$ or $G_j(a_j) \subseteq G_i(a_i)$ for all $i,j \in \Lambda$, $a_i \in B_i$, then the $\vee$-union $\bigvee_{i\in\Lambda}(G_i,B_i)$ is a soft subring of $\bigvee_{i\in\Lambda}(F_i,A_i)$*

Proof. Similar to the proofs of Theorem 3.15. and Theorem 3.16. □

4. Conclusion

The concept of soft set was first introduced by Molodtsov [25]. Following Molodtsov's approach, some basic algebraic structures on soft set were introduced in [1, 2, 7, 8, 12, 16, 19, 28, 29]. Acar *et al.* [1] applied the theory of soft sets to a ring. In this paper, some new properties of soft rings are investigated. Also the extended sum, restricted sum, extended product, restricted product of soft sets are established and their some basic properties are given. To extend this work, one could study the properties of soft sets in other algebraic structures such as modules.

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