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# Fuzzy Soft Cycles in Fuzzy Soft Graphs

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**ABSTRACT:** Fuzzy soft set theory is one among many topics which has been developed recently for dealing with uncertainties. In this paper, decomposition of complete fuzzy soft graphs into Hamiltonian fuzzy soft cycles is proposed and related properties are studied. Also, some results on complement of fuzzy soft cycles are presented with examples.

***Keywords –*** *Fuzzy soft graph, complete fuzzy soft graph, Hamilton fuzzy soft cycles, complement of fuzzy soft cycles.*

## 1. Introduction

Graph theory has numerous applications in many fields such as computer science, operations research, economics, etc. In 1890, Walecki constructed Hamiltonian decomposition of complete graphs (Alspach, 2008). Later various researchers extended his work on Hamiltonian decomposition. Graph-theoretic concepts can be applied when there is preciseness in data. Whenever there is uncertainty in the description of objects, fuzziness arises. A. Rosenfeld developed the theory of fuzzy graphs (Rosenfeld, 1975) based on fuzzy sets which were initiated in ( Zadeh, 1965). G. Nirmala and M. Vijaya discussed Hamiltonian fuzzy cycles on complete fuzzy graph (Nirmala and Vijaya, 2012). Two algorithms on finding Hamiltonian fuzzy cycle in a fuzzy network with an example that the air distance can be approximately evaluated from the total length covered by each fuzzy Hamiltonian cycle were illustrated in (Nagoor Gani and Latha, 2016). Molodtsov initiated the concept of soft sets (Molodtsov, 1999). P.K. Maji, A.R. Roy, R. Biswas introduced fuzzy soft sets which is more generalization than fuzzy sets and soft sets in order to solve problems dealing with uncertainties (Maji et al. 2001). Applications of fuzzy soft sets to decision making problems were presented in (Roy and Maji, 2007). The notions of fuzzy soft graph, complete fuzzy soft graph and some of the operations such as union, intersection of two fuzzy soft graphs were presented in (Mohinta and Samanta, 2015). M. Akram and F. Zafar introduced and investigated some of the properties of fuzzy soft cycles, fuzzy soft trees, fuzzy soft bridges (Akram and Zafar, 2016). Connectivity in fuzzy soft graphs in comparison with their complements through various examples were discussed in (Shashikala and Anil, 2016). Some applications of fuzzy soft graphs in social network and road network were described in (Akram and Nawaz, 2016). A new method was introduced for graph representation based on adjacency of vertices and soft set theory in (Ali et al. 2016). Some operations on type 2 soft graphs were introduced and its practical application in communication network was presented in (Hayat et al. 2017). Regular and irregular type 2 soft graphs, type 2 soft trees, type 2 soft cut nodes, type 2 soft cycles were introduced and some operations on type 2 soft trees were demonstrated in (Hayat et al. 2018). Different types of intuitionistic fuzzy soft graphs are defined and are also applied in agriculture field in decision making problems (Sarala and Deepa, 2018). Some properties of regular and totally regular fuzzy soft graphs are studied in (Akilandeswari, 2019). In this paper, Hamilton decomposition of complete fuzzy soft graph is presented and some results on fuzzy soft cycles with its complement through examples are discussed.

## 2. BASIC DEFINITIONS

Some of the definitions are reviewed below.

**Definition 2.1** (Akram and Nawaz, 2016)**:** A fuzzy soft graph $\tilde{G} = (G^*, \tilde{F}, \tilde{K}, A)$ is a 4-tuple such that

a) $G^* = (V, E)$ is a simple graph

b) A is a non-empty set of parameters

c) $(\tilde{F}, A)$ is a fuzzy soft set over $V$

d) $(\tilde{K}, A)$ is a fuzzy soft set over $E$

e) $\left((\tilde{F}, e), (\tilde{K}, e)\right)$ is a fuzzy graph of $G^*$ $\forall e \in A$

i.e. $\tilde{K}(e)(xy) \le \min\{\tilde{F}(e)(x), \tilde{F}(e)(y)\}$ for all $e \in A$ and $x, y \in V$. The fuzzy graph $\left((\tilde{F}, e), (\tilde{K}, e)\right)$ is denoted by $\tilde{H}(e)$ for convenience.

**Definition 2.2** (Mohinta and Samanta, 2015)**:** A fuzzy soft graph $\tilde{G}$ is a complete fuzzy soft graph if $\tilde{H}(e)$ is a complete fuzzy graph $\forall e \in A$ i.e. $\tilde{K}(e)(uv) = \min\{\tilde{F}(e)(u), \tilde{F}(e)(v)\}$

**Definition 2.3** (Nirmala and Vijaya, 2012)**:** A cycle C is called a fuzzy cycle if it contains more than one weakest arc and if a fuzzy cycle C traces all the vertices exactly once except the end vertices then it is called as Hamilton Fuzzy cycle.

**Definition 2.4** (Akram and Zafar, 2016)**:** A fuzzy soft graph $\tilde{G}$ is a fuzzy soft cycle if each fuzzy graph $\tilde{H}(e)$ $\forall e \in A$ is a fuzzy cycle.

**Definition 2.5** (Shashikala and Anil, 2016)**:** The complement of a fuzzy soft graph $\tilde{G}$ is a fuzzy soft graph denoted by $\tilde{G}^c$ where the fuzzy soft set over $V$ is same in both $\tilde{G}$ and $\tilde{G}^c$ and $\tilde{K}^c(e)(xy) = \min\{\tilde{F}(e)(x), \tilde{F}(e)(y)\} - \tilde{K}(e)(xy)$ $\forall e \in A, \forall x, y \in V$.

In what follows, Fuzzy soft vertices and Fuzzy soft edges are assumed to have membership values greater than zero, unless specified.

## 3. HAMILTONIAN FUZZY SOFT GRAPH

**Definition 3.1:** A fuzzy soft graph $\tilde{G}$ is said to be Hamiltonian if each $\tilde{H}(e)$ $\forall e \in A$ is Hamiltonian i.e. it contains Hamiltonian fuzzy cycles. If $\tilde{H}(e)$ contains n-Hamilton fuzzy cycles $\forall e \in A$ then $\tilde{G}$ also contains n-Hamilton fuzzy soft cycles.

**Example 3.1:** Consider a simple graph $G^* = (V, E)$ where $V = \{a, b, c, d, f\}$ and $E = \{ab, ac, ad, af, bc, bd, bf, cd, cf, df\}$. Let $A = \{e_1, e_2\}$ be a parameter set. Let $(\tilde{F}, A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\tilde{F} : A \to P(V)$ defined by

$$\tilde{F}(e_1) = \{ a/0.2, b/0.1, c/0.4, d/0.3, f/0.5 \}$$
$$\tilde{F}(e_2) = \{ a/0.1, b/0.3, c/0.4, d/0.2, f/0.4 \}$$

Let $(\tilde{K}, A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\tilde{K} : A \to P(E)$ defined by

$$\tilde{K}(e_1) = \{ ab/0.1, ac/0.2, ad/0.2, af/0.2, bc/0.1, bd/0.1, bf/0.1, cd/0.3, cf/0.4, df/0.3 \}$$
$$\tilde{K}(e_2) = \{ ab/0.1, ac/0.1, ad/0.1, af/0.1, bc/0.3, bd/0.2, bf/0.3, cd/0.2, cf/0.4, df/0.2 \}$$

$\tilde{H}(e_1) = (\tilde{F}(e_1), \tilde{K}(e_1))$ *and* $\tilde{H}(e_2) = (\tilde{F}(e_2), \tilde{K}(e_2))$ are complete fuzzy graphs. Hence, $\tilde{G}$ is a complete fuzzy soft graph as shown in Fig.1.

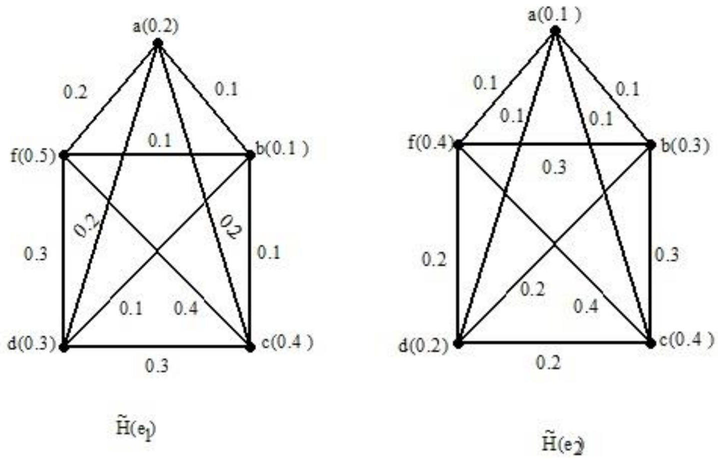


Figure1. Complete Fuzzy soft graph $\tilde{G}$

Now, consider $\tilde{H}(e_1)$ which has two Hamiltonian fuzzy cycles. Let it be *abcdfa* and *acfbda*. In a similar manner $\tilde{H}(e_2)$ also has Hamiltonian fuzzy cycles which visits every vertex exactly once. Hence, $\tilde{G}$ is Hamiltonian.

**Example 3.2:** Consider a simple graph $G^* = (V, E)$ where $V = \{a, b, c, d, f, g, h\}$ and $E = \{ab, ac, ad, af, ag, ah, bc, bd, bf, bg, bh, cd, cf, cg, ch, df, dg, dh, fg, fh, gh\}$. Let $A = \{e_1, e_2\}$, $\tilde{F} : A \to P(V)$ defined by

$$\tilde{F}(e_1) = \{ a/0.7, b/0.3, c/0.5, d/0.2, f/0.1, g/0.4, h/0.6 \}$$

$$\tilde{F}(e_2) = \{ a/0.2, b/0.2, c/0.1, d/0.5, f/0.6, g/0.3, h/0.4 \}$$

Let $\tilde{K} : A \to P(E)$ be defined by $\tilde{K}(e)(xy) = \min\{\tilde{F}(e)(x), \tilde{F}(e)(y)\}$.

We get $\tilde{H}(e_1) = (\tilde{F}(e_1), \tilde{K}(e_1))$ *and* $\tilde{H}(e_2) = (\tilde{F}(e_2), \tilde{K}(e_2))$ as complete fuzzy graphs which also contains Hamiltonian fuzzy cycles as shown in Fig.2.

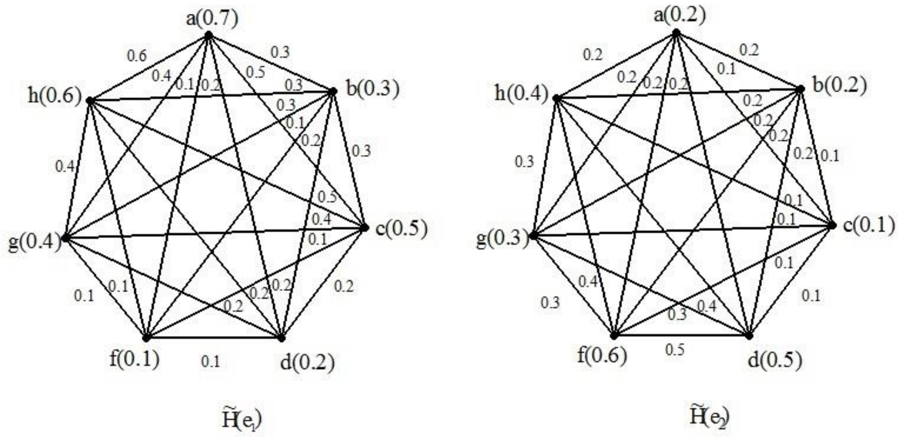


Figure 2. Complete Fuzzy soft graph $\widetilde{G}$

In $\widetilde{H}(e_1)$ *and* $\widetilde{H}(e_2)$, we get *abcdfgha*, *acfhbdga*, *adhcgbfa* Hamiltonian fuzzy cycles. Hence, $\widetilde{G}$ is Hamiltonian.

**Theorem 3.1:** If $\widetilde{G}$ is a complete fuzzy soft graph on $G^* = (V, E)$ with $|V| = 2n+1,$ for any $n \geq 1$ then $\widetilde{G}$ is Hamiltonian.

Proof: Let $\widetilde{G}$ be a complete fuzzy soft graph, which implies $\widetilde{H}(e)\ \forall e \in A$ is a complete fuzzy graph i.e. $\widetilde{K}(e)(v_i v_j) = \min\{\widetilde{F}(e)(v_i),\ \widetilde{F}(e)(v_j)\}\ \forall v_i, v_j \in V,\quad i \neq j.$

The vertex set V contains odd number of vertices and the membership degree of each edge is defined as the minimum of membership values of its vertices, for all vertices in V, in each fuzzy graph $\widetilde{H}(e)\ \forall e \in A$. This implies that each $\widetilde{H}(e)$ contains 2n+1 vertices, for $n \geq 1.$ By the assumption that each vertex has membership value > 0, we can say that there is an edge between every pair of vertices. Let us label the vertices of $\widetilde{H}(e)$ for some $e \in A$ as $v_\infty, v_0, v_1, v_2, v_3, \ldots\ldots.. v_{2n-1}$.

In each $\widetilde{H}(e)\ \ \forall e \in A$, Hamiltonian fuzzy cycles can be formed as follows.

$$1st\ cycle: \ v_\infty, v_0, v_{2n-1}, v_1, v_{2n-2}, v_2, v_{2n-3}, \ldots\ldots.. v_{n-1}, v_n, v_\infty$$
$$2nd\ cycle: \ v_\infty, v_1, v_0, v_2, v_{2n-1}, v_3, v_{2n-2}, \ldots\ldots.. v_n, v_{n+1}, v_\infty$$
$$3rd\ cycle: \ v_\infty, v_2, v_1, v_3, v_0, v_4, v_{2n-1}, \ldots\ldots.. v_{n+1}, v_{n+2}, v_\infty$$

.

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$$nth\ cycle: \ v_\infty, v_{n-1}, v_{n-2}, v_n, v_{n-3}, v_{n+1}, \ldots\ldots.. v_{2n-2}, v_{2n-1}, v_\infty$$

Therefore, every $\widetilde{H}(e)\ \forall e \in A$ is decomposable into n-Hamiltonian fuzzy cycles and hence $\widetilde{G}$ is Hamiltonian.

**Example 3.3:** Consider a complete fuzzy soft graph on $G^* = (V, E)$ with $|V| = 9.$ Let the membership values of all vertices and edges of $\tilde{H}(e)\ \ \forall e \in A$ be greater than zero. Denote the vertices of each fuzzy graph as $v_\infty, v_0, v_1, v_2, v_3, v_4, v_5, v_{6,}\ v_7.$
Here, 2n+1=9 which implies n=4.

Each $\tilde{H}(e)\ \ \forall e \in A$ can be decomposed into four Hamiltonian fuzzy cycles as follows.

$$v_\infty, v_0, v_7, v_1, v_6, v_2, v_5, v_{3,}\ v_4, v_\infty$$

$$v_\infty, v_1, v_0, v_2, v_7, v_3, v_6, v_{4,}\ v_5, v_\infty$$

$$v_\infty, v_2, v_1, v_3, v_0, v_4, v_7, v_{5,}\ v_6, v_\infty$$

$$v_\infty, v_3, v_2, v_4, v_1, v_5, v_0, v_{6,}\ v_7, v_\infty$$

Hence, $\tilde{G}$ is a Hamiltonian fuzzy soft graph.

**Corollary 3.1:** In any complete fuzzy soft graph $\tilde{G}$ on $G^* = (V, E)$ with $|V| = 2n,$ every $\tilde{H}(e)\ \forall e \in A$ is decomposable into Hamiltonian fuzzy paths $\tilde{P}_{2n-1}, n \geq 1.$

Proof: From Theorem 3.1, considering the composition of $\tilde{H}(e)$ which is a complete fuzzy graph with 2n+1 vertices. Remove the initial vertex $v_\infty$ then every $\tilde{H}(e)\ \forall e \in A$ will still be a complete fuzzy graph but with 2n vertices, while each Hamiltonian fuzzy path $\tilde{P}_{2n-1}$ of $\tilde{H}(e).$

**Theorem 3.2:** If $\tilde{G}$ is a complete fuzzy soft graph on $G^* = (V, E)$ with $|V| = 2n$ for any $n \geq 1$ then $\tilde{G}$ is decomposable into (2n-1) 1-factors of $\tilde{H}(e)\ \forall e \in A.$

Proof: Since $\tilde{G}$ is a complete fuzzy soft graph on $G^* = (V, E)$ with $|V| = 2n,$ each $\tilde{H}(e)\ \forall e \in A$ is a complete fuzzy graph with 2n vertices. Let us denote the vertices as $v_\infty, v_0, v_1, v_2, v_3, \ldots\ldots.. v_{2n-2}.$
1-factors can be formed as follows.

$$v_\infty, v_0; v_1, v_{2n-1}; v_2, v_{2n-2}; \ldots\ldots.. v_{n-1}, v_n$$
$$v_\infty, v_1; v_2, v_0; v_3, v_{2n-2}; \ldots\ldots.. v_n, v_{n+1}$$
.
.
.
$$v_\infty, v_{2n-2}; v_0, v_{2n-3}; v_1, v_{2n-4}; \ldots\ldots.. v_{n-2}, v_{n-1}$$

Hence, 1-factors for each $\tilde{H}(e)$ are obtained which is in total number 2n-1. Therefore, $\tilde{G}$ is decomposable into (2n-1) 1-factors of $\tilde{H}(e)\ \forall e \in A.$

**Theorem 3.3:** A complete fuzzy soft graph $\tilde{G}$ on $G^* = (V, E)$ with $|V| = 2n$ for any $n \geq 1$ can be decomposed into the integer value of (2n-1)/2 Hamiltonian fuzzy cycles with 1-factorization where the number of rest of the edges is n which cannot form Hamiltonian fuzzy soft cycles.

Proof: First we show that the complete fuzzy soft graph in which each fuzzy graph with 2n vertices can be decomposed into the integer value of (2n-1)/2 Hamiltonian fuzzy soft cycles.

From Theorem 3.1, if $\tilde{G}$ is a complete fuzzy soft graph on $G^* = (V, E)$ with 2n+1 vertices in V then $\tilde{G}$ is Hamiltonian. By the construction of $\tilde{H}(e)$, each vertex of $\tilde{H}(e)$ $\forall e \in A$ will have exactly 2n neighbors.

From the definition of Hamiltonian fuzzy soft cycle, each Hamiltonian fuzzy cycle covers all the vertices exactly once except the end vertices. Therefore, the complete fuzzy soft graph can be decomposed into (2n/2) = n Hamiltonian fuzzy soft cycles.

In a similar way, each vertex of $\tilde{H}(e)$ with 2n vertices has exactly (2n-1) neighbors. Hence, the complete fuzzy soft graph with $\tilde{H}(e)$ of 2n vertices $\forall e \in A$ can be decomposed into (2n-1)/2 Hamiltonian fuzzy soft cycles. But (2n-1)/2 yields a decimal value and Hamiltonian fuzzy soft cycles cannot be formed using decimal value. Therefore, only the integer value of (2n-1)/2 are considered without any loss of generality.

Hence, a complete fuzzy soft graph $\tilde{G}$ in which each $\tilde{H}(e)$ $\forall e \in A$ with 2n vertices is decomposable into integer values of (2n-1)/2 Hamiltonian fuzzy soft cycles with 1-factorization (using Theorem 3.2).

Next, it is proved that the number of rest of the edges is n.
From the result that if a fuzzy graph is complete with n vertices then it has n(n-1)/2 edges which is true for all odd and even number of n. Here, the number of vertices is considered to be even denoted as 2n.

The total number of edges in a complete fuzzy graph $\tilde{H}(e)$ $\forall e \in A$ of 2n vertices is given by 2n(2n-1)/2.

By the proof of the first part, the number of edges used in the Hamiltonian fuzzy cycles of 2n vertices is (n-1)2n.

Consider,
Total number of edges in $\tilde{H}(e)$ - Number of edges used in the Hamiltonian fuzzy cycles

$$= \frac{2n(2n-1)}{2} - (n-1)2n$$
$$= n$$
$$< 2n$$

which is true for each $\tilde{H}(e)$ $\forall e \in A$.
But, every Hamiltonian fuzzy cycle contains equal number of vertices and edges. Here, the number of remaining edges is n which is less than the number of vertices in Hamiltonian fuzzy cycle. Hence, it cannot form Hamiltonian fuzzy soft cycles.
Hence, proof of the theorem.

**Example 3.4:** Consider a complete fuzzy soft graph $\tilde{G}$ on $G^* = (V, E)$ with $|V| = 4$. Let $V = \{a,b,c,d\}$ and $E = \{ab, ac, ad, bc, bd, cd\}$. Let $A = \{e_1, e_2\}$.

By Theorem 3.2, $\tilde{G}$ can be decomposed into one Hamiltonian fuzzy soft cycle and the edges *ac* and *bd* in each $\tilde{H}(e)$ $\forall e \in A$ which is 2 in number cannot form Hamiltonian fuzzy soft cycles.

**Example 3.5:** Consider a complete fuzzy soft graph $\tilde{G}$ on $G^* = (V, E)$ with $|V| = 6$. Let $V = \{a,b,c,d,f,g\}$ and $E = \{ab, ac, ad, af, ag, bc, bd, bf, bg, cd, cf, cg, df, dg, fg\}$. Let $A = \{e_1, e_2\}$.

By Theorem 3.3, $\tilde{G}$ can be decomposed into (2n-1)/2 = two Hamiltonian fuzzy soft cycles with n=3 and the edges *ag, ad, cd* cannot form the cycle.

## 4. COMPLEMENT OF FUZZY SOFT CYCLES

In the following examples, it is illustrated that the complement of fuzzy soft cycles are fuzzy soft cycles for |V| = 3, 4, 5 by choosing the membership values of fuzzy soft arcs and fuzzy soft nodes suitably.

**Example 4.1:** Consider a simple graph $G^* = (V, E)$ where $V = \{a_1, a_2, a_3\}$ and $E = \{a_1a_2, a_2a_3, a_3a_1\}$. Let $A = \{e_1, e_2\}$ be a parameter set. Let $(\tilde{F}, A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\tilde{F} : A \to P(V)$ defined by

$$\tilde{F}(e_1) = \{ a_1/1, a_2/1, a_3/1\}$$

$$\tilde{F}(e_2) = \{ a_1/1, a_2/1, a_3/1\}$$

Let $(\tilde{K}, A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\tilde{K} : A \to P(E)$ defined by

$$\tilde{K}(e_1) = \{ a_1a_2 / 0.3, a_2a_3 / 0.3, a_3a_1 / 0.3 \}$$

$$\tilde{K}(e_2) = \{ a_1a_2 / 0.5, a_2a_3 / 0.5, a_3a_1 / 0.5 \}$$

Since $\tilde{H}(e_1) = (\tilde{F}(e_1), \tilde{K}(e_1))$ *and* $\tilde{H}(e_2) = (\tilde{F}(e_2), \tilde{K}(e_2))$ are fuzzy cycles for $e_1$ *and* $e_2 \in A$, $\tilde{G}$ is a fuzzy soft cycle and $\tilde{G}^c$ is also a fuzzy soft cycle as shown in Fig.3 and Fig 4.

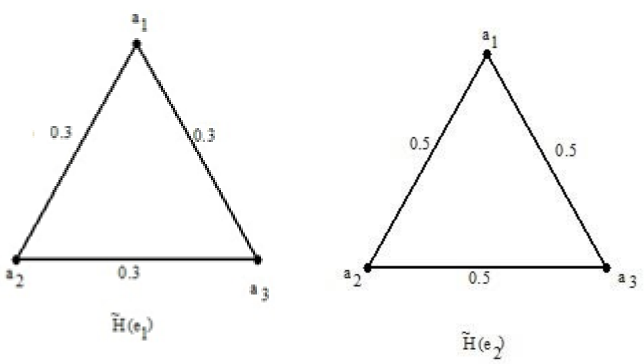

Figure 3. $\tilde{G}$

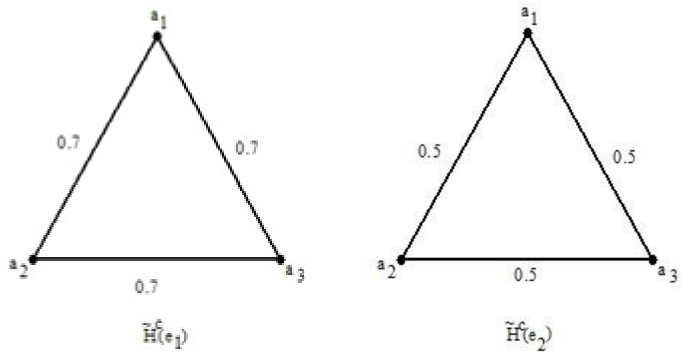


Figure 4. $\tilde{G}^c$

**Example 4.2:** Consider a simple graph $G^* = (V, E)$ where $V = \{a_1, a_2, a_3, a_4\}$ and $E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1\}$. Let $A = \{e_1, e_2\}$ be a parameter set. Let $(\tilde{F}, A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\tilde{F} : A \to P(V)$ defined by

$$\tilde{F}(e_1) = \{ a_1/1, a_2/1, a_3/1, a_4/1\}$$

$$\tilde{F}(e_2) = \{ a_1/1, a_2/1, a_3/1, a_4/1\}$$

Let $(\tilde{K}, A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\tilde{K} : A \to P(E)$ defined by

$$\tilde{K}(e_1) = \{ a_1a_2/1, a_2a_3/0.5, a_3a_4/1\ a_4a_1/0.5 \}$$

$$\tilde{K}(e_2) = \{ a_1a_2/0.5, a_2a_3/1, a_3a_4/0.5\ a_4a_1/1 \}$$

$\tilde{G}$ and $\tilde{G}^c$ are both fuzzy soft cycles as shown in Fig.5 and Fig 6.

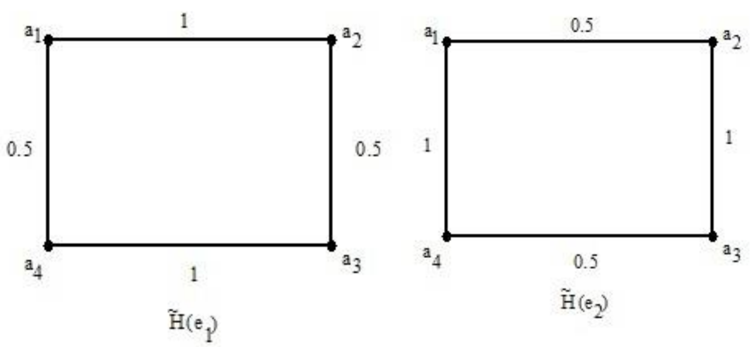


Figure 5. $\tilde{G}$

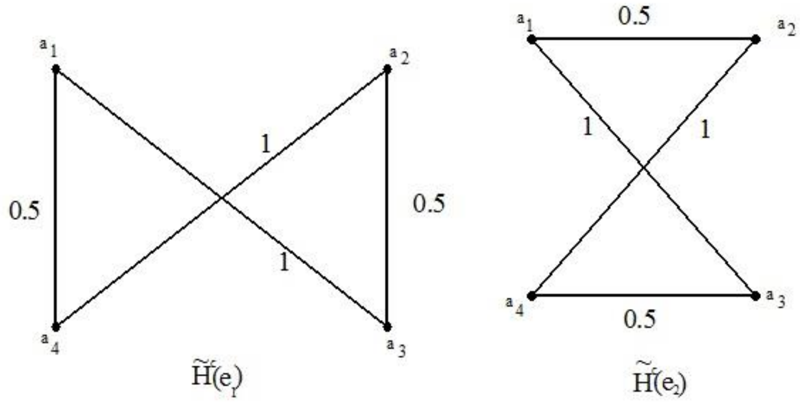


Figure 6. $\tilde{G}^c$

**Example 4.3:** Consider a simple graph $G^* = (V, E)$ where $V = \{a_1, a_2, a_3, a_4, a_5\}$ and $E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_5, a_5a_1\}$. Let $A = \{e_1, e_2\}$. Let $(\tilde{F}, A)$ be a fuzzy soft set over $V$ with its fuzzy approximate function $\tilde{F} : A \to P(V)$ defined by

$$\tilde{F}(e_1) = \{ a_1/1, a_2/1, a_3/1, a_4/1, a_5/1\}$$

$$\tilde{F}(e_2) = \{ a_1/1, a_2/1, a_3/1, a_4/1, a_5/1\}$$

Let $(\tilde{K}, A)$ be a fuzzy soft set over $E$ with its fuzzy approximate function $\tilde{K} : A \to P(E)$ defined by

$$\tilde{K}(e_1) = \{ a_1a_2/1, a_2a_3/1, a_3a_4/1\ a_4a_5/1\ a_5a_1/1\}$$

$$\tilde{K}(e_2) = \{ a_1a_2/1, a_2a_3/1, a_3a_4/1\ a_4a_5/1\ a_5a_1/1\}$$

$\tilde{G}$ and $\tilde{G}^c$ are both fuzzy soft cycles as shown in Fig.7 and Fig 8.

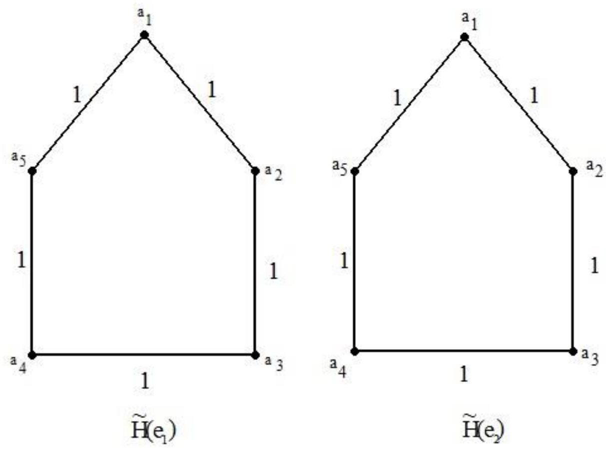


Figure7. $\tilde{G}$

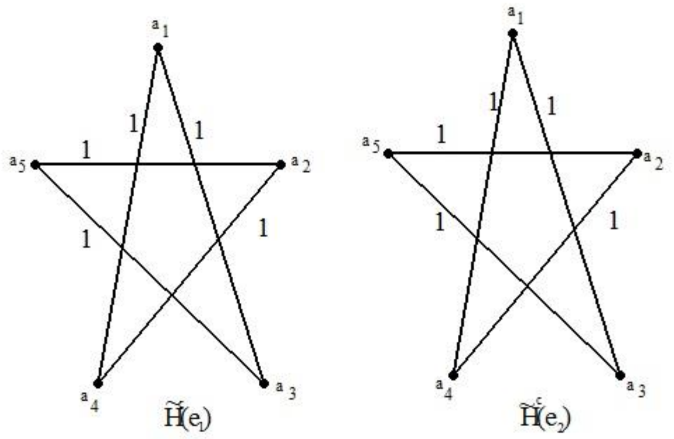


Figure8. $\tilde{G}^c$

**Theorem 4.1:** Let $\tilde{G}$ be a fuzzy soft cycle in which each fuzzy cycle has more than 5 vertices then $\tilde{G}^c$ cannot be a fuzzy soft cycle.

Proof: Let $\tilde{H}(e)$ be a fuzzy cycle with n nodes where $n \geq 6$ for some $e \in A$ then $\tilde{H}(e)$ will have exactly n arcs. $\tilde{H}^c(e)$ also will have n nodes. Let the nodes of $\tilde{H}(e)$ and its complement be $u_1, u_2, ..........u_n$ then $\tilde{H}^c(e)$ must contain atleast the following edges.

$(u_1, u_3), (u_1, u_4), ......(u_1, u_{n-1})$; $(u_2, u_4), (u_2, u_5), ......(u_2, u_n)$; $(u_3, u_5), (u_3, u_6), ......(u_3, u_n)$

Since $n \geq 6$, the total number of edges in $\tilde{H}^c(e)$ will be greater than n. Thus, it will not be a cycle. Therefore, $\tilde{H}^c(e)$ for some $e \in A$ is not a fuzzy cycle and hence $\tilde{G}^c$ is not a fuzzy soft cycle.

## 5. Conclusion

Fuzzy soft graph has emerged as a potential area of interdisciplinary research. In this paper, Hamiltonian fuzzy soft cycles are introduced and it is shown through examples that the decomposition of a complete fuzzy soft graph is decomposition into Hamiltonian fuzzy soft cycles. Also, it is proved that a fuzzy soft cycle with each fuzzy cycle having more than five vertices does not remain as a fuzzy soft cycle in its complement.

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