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THE CHAOS-BASED WHALE OPTIMIZATION ALGORITHMS GLOBAL OPTIMIZATION

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Abstract. Whale optimization algorithm (WOA) is one of the methods used effectively and efficiently to solve optimization problems in various fields. In the study, 5 different discrete chaotic maps were used in the aim of increasing the efficacy and efficiency of the WOA algorithm. CBWOA 1, CBWOA 2, CBWOA 3 and CBWOA 4 algorithms based on chaotic maps and whale optimization algorithm have been proposed for global optimization. In order to adjust the parameters contained in the WOA, random number sequences from Tinkerbell, Sine, Logistic, Tent and Henon chaotic maps were used. In order to test the accuracy of the developed algorithms and do the performance analysis, six different unimodal and multimodal benchmark functions were used. According to the simulation results, all the chaotic maps used in the study significantly improved the performance of the WOA algorithm.

Keywords: Chaos, Discrete time chaotic systems, Whale optimization algorithm

1. Introduction

Chaos is a global phenomenon that arises in specific nonlinear dynamic systems. In recent years, chaos has attracted widespread attention and has been widely applied in various disciplines such as synchronization, control and optimization [1]. Chaos is understood as the complex, bounded and unstable behavior caused by a simple deterministic non-linear system or chaotic map such that the generated sequences are irregular, quasi-random, semi- stochastic, very sensible to initial value [2]. Basically, meta-heuristic algorithms simulate biological or physical natural phenomena to solve complex optimization problems. These algorithms can be applied to various fields due to their simplicity, flexibility, robustness and efficiency [3]. Today, meta-heuristic algorithms such as Whale Optimization Algorithm (WOA) [14], Butterfly Optimization Algorithm (BOA) [13], Ant Lion Optimizer (ALO) [12], Gray Wolf Optimizer (GWO) [11], Cat Swarm Optimization 10, Artificial Bee Colony (ABC) [5], Firefly Algorithm (FA) [6], Biogeography-Based Optimization algorithm (BBO) [7], the Bat Algorithm [8], Krill-Herd (KH) [9] and Particle Swarm Optimization (PSO) [4] have been proposed and used by many researchers. Recently, chaos and meta-heuristic algorithms have been combined in many studies for different purposes. The combination of the methods can be performed in two different ways. They are using the chaos number sequences instead of random number generators in the structure of heuristic algorithms or including chaotic search in search behavior [15]. Meta-heuristic algorithms use randomness to discover new fields in the search space, basically to find better fit values. Algorithms generally use either uniform or Gaussian distributions while using randomness. Uniform distribution provides the same probability ratio while Gaussian distribution uses the partial probability ratio. Deciding on the type of randomness distribution that the algorithm should use depends on the problem to be solved [16]. Chaotic sequences are used instead of random number generators to develop meta-heuristic algorithms and avoid local minimum points. There are many studies in the literature on meta-heuristic algorithms and chaotic maps. In some of these studies, the effect of chaotic behaviors is emphasized in meta-heuristic algorithms and in the other ones, the usage of chaotic maps in algorithms to extend search is mentioned. Hosseinpourfard et al. developed three different chaotic map based particle swarm optimization algorithms for global optimization. Instead of random number generators found in PSO, random number sequences obtained from chaotic maps are used. They reported that the convergence rate of the method obtained from the combination of Lorenz chaotic map and PSO was better and reduced the number of iterations to find the global optimum value [17]. Abdullah et al. presented a new hybrid method based on genetic algorithm and chaotic function in order to encode the image. While the logistic chaotic map was used for the first encryption of the image, the same map was used in the genetic algorithm to improve the encryption process. They stated that the advantages of the proposed method were high efficiency and higher resistance to general attacks [18]. Afrabandpey et al. used chaotic sequences to improve the convergence rate of the bat algorithm. They formed the initial parameters of the algorithm using Gauss and Tent chaotic maps. Experimental results show that chaotic system integrated into algorithm provides avoidance of local solutions [19]. Huang et al. developed the initial positions of the nests in the structure of CS, the Levy flight parameters and the boundary transport mechanism using five different chaotic maps. They stated that the new algorithm that they developed according to the numerical results significantly improved the performance of the basic cuckoo search optimization algorithm [20]. Tian proposed a new algorithm by combining the chaotic maps and the gauss mutation parameter with the PSO algorithm. They used two different chaotic maps to generate initial values for the population of the PSO algorithm. In experimental results, they demonstrated the efficacy and efficiency of the proposed PSO [24]. In the study, five different discrete chaotic maps were used in the aim of increasing the efficacy and efficiency of the WOA algorithm. CBWOA 1, CBWOA 2, CBWOA 3 and CBWOA 4 algorithms based on chaotic maps and whale optimization algorithm have been developed for global optimization. In order to adjust the A,C and p parameters of WOA, random number sequences obtained from Tinkerbell, Sine, Logistic, Tent and Henon chaotic maps were used. In order to test the accuracy of the developed algorithms and do the performance analysis, six different unimodal and multimodal benchmark functions were used.

2. Whale Optimization Algorithm And Chaotic Maps

2.1. **Chaotic Maps.** Chaotic maps show complex and dynamic behaviors that occur in nonlinear systems and in determining system states [22]. In this study, logistic, henon, sine, tent and tinkerbell discrete chaotic maps are used.

2.1.1. *Logistic Map.* Logistics chaotic map is a dynamic system with discrete time. The logistic map is also one-dimensional and nonlinear. In equation 2.1, the logistic chaotic map formulation is given.

$$X_n + 1 = aX_n(1 - X_n) \tag{2.1}$$

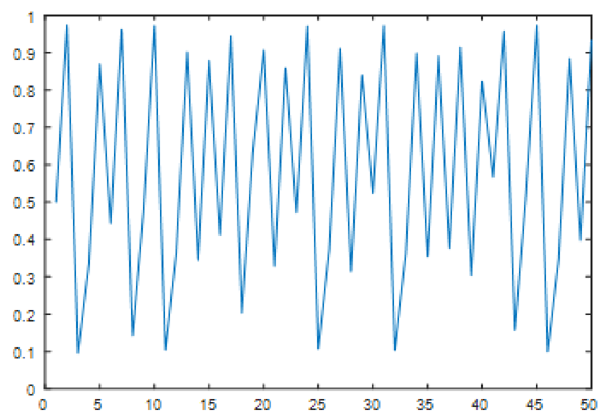

Figure 1. Logistic Map

In the equation, n is the number of iterations, xn n. is the chaotic number, a is the logistic map constant. Whether the system is chaotic is determined by the values that a can take. Generally, the a constant is in the range between [3.57-4]. In the study, the value of a constant was determined as 3.9 in order to have a chaotic map.

2.1.2. *Tent Map.* The tent map is topologically conjugate and therefore the behavior of the map is identical in iteration. The formulation of the chaotic map is given in Equation 2.2

$$\begin{aligned} x(i+1) &= f(x_i, \mu) \\ f(x_i, \mu) &= \begin{cases} f_L(x_i, \mu) = \mu x_i & x_i < 0.5 \\ f_R(1 - x_i, \mu) = \mu x_i & x_i \geq 0.5 \end{cases} \end{aligned} \tag{2.2}$$

x_0 is the initial value of the system. In this equation, x_i is in the range of [0, 1] and the control parameter in the map is in the range of μ [0, 2]. In this study, the μ constant was determined as 1.41.

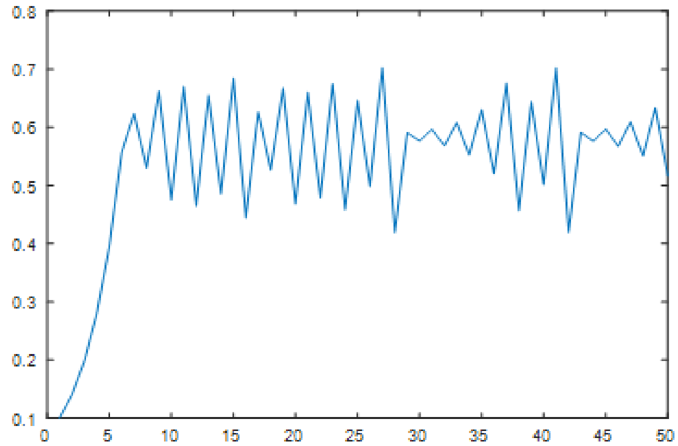

FIGURE 2. Tent Map

2.1.3. *Henon Map.* Henon maps are two dimensional maps developed by reducing the dynamics of the Lorenz system. The formulation of the Henon map is represented by Equation 2.3 and 2.4.The chaotic behavior of the Henon map depends on parameters a and b. In the system used in this study, a = 1.4 and b = 0.3. The chaotic behavior of the map is seen from the state space diagram in Figure 3.

$$x_(n+1) = 1 - ax_n^2 + by_n \tag{2.3}$$

$$y_(n+1) = x_n \tag{2.4}$$

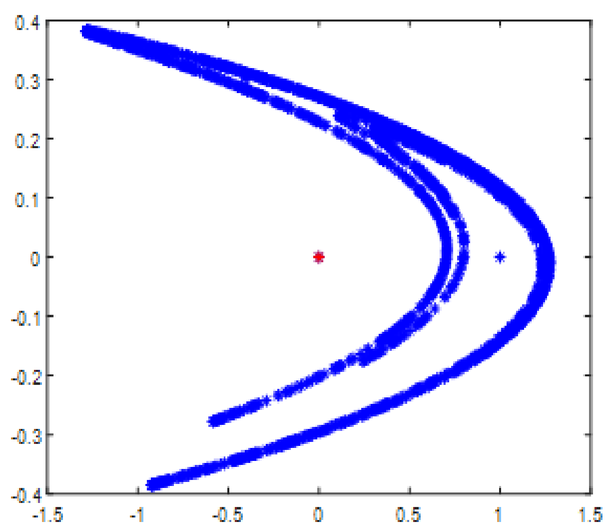

FIGURE 3. Henon Map

2.1.4. *Sine Map.* The basis of the sine map is based on the sine iteration function and is defined by Equation 2.5. The parameters r and x in the equation should be selected from the range 0≤r≤1, 0≤x_n≤1,to show a chaotic approach. In this study, r = 0.867 was chosen. The chaotic behavior of the map is shown in Figure 4.

$$x_(n+1) = r\sin(\pi x_n) \tag{2.5}$$

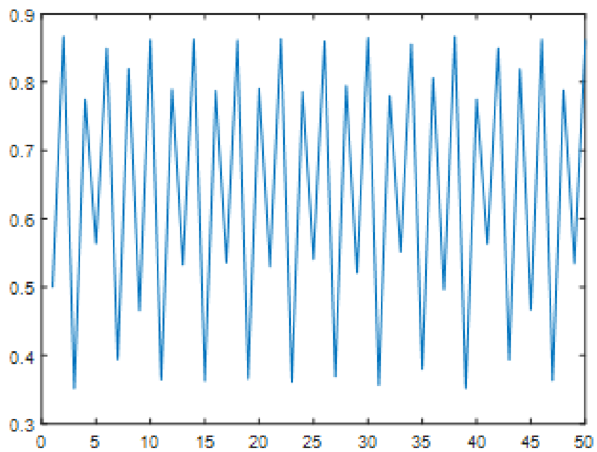

FIGURE 4. Sinus Map

2.1.5. *Tinkerbell Map.* The discrete-time tinkerbell chaotic map is defined by Equation 2.6 and 2.7. The non-zero parameters in the equation were determined as a = 0.9, b = -0.6013, c = 2.0, d = 0.50. In addition to these parameters, initial values of x and y parameters were taken as -0.72 and -0.64 respectively. The chaotic behavior of the map is shown in Figure 5.

$$x_(n+1) = x_n^2 - y_n^2 + ax_n + by_n \tag{2.6}$$

$$y_(n+1) = 2x_ny_n + cx_n + dy_n \tag{2.7}$$

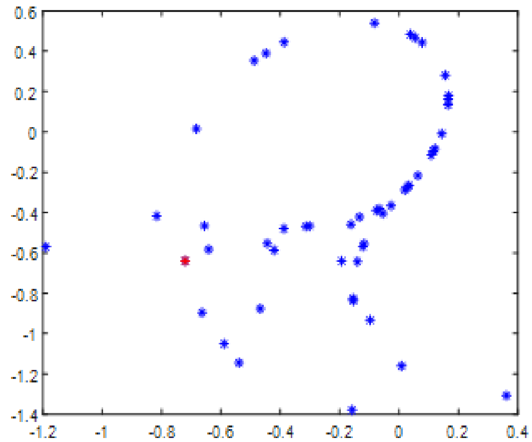

FIGURE 5. Tinkerbell Map

2.2. **Whale optimization algorithm.** The whale optimization algorithm (WOA) was proposed by Jalili and Lewis, inspired by the hunting behavior of humpback whales for use in optimization problems[14]. The foraging behavior observed only in humpback whales is the bubble-net feeding method. Whales create bubbles along a circular path while encircling the prey. The Pseudo code showing the processing steps of the spiral bubble-web feeding behavior is provided in Figure 6 for performing the optimization.

2.2.1. *Encircling prey.* Humpback whales can find and surround their prey in hunting. The WOA algorithm assumes that the best available solution is the target hunt or near the optimum situation, since the location of the optimal design in the search area is not known in advance. Once the best search agent is identified, other search agents will try to update their location towards the best search agent. The mathematical model of the behavior of humpback whales surrounding the prey is shown in Equations 2.8 and 2.9.

In equation 2.8 and 2.9. , $\vec{X}$ (t) represents the position of the agent, t represents the iteration and $\vec{X}*$ represents the best solution, while in equations 2.10 and 2.11, $\vec{A}$, $\vec{C}$ represents the convergence values. $\vec{r}$ [0,1] represents a random number and $\vec{a}$ represents the vector decreasing linearly from 2 to zero during iteration.

$$\vec{D} = |\vec{C}\vec{X}*(t) - \vec{X}(t)| \tag{2.8}$$

$$\vec{X}(t+1) = |\vec{X}^*(t) - \vec{A}.\vec{D}| \tag{2.9}$$

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \tag{2.10}$$

$$\vec{C} = 2\vec{r} \tag{2.11}$$

Initialize the whales population X_i=(i=1, 2, ... n)
Calculate the fitness of each search agent
X*=the best search agent
while (t<maximum number of iterations)
for each search agent
Update a, A, C, l and p
İf1 (p<0.5)
  if2 ($|A|$<1)
  Update the position of the current search agent by Eq. (2.8)
  else if2 ($|A|\geq1$)
  Select a random search agent (X_{rand})
  Update the position of the current search agent by Eq. (2.9)
  end if2
else if1 ($p \geq 0.5$)
Update the position of the current search agent by Eq. (2.14)
end if1
end for
Check if any search agent goes beyond the search space and amend it
Calculate the fitness of each search agent
Update X* if there is a better solution
t=t+1
end while
return X*

FIGURE 6. Pseudo-code of WOA

2.2.2. *Bubble-net attacking method.* In the bubble-net attack method of humpback whales, there are some features like shrinking the search environment while moving towards the prey and moving along a spiral path. By decreasing the$\vec{a}$ value in Equation 8, the whales exhibit prey-catching behavior by shrinking the search environments. Since $\vec{A}$ depends on$\vec{a}$, it decreases linearly from 2 to zero. The mathematical model of the spiral shape formed by humpback whales catching their prey is given in Equations 2.12 and 2.13.

$$\vec{D'} = |\vec{X}*(t) - \vec{X}(t)| \tag{2.12}$$

$$\vec{X}(t+1) = \vec{D}.e^{bl}.\cos(2\pi l) + \vec{X}^* \tag{2.13}$$

In Equation 2.12 and 2.13, D' is the distance between the whale and the best prey, b is the logarithmic spiral constant and l is the random number between [-1,1]. As the humpback whales move towards their prey, they choose either the shrinking movement model or the spiral movement model with a 50 percent probability. The parameter p in Equation 2.14 is a random number in the range of [0,1].

$$\vec{X}(t+1) = \begin{cases} \vec{X^*}(t) - \vec{A}.\vec{D} & p < 0.5 \\ \vec{D}.e^{bl}.\cos(2\pi l) + \vec{X^*} & p \geq 0.5 \end{cases}$$

2.3. **Developed Chaotic Based Whale Optimization Algorithms.** Equations 2.15 and 2.16 are obtained by replacing the random parameter r in the $\vec{A}$ and $\vec{C}$ convergence parameters found in the whale optimization algorithm with the sequence obtained from 5 different chaotic maps. The kp parameter in Equation 2.15 is a series of 5 different chaotic random numbers that take a value in the range of [0,1].

$$CBWOA1 : \vec{A} = 2\vec{a}.\vec{kp} - \vec{a} \tag{2.15}$$

$$CBWOA2 : \vec{C} = 2\vec{kp} \tag{2.16}$$

$$CBWOA3 : \vec{A} = 2\vec{a}.\vec{kp} - \vec{a}, \vec{C} = 2\vec{kp} \tag{2.17}$$

CBWOA3 has been obtained by using convergence values of developed CBWOA 1 and CBWOA 2 algorithms together. The new parameter is created by replacing the parameter p in equation 2.14 with chaotic map functions.

$$CBWOA4 : \vec{X}(t+1) = \begin{cases} \vec{X^*}(t) - \vec{A}.\vec{D} & kp < 0.5 \\ \vec{D}.e^{bl}.\cos(2\pi l) + \vec{X^*} & kp \geq 0.5 \end{cases}$$

3. Simulation Results

In order to perform function optimization, five different chaotic map based whale optimization algorithms were developed. Moreover, benchmark functions were selected to evaluate the performance of the developed chaotic based algorithms and show their effect. As shown in Table 1, a benchmark function with various characteristics and different levels of complexity was selected. Limit ranges of optimization were determined according to benchmark functions. Simulation studies were performed on a computer with Intel (R) Core (TM) i5-3470 CPU @ 3.20 Ghz, 64 Bit, 8GB RAM in Windows 10 operating system.
Table 1 shows the equations of functions, the number of variables in the function, the lower and upper limit values at which optimization will be performed, and the optimal result values.

Table 1. Unimodal and multimodal bencmark functions

Function	Variable	Range	fmin
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	[-100,100]	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	[-10,10]	0
$F_3(x) = \sum_{i=1}^{n} ([x_i + 0.5]^2)$	30	[-100,100]	0
$F_4(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1]$	30	[-1.28,1.28]	0
$F_5(x) = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\sqrt{\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)}) + 20 + e$	30	[-32,32]	0
$F_6(x) = \frac{\pi}{n} 10\sin(\pi y_1) + \sum_{i=1}^{n}(y_i - 1) [1 + 10\sin(\pi y_{i+1})] + (y_n - 1)^2 + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	30	[-50,50]	0

TABLE 2. Mean Error (ME) and Standard Deviation (STD) Results for Tinkerbell ans Sine used in Algorithms

Map	Algorithms		Functions					
			F1	**F2**	**F3**	**F4**	**F5**	**F6**
No Map	**WOA**	ME	2,26E-71	3,82E-49	3,64E-01	4,70E-03	5,51E-11	2,32E-02
		STD	7,15E-71	3,47E-45	0,241	0,0069	4,4E-11	0,0102
Tinkerbell	**KBWOA 1**	ME	5,06E-14	4,14E-09	4,11E-01	1,44E-02	3,45E-08	1,03E-01
		STD	7,97E-14	5,67E-09	0,253	0,0147	5,7E-08	0,1512
	KBWOA 2	ME	**1,1E-106**	**2,37E-72**	4,88E-01	8,58E-02	**3,38E-11**	3,05E-02
		STD	**3,09E-106**	**5,93E-72**	**0,157**	0,1439	**2,9E-11**	**0,0092**
	KBWOA 3	ME	2,08E-19	1,40E-14	**2,87E-01**	**4,03E-03**	**3,58E-11**	2,77E-02
		STD	4,52E-19	2,16E-14	**0,103**	**0,0045**	**2,2E-11**	0,0367
	KBWOA 4	ME	3,85E-58	3,26E-35	**2,06E-01**	1,27E-01	**4,44E-11**	**1,04E-02**
		STD	1,08E-57	1,02E-34	**0,070**	0,2643	**2,4E-11**	**0,0043**
Sine	**KBWOA 1**	ME	1,02E-69	**3,20E-50**	4,82E-01	**3,29E-03**	**5,15E-11**	2,70E-02
		STD	2,14E-69	**7,81E-50**	**0,156**	**0,0043**	**2,2E-11**	0,0121
	KBWOA 2	ME	7,58E-59	5,56E-41	3,90E-01	**4,01E-03**	**4,44E-11**	**1,72E-02**
		STD	2,4E-58	9,89E-41	0,257	**0,0067**	**2,4E-11**	**0,0089**
	KBWOA 3	ME	1,32E-56	3,08E-45	5,19E-01	**3,49E-03**	5,51E-11	3,31E-02
		STD	3,66E-56	6,12E-45	**0,191**	**0,0051**	4,4E-11	0,0192
	KBWOA 4	ME	3,33E-69	1,11E-44	4,96E-01	5,10E-03	**3,38E-11**	3,75E-02 E
		STD	1,02E-68	3,4E-44	**0,224**	**0,0044**	**2,4E-11**	0,0221

Each algorithm was run 10 times with the aim of comparing the performance of the algorithms. The results of Tinkerbell, Sine, Logistic, Tent and Henon maps applied from CBWOA 1 to CBWOA 4 are given in Table 2 and Table 3. Function values which have better conformity values than the optimum result of WOA are marked in bold.

TABLE 3. Mean Error(ME) and Standart Deviation(STD) Results for Logisti, Tent and Henon Maps Algorithms

Map	Algorithms		Functions					
			F1	**F2**	**F3**	**F4**	**F5**	**F6**
Logistic	**KBWOA 1**	ME	5,58E-59	3,85E-41	**3,14E-01**	**3,88E-03**	**4,80E-11**	7,05E-02
		STD	1,76E-58	1,21E-40	**0,116**	**0,00260**	**3,1E-11**	0,0569
	KBWOA 2	ME	**9,33E-77**	2,81E-48	**3,00E-01**	2,91E-02	**5,15E-11**	2,42E-02
		STD	**2,1E-76**	**6,93E-48**	**0,161**	0,0617	**4E-11**	0,0149
	KBWOA 3	ME	3,19E-67	2,64E-42	**3,03E-01**	**2,67E-03**	**2,31E-11**	4,99E-02
		STD	9,94E-67	8,34E-42	**0,168**	**0,0036**	**1,8E-11**	0,0435
	KBWOA 4	ME	**1,16E-73**	5,31E-48	4,05E-01	6,28E-02	**5,15E-11**	**1,85E-02**
		STD	**3,22E-73**	**8,67E-48**	**0,146**	0,1882	**2,8E-11**	0,0104
Tent	**KBWOA 1**	ME	9,76E-64	**1,10E-53**	1,17E+04	**3,98E-03**	7,28E-11	1,35E-01
		STD	3,09E-63	**2,23E-53**	10083,21	**0,0031**	**3,3E-11**	0,1358
	KBWOA 2	ME	5,62E-36	4,70E-29	5,49E-01	1,04E-02	9,47E-11	5,44E-02
		STD	1,44E-35	1,19E-28	0,25	0,0091	7,8E-11	0,03910
	KBWOA 3	ME	2,38E-43	9,83E-44	2,00E+04	1,22E-02	1,01E-10	7,44E-02
		STD	7,52E-43	2,13E-43	8021,04	0,0134	5,3E-11	0,0211
	KBWOA 4	ME	8,52E-68	1,09E-45	6,57E-01	9,73E-03	**4,09E-11**	3,51E-02
		STD	2,65E-67	**3,13E-45**	0,242	0,0093	2,6E-11	0,0217
Henon	**KBWOA 1**	ME	6,87E-33	3,55E-21	**2,70E-02**	1,08E-02	9,26E-11	**5,30E-03**
		STD	2,08E-32	5,44E-21	**0,017**	0,0087	6,2E-11	**0,0062**
	KBWOA 2	ME	**5,22E-89**	**1,05E-61**	**3,16E-01**	8,17E-02	**4,09E-11**	2,34E-02
		STD	**1,48E-88**	**2,51E-61**	**0,179**	0,2553	**2E-11**	0,0263
	KBWOA 3	ME	8,18E-43	1,58E-26	**5,82E-02**	**4,58E-03**	**4,44E-11**	**1,92E-02**
		STD	1,81E-42	4,88E-26	**0,044**	0,0070	**2,9E-11**	0,0141
	KBWOA 4	ME	2,42E-57	1,76E-36	4,96E-01	**1,42E-03**	**4,80E-11**	**1,73E-02**
		STD	7,48E-57	4,1E-36	**0,173**	**0,0013**	**2,6E-11**	**0,0063**

Comparing test functions optimized by WOA and developed chaotic map based whale optimization algorithms;
-In F1 function, Tinkerbell, Henon and Logistic map based CBWOA 2 algorithm together with Logistic map based CBWOA 4 algorithm gave better results than WOA algorithm.
-In F2 function, Tinkerbell and Henon map based CBWOA 2 algorithm together with Sine and Tent map based CBWOA 1 algorithm gave better results than WOA algorithm.
-In F3 function, Tinkerbell map based CBWOA 3 and CBWOA 4 algorithm together with Henon and Logistic map based CBWOA 4 algorithm gave better results than WOA algorithm.,
-In F4 function, Tinkerbell map based CBWOA 3, algorithms except Sine map based CBWOA4, Logistic map based CBWOA 1 and CBWOA 3 algorithms, Tent Map map based CBWOA 1 and Henon map based CBWOA 3 and CBWOA 4 algorithms gave better results than WOA algorithm.
-In F5 function, all developed the algorithms except Tinkerbell and Henon map based CBWOA1, all the developed algorithms except Sine map based CBWOA 3, all developed algorithms based on Logistic and Tent Map map based CBWOA 4 algorithm gave better results than WOA algorithm.
-In F6 function, all algorithms except Tinkerbell map based CBWOA4, Sine map based CBWOA 2, Logistic based CBWOA 4 and Henon map based CBWOA 2 gave better results than WOA algorithm.

According to the comparison results, F3 and F5 functions performed better than others in Tinkerbell, Sine, Logistic, Tent and Henon chaotic map based algorithms.

4. Conclusion

In the study, the performance of the algorithms created by combining the whale optimization algorithm (WOA) developed with the hunting feature of humpback whales with chaotic maps was investigated. In the study, chaotic values obtained from Tinkerbell, Sine, Logistic, Tent and Henon chaotic maps were replaced with random values found in WOA algorithm and four different chaotic based WOA approaches were developed. These approaches are CBWOA1, CBWOA2, CBWOA3 and CBWOA4 algorithms. Performance analysis and validation were performed on six different unimodal and multimodal benchmark functions in the literature by using the developed chaotic algorithms and original WOA algorithm. According to the simulation results, it was found that all the chaotic maps used in the study significantly improved the performance of the WOA algorithm in the optimization of functions. The algorithm developed with the Henon map was found to have a better performance than other chaotic based maps. Consequently, the most important reason for the higher performance of the developed chaotic based algorithms derived from the chaos in the chaotic maps in the search area. Therefore, it was observed that the performance of the algorithms developed using chaotic maps increased in general.

References

[1] N. Aslimani and R. Ellaia, "A new chaos optimization algorithm based on symmetrization and levelling approaches for global optimization" , Numer Algo.,Vol.79, pp. 10211047, 2018.
[2] J. D. Velsquez, "An introduction to chaos-based algorithms for numerical optimization", Avances en Sistemas e Informtica, Vol. 8, No. 3, 2011.
[3] G. Kaur and S. Aror, "Chaotic whale optimization algorithm", Journal of Computational Design and Engineering Vol.5, pp. 275284, 2018.
[4] R. Eberhart and J. Kennedy, " Particle swarm optimization", In Proceedings of IEEE international conference on neural networks,pp. 19421948, 1995.
[5] M. Dorigo and G. Di Caro, "Ant colony optimization: A new meta-heuristic", In CEC 99. Proceedings of the 1999 congress on evolutionary computation, Vol. 2, pp. 14701477, 1999.
[6] X. S. Yang, "Firefly algorithm, Levy flights and global optimization", In Research and development in intelligent systems XXVI, pp. 209218, 2010.
[7] D. Simon, "Biogeography-based optimization", IEEE Transactions on Evolutionary Computation, Vol. 12, No. 6, pp. 702713, 2008.
[8] A. H. Gandomi and X. S. Yang, "Chaotic bat algorithm", Journal of Computational Science, Vol. 5, No.2, pp. 224232 ,2014 .
[9] A. H. Gandomi and A. H. Alavi,"Krill herd: A new bio-inspired optimization algorithm", Communications in Nonlinear Science and Numerical Simulation, Vol. 17,No. 12, pp. 48314845, 2012.
[10] C. Shu-Chuan, T. Pei-Wei and P. Jeng-Shyang, "Cat swarm optimization", In 9th Pacific Rim international conference on artificial intelligence, LNAI Vol. 4099, pp. 854858, 2006.
[11] S. Mirjalili, S. M. Mirjalili and A. Lewis, "Grey wolf optimizer", Advances in Engineering Software, Vol. 69, pp. 4661, 2014.
[12] S. Mirjalili, "The ant lion optimizer", Advances in Engineering Software,Vol. 83,pp. 8098, 2015.
[13] S. Arora and S. Singh, " Butterfly algorithm with Lvy Flights for global optimization. In 2015 International conference on signal processing, computing and control, pp. 220224, 2015.
[14] S. Mirjalili and A. Lewis, "The whale optimization algorithm", Advances in Engineering Software, Vol. 95, pp. 5167, 2016.
[15] R. Sheikholeslami and A. Kaveh , "A Survey Of Chaos Embedded Meta-Heurstc algorthms", Int. J. Optim. Civil Eng., Vol.3 No.4, pp. 617-633, 2013;
[16] I. Fister, M. Perc and Sa.M. Kamal, "A review of chaos-based firefly algorithms: Perspectives and research challenges", Applied Mathematics and Computation, Vol. 252, pp.155165, 2015.
[17] R. Hosseinpourfard and M.M. Javidi, "Chaotic PSO using the Lorenz System: An Efficient Approach for Optimizing Nonlinear Problems", ankaya University Journal of Scien ce and Engineering, Vol. 12, No. 1, pp. 040-059, 2015.
[18] A. H. Abdullah , R. Enayatifar and M. Lee, "A hybrid genetic algorithm and chaotic function model for image encryption", Int. J. Electron. Commun.,Vol. 66, pp. 806816, 2012.
[19] H. Afrabandpey, M. Ghaffari, A. Mirzaei and M. Safayani ,"A Novel Bat Algorithm Based on Chaos for Optimization Tasks", Iranian Conference on Intelligent Systems,2014.
[20] L. Huang, S. Ding, S. Yu, J.Wang and K. Lu ," Chaos-enhanced Cuckoo search optimization algorithms for global optimization", Applied Mathematical Modelling Vol. 40, pp. 38603875, 2016.
[21] Q. V. Lawande , B. R. Ivan and S. D. Dhodapkar, "Chaos based ryptography: a new approach to secure communications", Barc new letter, Vol. 258,2005.
[22] L. M. Pecora and T. L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. Vol. 64,No. 821, 1990.
[23] F. S. Gharehchopogh and H. Gholizadeh," A comprehensive survey: Whale Optimization Algorithm and its applications", Swarm and Evolutionary Computation,Vol. 48,pp. 124, 2019.

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