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EFFICIENT ESTIMATORS FOR THE POPULATION MEAN

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Abstract

M. Khoshnevisan, R. Singh, P. Chauhan, N. Sawan and F. Smarandache (*A general family of estimators for estimating population mean using known value of some population parameter(s)*, Far East Journal of Theoretical Statistics **22**, 181–191, 2007) introduced a family of estimators using auxiliary information in simple random sampling. They showed that these estimators are more efficient than the classical ratio estimator and that the minimum value of the mean square error (*MSE*) of this family is equal to the *MSE* of the regression estimator. In this paper we propose another family of estimators using the results of B. Prasad (*Some improved ratio type estimators of population mean and ratio in finite population sample surveys*, Communications in Statistics: Theory and Methods **18**, 379–392, 1989). Expressions for the bias and *MSE* of the proposed family are derived. Besides, considering the minimum cases of these *MSE* equations, a comparison of the efficiency conditions between the Khoshnevisan and proposed families are obtained. The proposed family of estimators is found to be more efficient than Khoshnevisan's family of estimators under certain conditions. Finally, these theoretical findings are illustrated by a numerical example with original data.

Keywords: Ratio estimator, Product estimator, Regression estimator, Auxiliary information, Mean square error, Efficiency.

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1. Introduction and notation

When the study variable y is highly correlated with the auxiliary variable, the use of auxiliary information in the ratio and product estimators can increase the precision of the estimates. To obtain the most efficient estimator, many authors proposed ratio and product estimators using the standard deviation, coefficient of variation, skewness, kurtosis, correlation coefficient, etc. of the auxiliary variable. In this study, we suggest a new family of estimators to estimate the population mean of the study variable Y by using the estimators of Prasad [4] and Khoshnevisan *et al.* [2], and the optimum cases of the suggested family of estimators are also obtained.

Consider a finite population of size N from which a sample s of size n is drawn according to simple random sampling without replacement. Let Y_i and X_i denote the values of the study and auxiliary variables for the i-th unit, $(i = 1, 2, \ldots, N)$, of the population. Further, let $\bar{y}$ and $\bar{x}$ be the sample means of the study and auxiliary variables, respectively.

To obtain the bias and *MSE*, let us define $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$. Using these notations,

$$E(e_0) = E(e_1) = 0,\ E(e_0^2) = \lambda C_y^2,$$
$$E(e_1^2) = \lambda C_x^2,\ E(e_0 e_1) = \lambda C_{yx} = \lambda \rho C_y C_x,$$

where

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2},\ C_x^2 = \frac{S_x^2}{\bar{X}^2},\ C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}},$$
$$S_y^2 = \frac{\sum_{i=1}^N \left(y_i - \bar{Y}\right)^2}{N-1},\ S_x^2 = \frac{\sum_{i=1}^N \left(x_i - \bar{X}\right)^2}{N-1},\ S_{yx} = \frac{\sum_{i=1}^N \left(y_i - \bar{Y}\right)\left(x_i - \bar{Y}\right)}{N-1}$$

and $\lambda = \frac{N-n}{Nn}$.

2. The suggested family of estimators

Khoshnevisan *et al.* [2] defined a family of estimators for the population mean in simple random sampling as

$$t = \bar{y}\left[\frac{a\bar{X}+b}{\alpha\left(a\bar{x}+b\right)+\left(1-\alpha\right)\left(a\bar{X}+b\right)}\right]^g \tag{2.1}$$

where $a\,(\neq 0)$, b are either real numbers or functions of the known parameters of the auxiliary variable x such as the standard deviation (S_x), coefficient of variation (C_x), skewness $(\beta_1(x))$, kurtosis $(\beta_2(x))$ and the correlation coefficient (ρ) of the population. Here, g and α are suitably chosen scalars such that the mean square error of t is minimum.

The bias and *MSE* of this family of estimators are respectively given by

$$B(t) = \lambda\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\upsilon^2 C_x^2 - \alpha\upsilon g C_{yx}\right], \tag{2.2}$$

and

$$MSE(t) = \lambda\bar{Y}^2\left[C_y^2 + \alpha^2\upsilon^2 g^2 C_x^2 - 2\alpha\upsilon g C_{yx}\right], \tag{2.3}$$

where $\upsilon = \frac{a\bar{X}}{a\bar{X}+b}$.

The minimum value of *MSE* for $\alpha^* = \frac{K}{\upsilon g}$ $\left(\text{where } K = \rho\frac{C_y}{C_x}\right)$ is also given by

$$MSE_{\min}(t) = \bar{Y}^2\lambda C_y^2\left(1-\rho^2\right) \tag{2.4}$$

which is equal to the *MSE* of the regression estimator.

The ratio estimators, which are given in Table 1, are in the same family (2.1), and we can express the Mean Square Error in (2.3) for these estimators as

$$(2.5)\qquad MSE(t_i)=\begin{cases}\bar{Y}^2\lambda(C_y^2+C_x^2-2C_{yx}), & i=1\\ \bar{Y}^2\lambda\Big[C_y^2-2\upsilon_{\frac{(i-1)}{2}}C_{yx}+\upsilon^2_{\frac{(i-1)}{2}}C_x^2\Big], & i=3,5,7,\dots,17.\end{cases}$$

Table 1. Some members of the family of estimators of t

Ratio estimators ($g=1$)	Product estimators ($g=-1$)	α	a	b
$t_1=\bar{y}\left[\frac{\overline{X}}{\bar{x}}\right]$	$t_2=\bar{y}\left[\frac{\bar{x}}{\overline{X}}\right]$	1	1	0
$t_3=\bar{y}\left[\frac{\overline{X}+C_x}{\bar{x}+C_x}\right]$ Sisodia and Dwivedi [8]	$t_4=\bar{y}\left[\frac{\bar{x}+C_x}{\overline{X}+C_x}\right]$ Pandey and Dubey [3]	1	1	C_x
$t_5=\bar{y}\left[\frac{\beta_{2(x)}\overline{X}+C_x}{\beta_{2(x)}\bar{x}+C_x}\right]$ Upadhyaya and Singh [9]	$t_6=\bar{y}\left[\frac{\beta_{2(x)}\bar{x}+C_x}{\beta_{2(x)}\overline{X}+C_x}\right]$ Upadhyaya and Singh [9]	1	$\beta_{2(x)}$	C_x
$t_7=\bar{y}\left[\frac{C_x\overline{X}+\beta_{2(x)}}{C_x\bar{x}+\beta_{2(x)}}\right]$ Upadhyaya and Singh [9]	$t_8=\bar{y}\left[\frac{C_x\bar{x}+\beta_{2(x)}}{C_x\overline{X}+\beta_{2(x)}}\right]$ Upadhyaya and Singh [9]	1	C_x	$\beta_{2(x)}$
$t_9=\bar{y}\left[\frac{\overline{X}+S_x}{\bar{x}+S_x}\right]$	$t_{10}=\bar{y}\left[\frac{\bar{x}+S_x}{\overline{X}+S_x}\right]$ Singh [5]	1	1	S_x
$t_{11}=\bar{y}\left[\frac{\beta_{1(x)}\overline{X}+S_x}{\beta_{1(x)}\bar{x}+S_x}\right]$	$t_{12}=\bar{y}\left[\frac{\beta_{1(x)}\bar{x}+S_x}{\beta_{1(x)}\overline{X}+S_x}\right]$ Singh [5]	1	$\beta_{1(x)}$	S_x
$t_{13}=\bar{y}\left[\frac{\beta_{2(x)}\overline{X}+S_x}{\beta_{2(x)}\bar{x}+S_x}\right]$	$t_{14}=\bar{y}\left[\frac{\beta_{2(x)}\bar{x}+S_x}{\beta_{2(x)}\overline{X}+S_x}\right]$ Singh [5]	1	$\beta_{2(x)}$	S_x
$t_{15}=\bar{y}\left[\frac{\overline{X}+\rho}{\bar{x}+\rho}\right]$ Singh and Tailor [6]	$t_{16}=\bar{y}\left[\frac{\bar{x}+\rho}{\overline{X}+\rho}\right]$ Singh and Tailor [6]	1	1	ρ
$t_{17}=\bar{y}\left[\frac{\overline{X}+\beta_{2(x)}}{\bar{x}+\beta_{2(x)}}\right]$ Singh *et al.* [7]	$t_{18}=\bar{y}\left[\frac{\bar{x}+\beta_{2(x)}}{\overline{X}+\beta_{2(x)}}\right]$ Singh *et al.* [7]	1	1	$\beta_{2(x)}$

For the product estimators in Table 1, the *MSE* equation is

$$(2.6)\qquad MSE(t_j)=\begin{cases}\bar{Y}^2\lambda\left(C_y^2+C_x^2+2C_{yx}\right), & j=2\\ \bar{Y}^2\lambda\left[C_y^2+2\upsilon_{(\frac{j}{2})-1}C_{yx}+\upsilon^2_{(\frac{j}{2})-1}C_x^2\right], & j=4,6,8,\dots,18,\end{cases}$$

where

$$\upsilon_1 = \frac{\bar{X}}{\bar{X}+C_x},\ \upsilon_2 = \frac{\beta_{2(x)}\bar{X}}{\beta_{2(x)}\bar{X}+C_x},\ \upsilon_3 = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_{2(x)}},\ \upsilon_4 = \frac{\bar{X}}{\bar{X}+S_x},$$

$$\upsilon_5 = \frac{\beta_{1(x)}\bar{X}}{\beta_{1(x)}\bar{X}+S_x},\ \upsilon_6 = \frac{\beta_{2(x)}\bar{X}}{\beta_{2(x)}\bar{X}+S_x},\ \upsilon_7 = \frac{\bar{X}}{\bar{X}+\rho},\ \upsilon_8 = \frac{\bar{X}}{\bar{X}+\beta_{2(x)}}.$$

Motivated by Prasad [4] and Gandge *et al.* [1] we propose a new family of estimators as given below

$$\eta = \kappa\bar{y}\left[\frac{a\bar{X}+b}{\alpha\left(a\bar{x}+b\right)+\left(1-\alpha\right)\left(a\bar{X}+b\right)}\right]^g \tag{2.7}$$

where κ is a suitable constant to be determined later.

Expressing the estimator, η in terms of e_i, $(i=0,1)$, we can write (2.7) as

$$\eta = \kappa\bar{Y}\left(1+e_0\right)\left[1+\alpha\upsilon e_1\right]^{-g}. \tag{2.8}$$

Expanding the right hand side of (2.8) to a first order approximation and subtracting $\bar{Y}$ from both sides we get

$$\eta-\bar{Y} = \kappa\bar{Y}\left[1-g\alpha\upsilon e_1+\frac{g\left(g+1\right)}{2}\alpha^2\upsilon^2e_1^2+e_0-g\alpha\upsilon e_0e_1\right]-\bar{Y}. \tag{2.9}$$

Taking the expectation on both sides of equation (2.9), we get the bias of the estimator η as

$$B\left(\eta\right) = \kappa\bar{Y}\lambda\left[\frac{g\left(g+1\right)}{2}\alpha^2\upsilon^2C_x^2-g\alpha\upsilon C_{yx}\right]+\bar{Y}\left(\kappa-1\right) \tag{2.10}$$

Squaring both sides of equation (2.9) gives

$$\begin{aligned}\left(\eta-\bar{Y}\right)^2 = \kappa^2\bar{Y}^2&\left[1-g\alpha\upsilon e_1+\frac{g\left(g+1\right)}{2}\alpha^2\upsilon^2e_1^2+e_0-g\,\alpha\upsilon e_0e_1\right]^2\\ &+\bar{Y}^2-2\kappa\bar{Y}^2\left[1-g\alpha\upsilon e_1+\frac{g\left(g+1\right)}{2}\alpha^2\upsilon^2e_1^2+e_0-g\alpha\upsilon e_0e_1\right]\end{aligned} \tag{2.11}$$

and then taking the expectation, we get the *MSE* of the estimator η, to a first order approximation, as

$$\begin{aligned}MSE\left(\eta\right) = \bar{Y}^2\{&\kappa^2\lambda C_y^2+\left(\kappa^2\left(2g^2+g\right)-\kappa\left(g^2+g\right)\right)\alpha^2\upsilon^2\lambda C_x^2\\ &-2g\alpha\upsilon\left(2\kappa^2-\kappa\right)\lambda C_{yx}+\left(\kappa-1\right)^2\}.\end{aligned} \tag{2.12}$$

The minimum of *MSE*(η) is obtained for the optimal value of κ, which is

$$\kappa^* = \frac{A}{2B} \tag{2.13}$$

where

$$\begin{aligned}A &= \left(g^2+g\right)\alpha^2\upsilon^2\lambda C_x^2-2g\alpha\upsilon\lambda C_{yx}+2,\\ B &= \lambda C_y^2+\left(2g^2+g\right)\alpha^2\upsilon^2\lambda C_x^2-4g\alpha\upsilon\lambda C_{yx}+1\end{aligned}$$

Thus, the minimum *MSE* of the estimator η is obtained as

$$MSE_{\min}\left(\eta\right) = \bar{Y}^2\left\{1-\frac{A^2}{4B}\right\}. \tag{2.14}$$

For the ratio estimators as given in Table 2, we can express the Mean Square Error given in equation (2.12) by the following equation

$$(2.15)\quad MSE(\eta_i) = \begin{cases} \bar{Y}^2\{\kappa^{\bullet 2}\lambda C_y^2 + \left(3\kappa^{\bullet 2} - 2\kappa^{\bullet}\right)\lambda C_x^2 \\ \qquad - 2(2\kappa^{\bullet 2} - \kappa^{\bullet})\lambda C_{yx} + (\kappa^{\bullet} - 1)^2\}, & i = 1 \\ \bar{Y}^2\{\kappa^{+2}\lambda C_y^2 + (3\kappa^{+2} - 2\kappa^{+})\upsilon^2_{\frac{(i-1)}{2}}\lambda C_x^2 \\ - 2\upsilon_{\frac{(i-1)}{2}}(2\kappa^{+2} - \kappa^{+})\lambda C_{yx} + (\kappa^{+} - 1)^2\}, & i = 3, 5, \ldots, 17 \end{cases}$$

and for product estimators, the Mean Square Error is given by the following equation

$$(2.16)\quad MSE(\eta_j) = \begin{cases} \bar{Y}^2\{\kappa^{\circ 2}\lambda C_y^2 + \kappa^{\circ 2}\lambda C_x^2 + 2\left(2\kappa^{\circ 2} - \kappa^{\circ}\right)\lambda C_{yx}^2 \\ \qquad + (\kappa^{\circ} - 1)^2\}, & j = 2 \\ \bar{Y}^2\{\kappa^{\tau 2}\lambda C_y^2 + \kappa^{\tau 2}\upsilon^2_{\left(\frac{j}{2}\right)-1}\lambda C_x^2 \\ + 2\upsilon_{\left(\frac{j}{2}\right)-1}\left(2\kappa^{\tau 2} - \kappa^{\tau}\right)\lambda C_{yx}^2 + (\kappa^{\tau} - 1)^2\}. & j = 4, 6, \ldots, 18 \end{cases}$$

Table 2. Some members of the family of estimators of η

Ratio estimators ($g = 1$)	Product estimators ($g = -1$)	α	a	b
$\eta_1 = \kappa\bar{y}\left[\frac{\overline{X}}{\bar{x}}\right]$ Prasad [4]	$\eta_2 = \kappa\bar{y}\left[\frac{\bar{x}}{\overline{X}}\right]$ Gandge *et al.* [1]	1	1	0
$\eta_3 = \kappa\bar{y}\left[\frac{\overline{X} + C_x}{\bar{x} + C_x}\right]$	$\eta_4 = \kappa\bar{y}\left[\frac{\bar{x} + C_x}{\overline{X} + C_x}\right]$	1	1	C_x
$\eta_5 = \kappa\bar{y}\left[\frac{\beta_{2(x)}\overline{X} + C_x}{\beta_{2(x)}\bar{x} + C_x}\right]$	$\eta_6 = \kappa\bar{y}\left[\frac{\beta_{2(x)}\bar{x} + C_x}{\beta_{2(x)}\overline{X} + C_x}\right]$	1	$\beta_{2(x)}$	C_x
$\eta_7 = \kappa\bar{y}\left[\frac{C_x\overline{X} + \beta_{2(x)}}{C_x\bar{x} + \beta_{2(x)}}\right]$	$\eta_8 = \kappa\bar{y}\left[\frac{C_x\bar{x} + \beta_{2(x)}}{C_x\overline{X} + \beta_{2(x)}}\right]$	1	C_x	$\beta_{2(x)}$
$\eta_9 = \kappa\bar{y}\left[\frac{\overline{X} + S_x}{\bar{x} + S_x}\right]$	$\eta_{10} = \kappa\bar{y}\left[\frac{\bar{x} + S_x}{\overline{X} + S_x}\right]$	1	1	S_x
$\eta_{11} = \kappa\bar{y}\left[\frac{\beta_{1(x)}\overline{X} + S_x}{\beta_{1(x)}\bar{x} + S_x}\right]$	$\eta_{12} = \kappa\bar{y}\left[\frac{\beta_{1(x)}\bar{x} + S_x}{\beta_{1(x)}\overline{X} + S_x}\right]$	1	$\beta_{1(x)}$	S_x
$\eta_{13} = \kappa\bar{y}\left[\frac{\beta_{2(x)}\overline{X} + S_x}{\beta_{2(x)}\bar{x} + S_x}\right]$	$\eta_{14} = \kappa\bar{y}\left[\frac{\beta_{2(x)}\bar{x} + S_x}{\beta_{2(x)}\overline{X} + S_x}\right]$	1	$\beta_{2(x)}$	S_x
$\eta_{15} = \kappa\bar{y}\left[\frac{\overline{X} + \rho}{\bar{x} + \rho}\right]$	$\eta_{16} = \kappa\bar{y}\left[\frac{\bar{x} + \rho}{\overline{X} + \rho}\right]$	1	1	ρ
$\eta_{17} = \kappa\bar{y}\left[\frac{\overline{X} + \beta_{2(x)}}{\bar{x} + \beta_{2(x)}}\right]$	$\eta_{18} = \kappa\bar{y}\left[\frac{\bar{x} + \beta_{2(x)}}{\overline{X} + \beta_{2(x)}}\right]$	1	1	$\beta_{2(x)}$

The expressions $MSE(\eta_i)$ and $MSE(\eta_j)$ are minimized for the optimal values of κ given by

$$\kappa^{\bullet} = \frac{1 + \lambda C_x^2 - \lambda C_{yx}}{1 + 3\lambda C_x^2 - 4\lambda C_{yx} + \lambda C_y^2} = \frac{A^{\bullet}}{B^{\bullet}},$$

$$\kappa^{+} = \frac{\upsilon^2_{\frac{(i-1)}{2}} \lambda C_x^2 - \upsilon_{\frac{(i-1)}{2}} \lambda C_{yx} + 1}{\lambda C_y^2 + 3\upsilon^2_{\frac{(i-1)}{2}} \lambda C_x^2 - 4\upsilon_{\frac{(i-1)}{2}} \lambda C_{yx} + 1} = \frac{A^{+}}{B^{+}},$$

$$\kappa^{\circ} = \frac{1 + \lambda C_{yx}}{1 + \lambda C_y^2 + \lambda C_x^2 + 4\lambda C_{yx}} = \frac{A^{\circ}}{B^{\circ}},$$

$$\kappa^{\tau} = \frac{\upsilon_{(\frac{j}{2})-1} \lambda C_{yx} + 1}{\lambda C_y^2 + \upsilon^2_{(\frac{j}{2})-1} \lambda C_x^2 + 4\upsilon_{(\frac{j}{2})-1} \lambda C_{yx} + 1} = \frac{A^{\tau}}{B^{\tau}}.$$

Substituting these optimal values in (2.15) and (2.16) we get the minimum MSE 's as

$$(2.17)\qquad MSE_{\min}(\eta_i) = \begin{cases} \bar{Y}^2 \left\{ 1 - \dfrac{A^{\bullet 2}}{B^{\bullet}} \right\}, & i = 1 \\ \bar{Y}^2 \left\{ 1 - \dfrac{A^{+2}}{B^{+}} \right\}, & i = 3, 5, \ldots, 17 \end{cases}$$

$$(2.18)\qquad MSE_{\min}(\eta_j) = \begin{cases} \bar{Y}^2 \left\{ 1 - \dfrac{A^{\circ 2}}{B^{\circ}} \right\}, & j = 2, \\ \bar{Y}^2 \left\{ 1 - \dfrac{A^{\tau 2}}{B^{\tau}} \right\}, & j = 4, 6, \ldots, 18. \end{cases}$$

3. Efficiency comparisons

The t-family of estimators is more efficient than the classical ratio estimator if

$$MSE(t_i) < MSE(t_1), \; i = 3, 5, 7, \ldots, 17,$$

that is,

$$(3.1)\qquad \begin{array}{l} \upsilon_{\frac{(i-1)}{2}} < \dfrac{2C_{yx}}{C_x^2} - 1 \text{ for } \upsilon_{\frac{(i-1)}{2}} > 1 \\ \upsilon_{\frac{(i-1)}{2}} > \dfrac{2C_{yx}}{C_x^2} - 1 \text{ for } \upsilon_{\frac{(i-1)}{2}} < 1 \end{array}$$

When condition (3.1) is satisfied, we can infer that the t-family is more efficient than the classical ratio estimator.

The suggested family of estimators as given in Table 2 is more efficient than the classical ratio estimator if

$$MSE_{\min}(\eta_i) < MSE(t_1), \; i = 3, 5, 7, \ldots, 17,$$

that is

$$(3.2)\qquad \left\{ 1 - \frac{A^{+2}}{B^{+}} \right\} < \lambda \left(C_y^2 + C_x^2 - 2C_{yx} \right)$$

When condition (3.2) is satisfied, we can infer that the suggested family is more efficient than the classical ratio estimator.

The suggested family of estimators is more efficient than the ratio estimator proposed by Prasad [4] if

$$MSE_{\min}(\eta_i) < MSE_{\min}(\eta_1), \; i = 3, 5, 7, \ldots, 17,$$

that is

$$\frac{A^{\bullet 2}}{B^{\bullet}} - \frac{A^{+2}}{B^{+}} < 0 \tag{3.3}$$

When condition (3.3) is satisfied we can infer that the suggested family of estimators is more efficient than the ratio estimator proposed in [4].

The suggested family of estimators as given in Table 2 is more efficient than the t-family of estimators given in Table 1 if

$$MSE_{\min}(\eta_1) < MSE(t_1),$$

that is

$$\left\{1 - \frac{A^{\bullet 2}}{B^{\bullet}}\right\} < \lambda\left(C_y^2 + C_x^2 - 2C_{yx}\right), \tag{3.4}$$

also

$$MSE_{\min}(\eta_i) < MSE(t_i), \; i = 3, 5, 7, \ldots, 17,$$

that is

$$\left\{1 - \frac{A^{+2}}{B^{+}}\right\} - \lambda\left[C_y^2 - 2\upsilon_{\frac{(i-1)}{2}} C_{yx} + \upsilon^2_{\frac{(i-1)}{2}} C_x^2\right] < 0. \tag{3.5}$$

It is clear that for the product estimators similar comparisons can be made and the related conditions can also be obtained. We would also like to note that the comparison between the minimum MSE of the proposed and t-families of estimators is obtained as

$$MSE_{\min}(\eta) < MSE_{\min}(t),$$

that is

$$\left\{1 - \frac{A^2}{4B}\right\} < \lambda C_y^2\left(1 - \rho^2\right).$$

4. A numerical example

In this section, we use data concerning primary and secondary schools for 923 districts of Turkey in 2007 (Source: Ministry of Education, Republic of Turkey), taking the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary schools.

Note that we take a sample of size $n = 180$, and we observe that the correlations between auxiliary and study variables are positive. Therefore, we use ratio estimators for the estimation of the population mean in this section. The summary statistics about the population is given in Table 3.

Table 3. Data Statistics

$N = 923$	$n = 180$
$S_y = 749.9395$	$S_x = 21331.1315$
$\overline{Y} = 436.4345$	$\overline{X} = 11440.4984$
$\rho = 0.9543$	$\beta_{2(x)} = 18.7208$
$\beta_{1(x)} = 3.9365$	

The MSE values of the t and the η estimators have been obtained using (2.5) and (2.17), respectively. These values are given in Table 4.

Table 4. Mean square error of the t and η families

t-family		η-family	
Estimator	MSE	Estimator	MSE
t_1	267.6354	η_1	265.5981
t_3	267.5192	η_3	265.4861
t_5	267.6292	η_5	265.5921
t_7	267.0118	η_7	264.9974
t_9	1057.6557	η_9	1056.2934
t_{11}	344.1315	η_{11}	344.1128
t_{13}	227.2753 $*$	η_{13}	226.7785 $**$
t_{15}	267.5759	η_{15}	265.5408
t_{17}	266.4772	η_{17}	264.4824

$*$ represents the most efficient estimator among the t_i estimators.
$**$ represents the most efficient estimator among the η_i estimators.

Table 5. Efficiency Conditions

t, η	(3.1)	(3.2)	(3.3)	(3.4)	(3.5)
t_1				0.001394 < 0.001405	
t_3	0.99983 $*$				
t_5	0.99999 $*$				
t_7	0.99912 $*$				
t_9	0.34909				
t_{11}	0.67858				
t_{13}	0.90942 $*$				
t_{15}	0.99992 $*$				
t_{17}	0.99837 $*$				
η_1					
η_3		0.001393 $*$	-5.87835E-07 $*$		-0.10673E-04 $*$
η_5		0.001394 $*$	-3.14250E-08 $*$		-0.10695E-04 $*$
η_7		0.001391 $*$	-3.15391E-06 $*$		-0.10575E-04 $*$
η_9		0.005545	0.004151176		-0.71520E-05 $*$
η_{11}		0.001807	0.000412205		-0.98000E-07 $*$
η_{12}		0.001191 $*$	-0.000203804 $*$		-0.26090E-05 $*$
η_{13}		0.001394 $*$	-3.00989E-07 $*$		-0.10685E-04 $*$
η_{15}		0.001389 $*$	-5.85766E-06 $*$		-0.10473E-04 $*$
	$0.759 < v_{\frac{(i-1)}{2}} < 1$	< 0.00141	< 0		< 0

$*$ shows that the stated condition is satisfied.

When we examine Table 4, we observe that the 13 th t estimator (t_{13}) and the proposed estimator (η_{13}) have the smallest *MSE* values within their own family of estimators. From this result, we can infer that the 13 th t and η estimators are more efficient than both the classical ratio (k_1) and the Prasad [4] estimator (η_1) for this data set. When we further examine Table 4, we see that $MSE(\eta_{13}) < MSE(t_i)$, where $i = 1, 3, 5, \ldots, 17$. From this result, we can conclude that the proposed estimators are more efficient than the adapted estimators for this data set. However, these results are expected as the conditions (3.1) – (3.5) are satisfied, as shown in Table 5.

Khoshnevisan *et al.* [2] have found that the minimum value of the *MSE* of the t-family is equal to the value of the *MSE* of the regression estimator when α takes the value $\alpha^* = \frac{K}{vg}$ (where $K = \rho \frac{C_y}{C_x}$). For example, we can also obtain the minimum *MSE* when $g = 1$ (ratio estimator) and $\alpha v = K$. For these values, the *MSE* of t is equal to the *MSE* of the regression estimator (224.619), and when we take the same values for the η estimator we get the *MSE* of estimator (224.355), which is slightly less than the *MSE* of the regression estimator. Moreover, there are various combinations of g and αv that we can have smaller *MSE* values than the regression estimator. Consequently, under various conditions, the *MSE* of our proposed estimators can be smaller than the *MSE* of the regression estimator.

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