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# Steam generator identification using piecewise affine model

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**Abstract**

The role of steam generators in power plants is to remove the thermal power produced in the first cycle and provide the turbine intake steam. Controlling the steam generator water level has an important role in preventing unwanted turbine trips and decreasing power plant operability. In this research, a steam generator is identified in the form of a Piecewise Affine AutoRegessive eXogenous (PWARX) structure. The real data for identification is extracted from RELAP5 thermo–hydraulic code. In addition, the minimum number of feasible subsystems (MIN PFS) algorithm is applied for the steam generator piecewise affine identification. The results show good ability of the identified model to track the RELAP5 data. The proposed model can be used in future researches for controlling and analyzing the steam generator. Also, a corresponding feedback controller can be designed for each subsystem in steam generator piecewise affine model.

*Keywords:* System identification Piecewise affine model Thermo–hydraulic code Steam generator
*2010 MSC:* 93B30; 93E12 93A30 93C10.

## 1. Introduction

Identification and modelling of dynamical systems using experimental data is the regularity interested in mathematical models construction of nonlinear systems[1, 2]. The identified model can be considered for the analysis and design of efficient controllers [3]. Acquiring an identified model from real data has been done in several different approaches [4]. Many nonlinear model schemes have been taken into account and their properties have been perused in the literature (see, e.g., the survey papers [5, 6], and references therein).

This paper for identification of steam generator applies a piecewise affine (PWA) model technique using input–output real data. In PWA scheme, the state set is partitioned into polyhedral regions and linear/affine

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subsystems are obtained for each region [7]. PWA systems present themselves to be a powerful class for identifying or approximating generic nonlinear systems via multiple linearization at different operating points [8]. Moreover, PWA system identification is more practical for identifying and estimating hybrid models from given experimental data compared to several classes of hybrid techniques [9, 10]. Note that, hybrid systems are systems with both continuous and discrete dynamics, the former typically associates with physical principles, the latter with logic devices [11].

In this research, we use the minimum number of feasible subsystems (MIN PFS) algorithm for steam generator PWA identification. The algorithm had been introduced by Amaldi and Roll in the references [12, 4], and then was refined by Bemporad in [8]. The data classification and estimation of the number of sub-systems are carried out simultaneously by real data partitioning into a minimum number of feasible subsystems (MIN PFS problem) via related linear complementary inequalities. In summary, the main contributions of this paper are as follows:

1. Identification of nonlinear dynamical systems using a PWA model demonstrates the original system as a set of linear subsystems.
2. Applying the MIN PFS algorithm, a piecewise affine model of a steam generator is proposed, which can be exerted to analyze and control the considered steam generator.

## 2. PWARX system identification

A robust identification scheme for a class of switching processes is known as Piecewise AutoRegressive eXogenous (PWARX) approach [13]. Consider a nonlinear discrete-time system with input $\mathbf{u}_k \in \mathbb{R}^p$ and output $y_k \in \mathbb{R}$. The past inputs and outputs up to time instant $k-1$ are denoted by $\mathbf{u}^{k-1}$ and $\mathbf{y}^{k-1}$, respectively [8]. Then a PWARX model is defined as follows

$$y_k = \Gamma(\mathbf{h}_k) + e_k, \tag{2.1}$$

where $e_k \in \mathbb{R}$ is the error, and $\mathbf{h}_k \in \mathbb{R}^n$ is the vector regression of $n_a$ past outputs and $n_b$ past inputs given as

$$\mathbf{h}_k = [y_{k-1} \dots y_{k-n_a} \mathrm{u}'_{k-1} \dots \mathrm{u}'_{k-n_b}]'. \tag{2.2}$$

In the PWA model, $\Gamma : \chi \to \mathbb{R}$ is considered over the regressor set $\chi \subseteq \mathbb{R}^n$ as given by

$$\Gamma(\mathbf{x}) = \left\{ \begin{array}{l} \nu'\theta_1 \ if \ \mathbf{h} \in \chi_1 \\ \vdots \quad \vdots \\ \nu'\theta_s \ if \ \mathbf{h} \in \chi_s \end{array} \right\}. \tag{2.3}$$

In Eq. (2.3), the parameter $s$ is the number of sub-models, $\nu = [\mathbf{h}' \quad 1]'$, and for each affine sub-model, the vector parameters are defined by $\theta_i \in \mathbb{R}^{n+1}$, $i = 1, \dots, s$ [8]. The regions $\chi_i : \ i = 1, \dots, s$ form a complete partition of $\chi$, i.e. $\bigcup_{i=1}^{s} \chi_i = \chi$ and $\chi_i^o \bigcap \chi_j^o = \varnothing$, $i \neq j$, where $\chi_i^o$ denotes the interior convex polyhedral of $\chi_i$ defined by

$$\chi_i = \left\{ \mathbf{h} \in \mathbb{R}^n : H_i \nu \preceq 0 \right\}, \tag{2.4}$$

and $H_i \in \mathbb{R}^{q_i \times (n+1)}$, $i = 1, \dots, s$ [8]. The parameter $q_i$ is the number of linear inequalities defining the $i$th polyhedral region, and $q_i \leq s-1$ in the identified model [8].

In this paper, the method that is applied for the steam generator PWARX identification, is based on bounded-error approach that has been introduced by Bemporad et al. [8]. In this approach, $\gamma \succ 0$ is considered as a bound on $e_k$ in (2.1) and constrained for all the samples such as all feasible PWARX model (2.1)–(2.4) satisfies

$$|y_k - \Gamma(\mathbf{h}_k)| \leq \gamma, \ \forall \ k = 1, \dots, N. \tag{2.5}$$

It is supposed that the minimum number of sub-models $s$ and the order of each sub-model are considered such as the condition (2.5) is satisfied [13]. Accordingly, the considered identification problem is expressed by [8]:

- Estimate an integer $s > 0$, a polyhedral partition $\{\chi_i\}_{i=1}^s$, and the related parameter vectors $\{\theta_i\}_{i=1}^s$ for given $N$ data points $(x_k, y_k)$, $k = 1, 2, \ldots, N$, such that the identified PWARX model (2.1)–(2.4) satisfies the condition (2.5) [8, 13].

Solving this problem involves classifying the available data points into clusters $\{D_i\}_{i=1}^s$ such that $(x_k, y_k) \in D_i$ if and only if $(x_k, y_k)$ is ascribed to the $i$th mode [8]. The procedure includes of three steps:

1. *Data classification and estimation of pointwise parameters via MIN PFS algorithm.*
2. *Refinement, improvement and update parameters via an iterative procedure.*
3. *Region estimation through two-class or multi-class linear separation techniques.*

Estimation of parameters will be done via $\ell_\infty$ projection estimator [14, 8], which for an $\ell_\infty$ set of data points is determined by

$$\phi_p(S) = \arg\min_{\theta} \max_{(y_k, \mathbf{h}_k) \in S} \left| y_k - \nu_k' \theta \right|, \tag{2.6}$$

where $\nu_k = [\mathbf{x}'_k \ \ 1]'$. Problem (2.6) can be solved via linear programming [8].

## 3. Data classification and estimation via MIN PFS

Given a possibly infeasible linear system $A\mathbf{x} = b$ with $A \in \mathbb{R}^{p \times n}$ and $b \in \mathbb{R}^p$, finding a partition of this system into a MINimum number of Feasible Subsystems is the MIN PFS problem [12]. The condition (2.5) is accomplished thorough finding the smallest number $s$ of vectors $\theta_i$, $i = 1, \ldots, s$, and a mapping $k \to i(k)$ for a given $\gamma \succ 0$, such that $|y_k - \nu_k' \theta| \leq \delta$ for all $k = 1, \ldots, N$.

In the MIN PFS problem, partitioning the system in the form of the following linear supplementary inequalities is done such that the feasible subsystems are in a minimum number

$$\left| y_k - \nu_k' \theta \right| \leq \gamma, \ , \forall \ k = 1, \ldots, N. \tag{3.1}$$

Each subsystem determines the set of parameter vectors of the related affine sub-model [14, 8].

## 4. Steam generator

In figure 1, the structure for Russian designed steam generator is depicted. The generator consists of U-shaped pipes, which are placed between inputs and vertical output collectors. Relative to wide expansion of U-shaped pipes, cold and hot collectors are placed in a class distance [15, 16].

Controlling the water level of steam generator plays a significant role in preventing unwanted turbine stoppage, and as a result, decreases in power plant's performance. Although many controllers have been presented for controlling the U-shaped steam generator in west designed PWR power plants, no suitable controller has been designed for steam generator of VVER Russian-designed reactors in load following operation. For controlling applications, a simple dynamic system is required for steam generator with input reactor's power and output feed water level. In this research, we identify the steam generator in the form of PWARX model as a single-input single-output system. To obtain the PWARX model based on the MIN PFS algorithm, the system input-output data set of various times must be used. In this research, first we model the steam generator by RELAP5 thermohyraulic code, and we use the data as the system's actual data. The first step in modelling every thermohydraulic system by RELAP5 is to gather the geometric and shape related data. The next step is to identify the system's main parameters and their boundaries. These boundaries define the system's limit. A model is a combination of control volumes, which are defined by the designer to present the best analysis of the system's conditions. To create control volumes, which are called model nodalization, each part of the system model is partitioned into discrete parameters. The volumes for controlling the considered steam generator are steam pipes with input and output. RELAP5 coding calculates the liquid average specification in center of volume controls and in the model's boundaries. The applied nodding in this steam power is shown in figure 2 [4, 16].

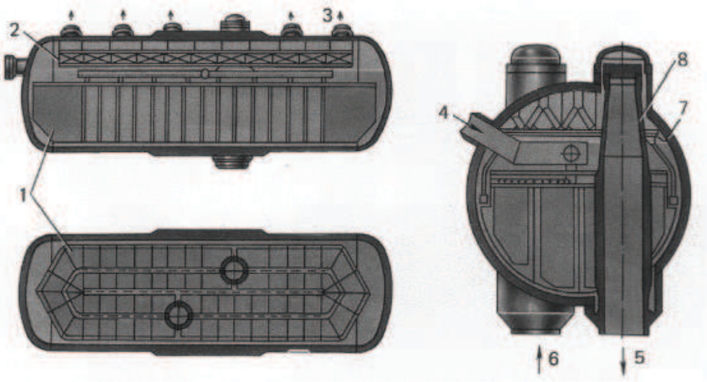


Figure 1: Steam generator VVER-1000 [15].

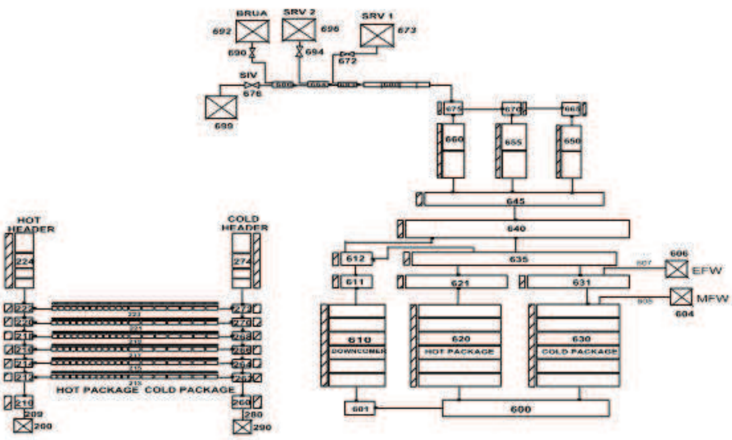


Figure 2: Steam generator nodalizing for the thermohydraulic code.

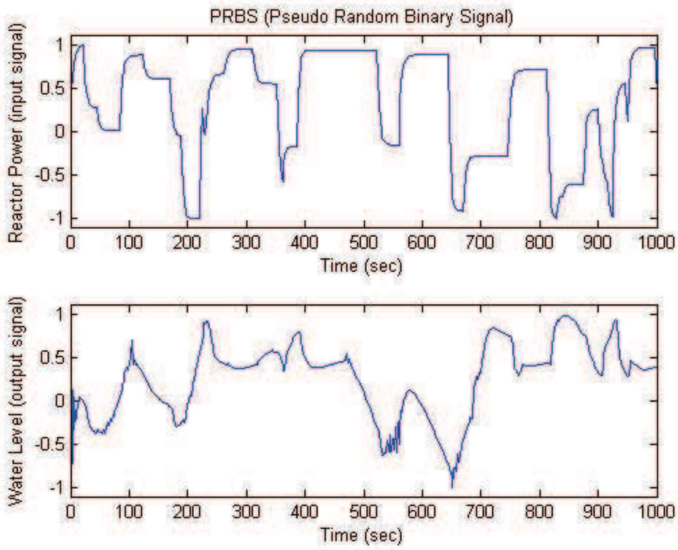


Figure 3: Input signal (top) and output signal (bottom) for producing data set.

## 5. Steam generator PWARX identification

In this project, we use the MIN PFS algorithm to construct the PWARX model thorough the real data, which is taken from the RELAP5 code. For endorsement of the obtained simulation outcomes of the PWARX with the real data of steam generator, we consider the degree of fitness (DF) as follows

$$DF = (1 - \frac{1}{N} \times \sum_{m=1}^{N} (|WL_{RELAP}(t(n)) - WL_{PWARX}(t(n))|)) \times 100, \tag{5.1}$$

where $N$ is the total number of time intervals, $WL_{RELAP}(t(n))$ is the real level of water, which thermo-hydraulic RELAP5 code represents them at the $n$th interval, and $WL_{PWARX}(t(n))$ is output of PWARX model at the $n$th interval [16]. A powerful signal to stimulate all possible dynamic conditions for a better identification is a random signal with a random period and amplitude called Pseudo Random Binary Signal (PRBS). For this purpose, in the RELAP model of steam generator, we modify the reactor power with a PRBS for up to 1000 s, while the other conditions are at operational point. This input signal (reactor power) and the output signal (water level) for identification are plotted in figure 3. The considered data points $(x_k, y_k)$, $k = 1, 2, \ldots, 500$, are sampled by 2 s step time over an interval 1000 s. These data points are shown in figure 4, that are classified into 2 clusters by the first. This classification and estimation of parameters are carried out by the first step of the PWARX identification procedure. Figure 5 represents the classification. A PWARX identified model of order $n_a = n_b = 1$ is supposed. Considering $\gamma = 0.05$, a model with $s = 2$ are identified from $N = 500$ estimation data. The steam generator PWARX model with 2 sub-models that is obtained by the data classification and parameter estimation during the identification procedure is in the form below

$$WL_k = \begin{cases} 0.9435WL_{k-1} + 0.02173RP_{k-1} & \text{if } RP_{k-1} \leq (0.2128WL_{k-1} + 0.3154), \\ 0.9804WL_{k-1} + 0.03108RP_{k-1} & \text{if } RP_{k-1} > (0.2128WL_{k-1} + 0.3154). \end{cases} \tag{5.2}$$

For completion, an ARX model with $s = 1$ and models of the same orders is identified that is described by

$$WL_k = 1.0983WL_{k-1} + 0.0231RP_{k-1}. \tag{5.3}$$

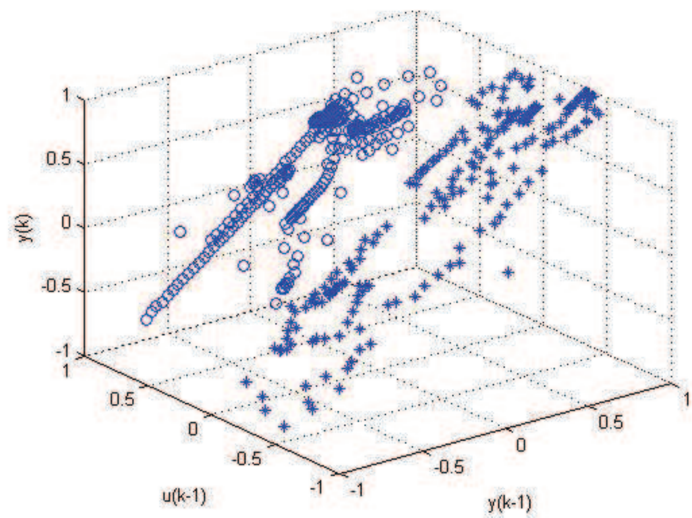


Figure 4: Available data points that are classified into 2 classes: class 'o' and class '*'.

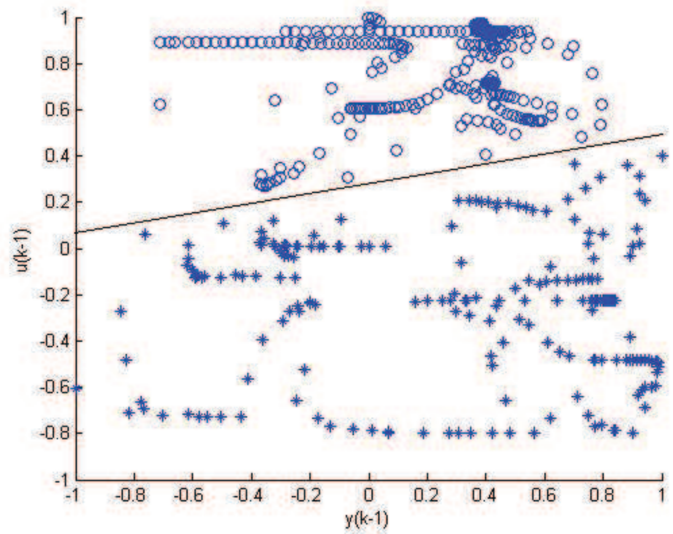


Figure 5: Classification of the regression vectors. Each mark corresponds to a different class. The top class consists of 297 data points and the bottom class consists of 203 data points. The line represents partition of the regressor set.

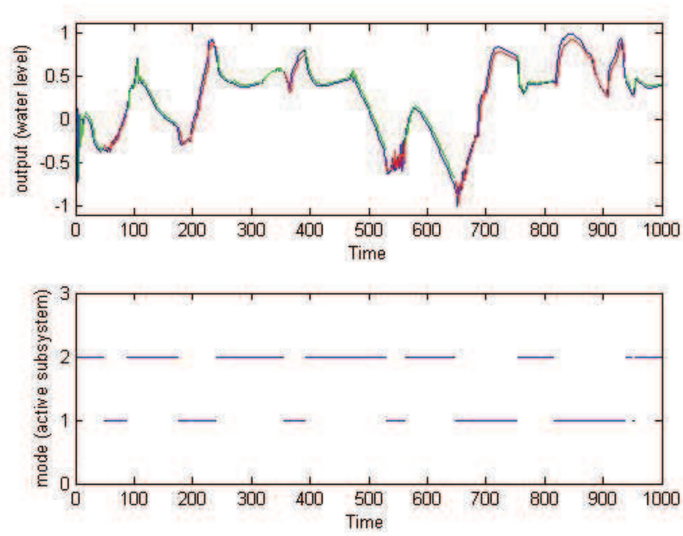


Figure 6: Simulation results for the data points: Top: RELAP output (blue) and PWARX output (red: mode 1, green: mode 2). Bottom: The evolution of discrete mode.

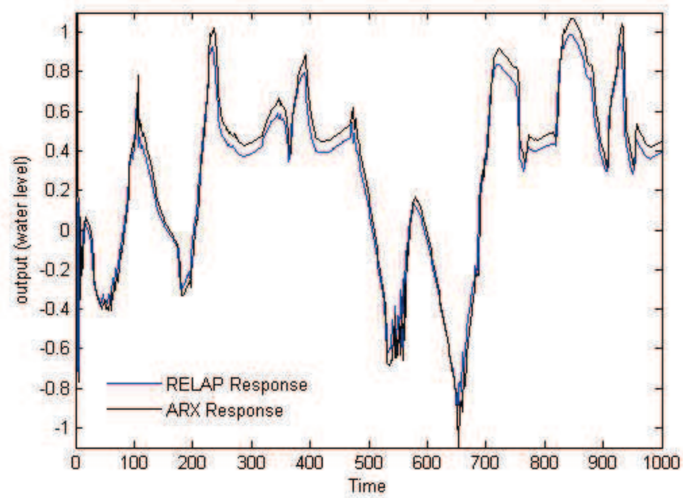


Figure 7: Simulation results for the data points, which are applied for the ARX model identification.

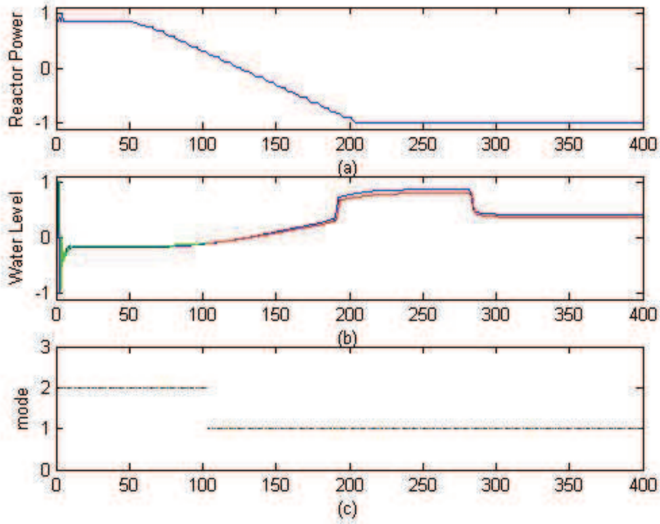


Figure 8: Comparison of PWARX and RELAP5 results (Test 1): $(a)$ reactor power, $(b)$ RELAP output (blue) and PWARX output (red: mode 1, green: mode 2), $(c)$ active subsystem.

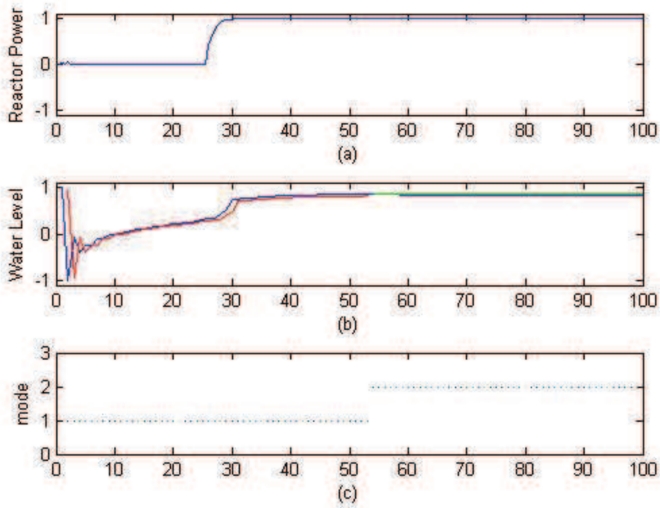


Figure 9: Comparison of PWARX and RELAP5 results (Test 2): $(a)$ reactor power, $(b)$ RELAP output (blue) and PWARX output (red: mode 1. green: mode 2), $(c)$ active subsystem.

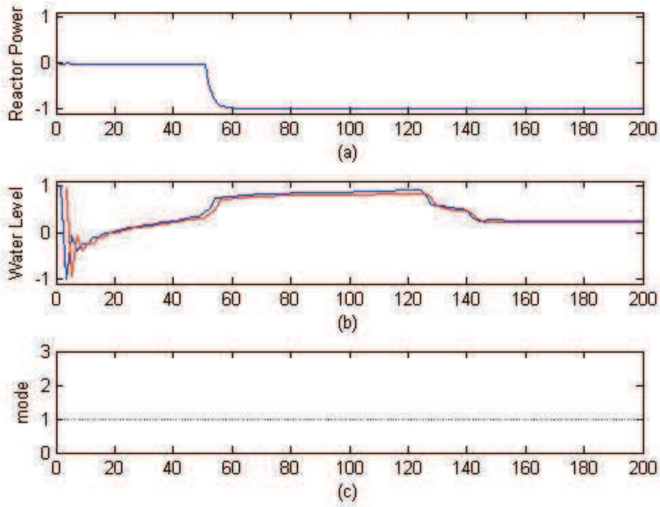


Figure 10: Comparison of PWARX and RELAP5 results (Test 3): $(a)$ reactor power, $(b)$ RELAP output (blue) and PWARX output (red: mode 1, green: mode 2), $(c)$ active subsystem.

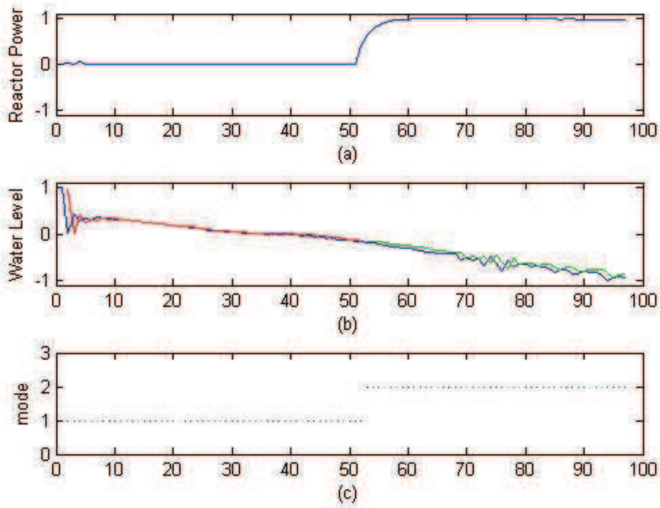


Figure 11: Comparison of PWARX and RELAP5 results (Test 4): $(a)$ reactor power, $(b)$ RELAP output (blue) and PWARX output (red: mode 1. green: mode 2), $(c)$ active subsystem.

## 6. Implementation and results

Figure 6 shows the simulation results for the data points, which are applied for steam generator using PWARX model with $s = 2$. Comparison of the results of PWARX model and RELAP5 code response results in a degree of fitness (DF) equal to 96.80%. The ARX model simulation results are evaluated with the response of RELAP5 data in figure 7. In this case, we have $DF$=87.66%. To accredit the PWARX model, the following test transients are chosen:

1. *Test 1*: The mass flow rate of feed water is at its operational point, and the power of reactor varies from 120% to 80% of its operational point as shown in figure 8, the first plot. The PWARX model results are compared with real data taken from RELAP5 in figure 8, second plot. Also, the active subsystems are shown in the third plot. In this test DF=95.17%.

2. *Test 2*: The mass flow rate of feed water is at 120% of its operational point, and the power of reactor varies from 100% to 120% of its operational point as shown in figure 9. In this test DF=96.08%.

3. *Test 3*: The mass flow rate of feed water is at 120% of its operational point, and the power of reactor varies from 100% to 80% of its operational point as shown in figure 10. In this test DF=94.18%.

4. *Test 4*: The mass flow rate of feed water is at 80% of its operational point, and the power of reactor varies from 100% to 120% of its operational point as shown in figure 11. In this test DF=94.05%.

## 7. Conclusion

Piecewise Affine AutoRegessive eXogenous (PWARX) models are efficient tools for identifying and modelling of a diverse class of nonlinear dynamical systems. In this paper, MIN PFS algorithm was used for PWARX model identification of a complex nonlinear plant. The proposed structure modeled and predicted the dynamical behaviours of the steam generator faster than the thermohyraulic code with a lower error. This PWARX model will be exerted in future works for controlling and analyzing the steam generator. Also, a corresponding feedback controller can be designed for each subsystem in the steam generator PWARX model.

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