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PRESERVICE TEACHERS' KNOWLEDGE OF STUDENTS' COGNITIVE PROCESSES ABOUT THE DIVISION OF FRACTIONS*

ÖĞRETMEN ADAYLARININ KESİRLERDE BÖLMEYE İLİŞKİN ÖĞRENCİLERİN BİLİŞSEL SÜREÇLERİ HAKKINDAKİ BİLGİLERİ

Mine IŞIKSAL**, Erdinç ÇAKIROĞLU***

ABSTRACT: The purpose of this study was to examine preservice mathematics teachers' knowledge about common (mis)conceptions and difficulties of elementary students. In addition, it was aimed to investigate preservice teachers' knowledge about the possible sources of these misconception/difficulties, and their suggested strategies to overcome those difficulties. Data was collected from senior preservice elementary mathematics teachers enrolled in a teacher education program at a public university at the end of the spring semester of 2004-2005. Qualitative case study design was used to collect the data. Results revealed that preservice teachers' knowledge on difficulties that elementary students might have could be grouped under four headings. In addition, preservice teachers suggested various strategies that can be used to overcome students' difficulties on division of fractions. Research study revealed the importance of content-pedagogy rich courses on preservice teachers' knowledge related to students' understanding of mathematics concepts.

Keywords: mathematics education, preservice elementary mathematics teachers, pedagogical content knowledge, division of fractions

ÖZET: Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının ilköğretim öğrencilerinin kesirlerde bölmeye ilişkin sahip olabilecekleri kavram yanılgıları ve karşılaştıkları zorluklar hakkındaki bilgilerini incelemektir. Ayrıca, öğretmen adaylarının bu kavram yanılgıları ve zorlukların olası sebepleri hakkındaki bilgileri ve bu zorlukları gidermeye yönelik sundukları stratejileri incelemektir. Çalışma 2004-2005 bahar dönemi sonunda bir devlet üniversitesinde öğretmen yetiştirme programına devam eden son sınıf ilköğretim matematik öğretmen adayları ile nitel durum çalışması yapılarak gerçekleştirilmiştir. Çalışmanın sonuçları, öğretmen adaylarının öğrencilerin kesirlerde bölmeyle ilgili kavram yanılgılarının dört ana başlıkta toplanabileceğini göstermiştir. Ayrıca, sonuçlar öğretmen adaylarının öğrencilerin karşılaştıkları zorlukları giderebilmek için birçok strateji geliştirdiklerini göstermiştir. Bulgular, içerik ve pedagoji ağırlıklı derslerin, öğretmen adaylarının öğrencilerin matematiksel kavramları anlamaya ilişkin bilgilerinin gelişmesindeki önemini ortaya koymuştur.

Anahtar sözcükler: matematik eğitimi, ilköğretim matematik öğretmen adayları, pedagojik içerik bilgisi, kesirlerde bölme

1. INTRODUCTION

Good teaching demands that teachers should know many things about teaching; about their students; and about the cultural, political, and social context within which they work (Ball & McDiarmid, 1990). Teachers who have conceptual understanding could answer students' questions about the meaning behind symbolic manipulations. However, subject matter knowledge is not enough to achieve this goal. Teachers should also transform the content into representations that help students develop understanding (Shulman, 1986).

Shulman (1986) mentioned three types of content knowledge: (a) subject matter knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. Shulman defined pedagogical content knowledge as subject matter knowledge for teaching. According to him, pedagogical content knowledge includes:

> The most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to other….it also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and

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** Dr., Middle East Technical University, misiksal@metu.edu.tr
*** Assist. Prof. Dr., Middle East Technical University, erdinc@metu.edu.tr.

preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

Pedagogical Content Knowledge (PCK) differentiates expert teachers in a subject area from the subject area experts (Shulman, 1987). The PCK concerns how teachers related their subject matter knowledge (what they know about what they teach) to their pedagogical knowledge (what they know about teaching) and how subject matter knowledge is related to the process of pedagogical reasoning (Shulman, 1987).

Grossman (1989, 1990) expanded Shulman's definition and stated that pedagogical content knowledge includes four main components: (a) an overarching conception of what it means to teach a particular subject, (b) knowledge of instructional strategies and representations for teaching particular topics,(c) knowledge of students' understanding and potential misunderstandings of a subject area, and (d) knowledge of curriculum and curricular materials. Ball (1990) reported that a teacher with pedagogical content knowledge use manipulatives in mathematics classes and is ready for possible student misconceptions on topics being taught.

Mack (1990) stated that fractions and rational numbers are considered as the most complex mathematical domains in elementary school mathematics. In their study, Şandır, Ubuz, and Argün (2007) mentioned that nine grade students had difficulty and made errors while performing arithmetic operations involving rational numbers. Although many students understand the rote algorithm needed to manipulate the symbols, they soon forget the procedures and thus find it difficult to learn operations on fractions and rational numbers (Ball, 1990; Tirosh, 2000). In their study with fifth graders, Haser and Ubuz (2003) mentioned that students' have difficulty while solving problems related to fractions because of their inadequate conceptual understanding of part and a quantity. Many students' understanding of fractions is characterized by knowledge of rote procedures, that are often incorrect, rather than by the concepts underlying the procedures (Behr, Lesh, Post, & Silver, 1983). For instance, algorithmically based mistakes were popular among children that arose because of the rote memorization of the algorithm while performing division of fractions (Ashlock, 1990). The most common error in this category was inverting the dividend instead of the divisor or inverting both the dividend and the divisor before multiplying numerators and denominators while dividing two fractions (Ashlock, 1990). Another common mistake mentioned in the literature was intuitively based mistakes result from intuitions held about division. Students tend to overgeneralize properties of operations with whole numbers to fractions and interpret division primarily using a primitive model of division. In the literature, basic intuitions on division of fractions were stated as follows: the dividend is always bigger than the divisor, the dividend is always bigger than the quotient, the quotient must be integer, and divisor must be a whole number (Ashlock, 1990; Barash & Klein, 1996; Fischbein, 1987; Graeber, Tirosh, & Glover, 1989; Tirosh, 2000). For instance, students stated that "one can not divide a small number by a large number because it is impossible to share less among more" (Tirosh, Fischbein, Graeber, & Wilson, 1993, p. 18). Thus, the predominance of these primitive conceptions limited children's and prospective teachers' abilities to correctly solve the division problems involving fractions (Fischbein, Deri, Nello, & Marino, 1985; Graeber, Tirosh, & Glover, 1989). Operations with fractions were often taught using procedures instead of allowing students to experience multiple ways of the meaning of operations (Tirosh, 2000). Ball (1991) stated that prospective teachers did not connect the concept of division across the different context like division of fractions, division by zero, and division in algebra. They treat each division as a separate topic without any relationship to the others and cite a particular procedure or rule for each of them. Researchers emphasized that when asked by a student why you could get a bigger value when dividing by fractions, teacher's most typical response was to ignore the question or simply told the student it is because of invert and multiply rule (Ball, 1990, 1991; Tirosh, 2000). That is; most of the teachers knew how to divide fractions but could not explain the procedure (Tirosh, 2000). In addition, Mack (1990) stated that prospective teachers were unaware of the major sources of students' incorrect responses in this domain. From an instructional aspect, teachers lack a deep understanding of fractions, which inhibits them or puts them at a disadvantage in using multiple strategies in their instruction.

Based on the literature, researchers agreed that teacher education programs should attempt to make preservice teachers aware of the common, sometimes erroneous, cognitive processes used by students in dividing fractions (Ball, 1990; Tirosh, 2000). Since today's preservice teachers are tomorrow's mathematics teachers, their learning and teaching cycle might effect their students' misconceptions and misunderstandings about division. Thus, in this study our aim is to investigate the aspects of preservice mathematics teachers' pedagogical content knowledge involving students' conceptions and cognitive processes related to division of fractions. More specifically, this research study aims to answer the following research questions:

- What do preservice elementary mathematics teachers know about common conceptions and misconceptions/difficulties held by upper elementary (6^{th} and 7^{th} grade) students related to division of fractions?
- What do preservice elementary mathematics teachers know about the possible sources of misconceptions/difficulties held by upper elementary (6^{th} and 7^{th} grade) students related to division of fractions?
- What kind of strategies do preservice elementary mathematics teachers suggested to use in order to overcome the misconceptions/difficulties held by upper elementary (6^{th} and 7^{th} grade) students on division of fractions?

2. METHODOLOGY

Qualitative case study design was used to support methodological perspective and findings of the research study. Merriam (1998) emphasized the importance of the case as a *thing*, a *single entity*, and a *unit* that has boundaries. Stake (1995) mentioned that "case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances" (p. 6). In this research design, preservice teachers' pedagogical content knowledge was analyzed within the context of elementary mathematics teacher education program.

Seventeen senior preservice elementary mathematics teachers in an undergraduate teacher education program in Ankara were selected by using purposive sampling at the end of the spring semester of 2004-2005 academic year. Data were collected at the end of the last semester of participants' undergraduate course work. Thus, at the time of the data collection, all of the participants had completed, or were about to complete, all of their required coursework. This was one of the underlying rationales for choosing senior PSTs, in the sense that their experiences in the program make them potential participants to study what sort of knowledge, thought, understanding, and experiences were critical in their conceptions on division of fractions. In addition, although those preservice teachers had completed school experience and practice teaching courses offered by the program they had no direct intense interaction with elementary students. They had been involved in several curricular activities in schools. However, none of those experiences were directly related to the division of fractions.

2.1. Data Sources

Participants were first administered a written question related to the division of fractions. In addition, semi-structured interview protocol was used to get indepth understanding on preservice teachers' knowledge on common conceptions and misconceptions/difficulties held by the elementary (sixth & seventh grade) students. In addition, the possible sources of these misconceptions/difficulties, and the strategies that PSTs suggested to use in order to overcome these misconceptions/difficulties were investigated. The question was taken and adapted from Tirosh (2000) which aimed to examine the preservice teachers' conceptions on partitive and quotitive division (Appendix A). Necessary revisions and additions were made in order to understand preservice teachers' nature of the pedagogical content knowledge. A written question was administered to the PSTs at their regular course hours, and all the preservice teachers who attended to the course on that day volunteered to participate in study.

After administering the written question, semi-structured interviews were conducted with 17 senior PSTs in order to get in-depth exploration on their pedagogical content knowledge. The interview

consisted of three parts: (1) Background questions, (2) Questions based on general knowledge on division operation, and (3) Questions based on the review of the responses to the written question. Sample items from the semi-structured interview protocol were given in Appendix B.

Ensuring trustworthiness in qualitative research is important in judging the quality of the study (Lincoln & Guba, 1985; Seale, 1999; Stenbacka, 2001). In this study, data triangulation and method triangulation were used for increasing the credibility and dependability of the research study. Seventeen senior PSTs that is more than one individual as a source of data were used. Different types of data collection tools including both questionnaire and interview protocols were prepared to have consistency among the cases. In addition to triangulation, we used member checking, where we had the participants view the raw data (questionnaires and transcriptions) and asked them to comment on their accuracy. We also used low inference descriptors, where we always tried to use the phrases that were very close to the participants' wordings and verbatim in reporting the analysis of the research findings. These were the evidences on increasing the credibility of the given case study. In addition, second coder was recruited for data analysis in order to have consensus on findings and reduce the researchers' bias. During the data coding both coders tried to identify the patterns and themes to increase the quality of the research study. Both coders analyzed the transcribed data with pseudonym names and coding were prolong until there was % 90 agreement between two coders.

3. RESULTS

Preservice teachers' own experiences as learners and their knowledge on students' cognitive thinking process were searched systematically. Based on the analysis of the data, preservice teachers' knowledge on students' misconceptions and difficulties was grouped under the four headings namely: (1) algorithmically based mistakes, (2) intuitively based mistakes, (3) mistakes based on formal knowledge of fractions, and (4) misunderstanding of the problem. Additionally, possible sources of these misconceptions/difficulties were also discussed within the circumstances of these four headings. These headings were based on available literature, participants' statements, and our own experiences with the data.

Results revealed that sixteen PSTs emphasized the importance of *algorithmically based mistakes* stemmed from students' rote-memorization and inadequate knowledge on four operations. Those preservice teachers mentioned that students may symbolize the solution by using multiplication, subtraction, or addition instead of division operation. More specifically, for the first part of the question, PSTs mentioned that instead of $\frac{1}{4} \div 4$, students could symbolize the given expression as $4 \div \frac{1}{4}$, $\frac{1}{4} \times 4$, $4 \div 4$, or $4 - \frac{1}{4}$ because of their inadequate knowledge. In addition to given misinterpretations, preservice teachers stated that students could have difficulties even they decided to solve the problem by using division operation. In other words, participants believed that students may invert the dividend instead of the divisor, invert both the dividend and the divisor, or simplify the numerator and denominator while doing division operation.

> Participant 11: "For the given division problem students can perform division operation like $\frac{1}{4} \div 4 = \frac{1}{4} \times \frac{4}{1}$, $\frac{1}{4} \div 4 = \frac{4}{1} \times \frac{1}{4}$, $\frac{1}{4} \div 4 = 1 \times \frac{4}{1}$ or $4 \div \frac{1}{4} = \frac{4 \div 1}{4 \div 4}$. These are because of students' lack of adequate knowledge on division. Students have confusion while performing the given operation since they do not know the meaning of division and concept of how to perform division operation.

Although this participant mentioned an algorithmically based mistake, she made an attempt to connect it to the conceptual meaning of division. Similarly twelve preservice teachers emphasized the role of memorization of the procedure and inadequate formal knowledge on four operations as the bases of misconceptions and difficulties. In other words, preservice teachers mentioned that students' lack of

knowledge to discuss what 'sharing' means and their incorrect "memorization" of how to perform the rule could be the main sources in making algorithmically based mistakes.

In addition to algorithmically based mistakes, three preservice teachers emphasized the importance of *intuitively based mistakes* while performing division of fraction operation. Those PSTs stated that students may have misconceptions that in a division problem divisor must be a whole number. Thus, in a given problem students may perform the operation incorrectly since to divide by the fraction is not meaningful to them. That is, students' overgeneralization and misinterpretation of primitive division models could be the main sources for intuitively based mistakes. For instance:

> Participant 6: "For the given sharing problem where $\frac{1}{4}$ will be shared among 4 people, students could perform $4 \div \frac{1}{4}$, in other words, they switch the places of dividend and the divisor since the dividend is less than the divisor. These are because in elementary schools students learn division by dividing bigger number by the smaller one. In addition, teacher can be one of the sources for this mistake. If teacher do not explain the properties of the division operation properly, students can have difficulties in performing operations"

Data analysis revealed another dimension of difficulties based on students' *formal knowledge* on fractions. Two preservice teachers stated that the errors under this category are due to both limited conceptions of the notion of fractions and inadequate knowledge related to the properties of the fractions. Those preservice teachers mentioned that for the given division problem, students may have difficulties since they do not have the conception of one-fourth. They also added that students' informal knowledge on the concept of 'whole' could lead them to make errors.

Regarding the source of students' common mistakes, analysis revealed that two of the PSTs emphasized that education system policies or teacher herself as a major factor for algorithmically based mistakes, intuitively based mistakes, and mistakes based on formal knowledge. More specifically, those preservice teachers believed that if teachers do not have enough content knowledge to teach conceptually and direct the students to memorize the given rules without emphasizing the logic behind the algorithmic operations, students do not learn the concepts well and have difficulties while solving questions.

The last category on students' misconceptions and difficulties mentioned by the PSTs was *misunderstanding of the problem*. Two of the PSTs stated that students can make errors since they do not understand the problem. Students could have difficulties in understanding what is being given and what is being asked in the given problem. Those preservice teachers added that, lack of care, lack of adequacy in mathematical knowledge, and lack of self-efficacy could be the sources for these difficulties.

Our other aim in the present study was to investigate the strategies that PSTs suggested to use in order to overcome the misconceptions and difficulties held by the sixth and seventh grade students on division of fractions. The analysis of data revealed that these strategies could be grouped under three headings: strategies based on teaching methodologies, strategies based on formal knowledge on fractions, and strategies based on psychological constructs.

Using multiple representations is one of the strategies offered by the PSTs. Eleven preservice teachers stated that they can use figures, verbal expressions, visual materials, and daily life examples to overcome the misconceptions and difficulties held by the elementary students. For instance:

> Participant 3: "I can make a model for the question. For example, I could bring a cheese of 4 kg and also distribute packages where each of them is one-fourth kilogram. Then, I can ask how many packages I need to put this 4 kilogram cheese. I think it's better to use concrete materials to reduce the misunderstandings. By this way student do not have confusions".

Those PSTs believed that with the help of the mathematical language and mathematical models, teacher could overcome the difficulties held by the elementary grade level students while studying division of fractions. Using different teaching methods is another strategy suggested by the PSTs in order to overcome the difficulties. Four of the PSTs emphasized that teacher should solve problems step by step

as mentioned in Polya's problem solving strategies. Teacher's description of question like what is being given and what is being asked could enable students to understand the question clearly. For instance:

> Participant 13: "We should avoid memorization of the rules. I mean we should not say like, 'invert and multiply while dividing fractions'. We should use manipulatives to make the concept understandable. We should reinforce the students to solve the problems according to Polya's steps. We should make the students to read the question carefully until they understand what is being given and asked to them".

Using drill and practice in mathematics classes is another strategy suggested by three of the PSTs. Those preservice teachers stated that teachers should solve many questions and suggest alternative solutions to the given questions in order to overcome difficulties and misconceptions held by the students.

In addition to the strategies based on teaching methodologies, most of the PSTs emphasized the importance of strategies based on teachers' formal knowledge in order to overcome the difficulties and misconceptions held by the elementary school students. Thirteen of the PSTs agreed that teachers should know the concepts well and make students understand the concept first before letting them to solve the related questions. Those preservice teachers stated that before proceeding to the traditional symbolism – invert the second fraction and multiply with the first one- for the given division operation, teachers should focus on the meaning of the division concept and relationship among the basic operations on fractions. They added that if teachers explain the logical relationships among the mathematical concepts students do not have difficulties in performing the operations. In addition, those PSTs emphasized the importance of making students explain their rationale while they are solving the given problems. In other words, students should be given extra time to explore and reason multiplicative relationships by themselves before solving the given problem. For instance,

> Participant 3: "In order to overcome the difficulties, we should let the student explain what he did. While students performing the operation, we can ask them why they solve the problem in a given way. By this way, we can understand their thinking process and whether they know the logic of operation that they are doing."

Apart from these strategies, one of the PSTs mentioned that teacher should inform the students about the misconceptions and difficulties that other students have while teaching the topic. PST believed that if students are informed about the common errors or difficulties held by the others they can get rid of making the same mistakes while performing the operation.

The importance of hierarchical sequence in teaching concepts was stated by two of the PSTs in order to overcome the misconceptions and difficulties held by the students. Those PSTs believed that teachers should teach the concepts by using easy examples and then move to the harder ones. State differently, PSTs emphasized that teachers should teach basic concepts by using concrete objects and then should move to the abstract ideas. By this way, students could understand the topic clearly and they do not made a mistake while performing operations.

The last strategy mentioned by PSTs was related to the affective domains. Two of the preservice teachers stated that teachers should also concentrate on students' needs in addition to improving their cognitive skills. Students may perform the operations incorrectly because of their inadequate self-concept or high level of anxiety apart from their subject matter knowledge. Those PSTs mentioned that teachers should focus on enhancing students' efficacy beliefs and decreasing their anxieties to overcome students' misconceptions and difficulties.

4. CONCLUSION AND DISCUSSION

In this study, our aim was to investigate preservice teachers' knowledge about common misconceptions and difficulties held by elementary (6^{th} and 7^{th} grade) students on division of fractions. In addition, preservice teachers' knowledge about possible sources of these misconceptions/difficulties and their suggested strategies to overcome these misconceptions were examined.

Research findings revealed that preservice teachers' knowledge on common conceptions and difficulties that elementary grade level students might have could be grouped under four headings

namely; algorithmically based mistakes, intuitively based mistakes, mistakes based on formal knowledge, and misunderstanding on problem. PSTs stated that rote memorization and inadequate knowledge on four operations could be the main sources for algorithmically based mistakes. These opinions of the preservice teachers were parallel with the literature where researchers mentioned that algorithmically based mistakes arouse because of rote memorization of the algorithm (Ashlock, 1990; Tirosh, 2000). On the other hand, students' conceptions on primitive models were stated as the main source for the intuitively based mistakes. Analysis of the data revealed that students' intuitive beliefs on division operation were as follows: in a division problem quotient should be whole number, dividend should be completely divided by divisor, divisor must be a whole number, and dividend should be bigger than the divisor. These beliefs were also consistent with the intuitive beliefs mentioned in the various research studies (Barash & Klein, 1996; Fischbein, 1987; Graeber et al., 1989; Tirosh, 2000). Inadequate formal knowledge and limited conceptions on the notion of fractions were identified as two important sources for the mistakes based on formal knowledge on fractions. In addition to these findings, two of the PSTs emphasized that teachers play important role in the development of misconceptions among students. To state differently, PST agreed that if teacher do not have enough competency in the given subject area then s/he could be the major source for these difficulties since teacher is responsible to overcome the difficulties held by the students. Thus, these findings were consistent with the research studies that emphasize the importance of teacher knowledge on subject area (Hill, Rowan, & Ball, 2005; Crespo & Nicol, 2006).

In line with these findings, PSTs suggested various strategies that can be used to overcome students' misconceptions and difficulties on division of fractions. PST suggested strategies for teaching such as using multiple representations of the concepts (e.g. verbal expressions, figures, and graphics), using different teaching methods (e.g. problem solving), emphasizing practice of computational skills, and focusing on understanding the problem. In addition to these approaches, PSTs suggested strategies based on formal knowledge such as emphasizing logical relationship among operations and suggesting alternative solutions to the problems.

Preservice teachers agreed that teachers should use various strategies in order to make students understand the topic. They suggested using concrete materials or models from daily life in order to make students familiar with the concepts. In other words, results revealed that PSTs planned to create learning based classrooms where they make students express their reasoning behind their calculations. Additionally, they suggested enhancing students' efficacy beliefs in order to overcome the difficulties that students have on division of fractions. That is, PST believed that teacher should not only focus on teaching concepts but also take into consideration students' needs in their classroom practices. Here, we could deduce the effect of teaching method courses in teacher education program on preservice teachers' pedagogical content knowledge. That is; in method courses much more emphasis is given on how to use various teaching strategies effectively, how to cooperate with students, and how to create learning based environments in order to increase students' learning and understanding. As mentioned above, participants of the study completed all the courses that the program offers and they were competent in using various teaching strategies to develop students' understanding. Thus, preservice teachers' experiences in these courses could enhance their knowledge on students' understanding and to suggest various strategies to overcome students' misconceptions. In other words, it is believed that those preservice teachers had enough competencies both in content and pedagogical content knowledge areas and thus proposed conceptual strategies in order to overcome students' difficulties.

Mathematics teacher educators are constantly faced with the question of how to help PSTs to develop deeper understanding of mathematics while learning about teaching and learning (Crespo & Nicol, 2006). Tirosh (2000) stated that a major goal in teacher education programs should be to promote development of preservice teachers' knowledge of common ways that children think about the mathematics topics. Findings of this research study both support and challenge recent policy initiatives' effort to improve teacher education programs in terms of content knowledge and professional development (Lederman, Gess-Newsome, & Latz, 1994). Teacher education programs should familiarize PSTs with various, and sometimes erroneous, common types of cognitive process and how they may lead to various ways of thinking (Tirosh, 2000). In other words, mathematics education programs should offer content-pedagogy rich courses related to the mathematics and mathematics education in order to enhance

the qualification of the future teachers (Lederman, Gess-Newsome, & Latz, 1994; Crespo & Nicol, 2006). In those courses, PSTs should have opportunities to share their ideas, communicate with their peers, discuss and interpret among concepts, struggle with definitions and concepts, and acquire knowledge on students' thinking. By this way, preservice teachers could have deep subject matter and pedagogical content knowledge before they graduate from the program and involve in actual teaching practices in classrooms.

Based on the findings above we could deduce well-defined implications for the further studies. Further research studies could be carried out to investigate the effectiveness of method and teaching practice courses on PSTs' subject matter knowledge and pedagogical content knowledge on various topics in mathematics. The continuum of developmental process of the knowledge structures of PSTs' could be examined at various time intervals during their involvement in teacher education program. In addition, effectiveness of instruction cannot be assessed without students' learning being measured (Hill, Rowan, & Ball, 2005). Thus, further studies need to be done to explore in-service teachers' subject matter and pedagogical content knowledge and how these knowledge structures affect students' learning in various topics in mathematics.

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APPENDICES:

APPENDIX A

The Division of Fractions Questionnaire (DFQ)

For each of the following word problems **(a)** Write an expression that will solve the problem **(b)** List two common mistakes students in sixth or seventh grade may make, **(c)** Describe possible sources for each of these mistakes **(d)** What kind of strategies will you use to overcome these difficulties?

(i) Four friends bought $\frac{1}{4}$ kilogram of sweets and shared it equally.

How much sweet did each person get?

(ii) Four kilograms of cheese were packed in packages of $\frac{1}{4}$ kilogram each. How many packages were needed to pack all the cheese?

APPENDIX B

Interview Protocol

Part I.

1. Tell me about your educational background?
   Which type of high school did you graduate from?
   What mathematics/mathematics education courses have you taken so far in the University?
2. Have you had any teaching experience? Where? When? How long? Grade level?

…

Part II.

1. How do you define division?
2. Does division with natural numbers relate to division with fractions? In what aspects?
3. What kind of difficulties/misconceptions can elementary school students have while working with division of fractions? Why?
4. What could be the reasons for those difficulties/misconceptions?
5. Which strategies will you use to overcome these difficulties?

……..

Part III.

The researchers ask the following questions based on the answers given in the DFQ.

- In investigating changes in the responses of preservice teacher, ask the following questions:
  "I see that your answer for the question *** has changed, can you explain to me the reason for the change?"
- In general, the phases that are used during the interview include:

" What do you mean by….."
"Here you mentioned that ….'
" Tell me more on……" (if there is something that is not clear to the researcher on the questionnaire"
" Why do you think so….." (if there is something that is not clear to the researcher on the questionnaire
"Is this what you mean…" (In response to the preservice teacher)
…..

GENİŞLETİLMİŞ ÖZET

İyi bir öğretim, öğretmenlerin öğretim verdikleri alan, öğrenciler ve çalıştıkları çevreye ilişkin kültürel, politik, ve sosyal değerlerle ilgili birçok şeyi bilmesini gerektirir (Ball & McDiarmid, 1990). Kavramsal bilgiye sahip öğretmenler, öğrencilerin sembolik işlemlerin ardındaki mantığa ilişkin sorularını rahatlıkla cevaplayabilirler. Ancak, yalnızca alan bilgisine sahip olmak bu amacı gerçekleştirmek için yeterli değildir. Öğretmenler, bu bilgiyi öğrencilerin anlamlı öğrenmesine yardımcı olacak gösterimlere dönüştürmelidirler (Shulman, 1986).

Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının ilköğretim öğrencilerinin (6. ve 7. sınıf) kesirlerde bölmeye ilişkin sahip olabilecekleri kavram yanılgıları ve karşılaştıkları zorluklar hakkındaki bilgilerini incelemektir. Ayrıca, öğretmen adaylarının bu kavram yanılgıları ve zorlukların olası sebepleri hakkındaki bilgileri ve bu zorlukları gidermeye yönelik sundukları stratejilerilerin incelenmesi de amaçlanmıştır. Çalışma, 2004-2005 bahar dönemi sonunda bir devlet üniversitesinde öğretmen yetiştirme programına devam eden 17 son sınıf ilköğretim matematik öğretmen adayı ile amaca dayalı örneklem yöntemi kullanılarak ve nitel durum çalışması yapılarak gerçekleştirilmiştir. Çalışmada son sınıf öğretmen adaylarının seçilmesindeki en temel amaç bu adayların programdaki öğretim seviyeleridir. Diğer bir deyişle, öğretmen adaylarının öğretmen yetiştirme programının öngördüğü tüm dersleri tamamlamış veya tamamlamak üzere olmalarıdır. Böylece, bu katılımcıların, öğretmen adaylarının kesirlerde bölmeye ilişkin kavramlaştırmalarında ne tür bilgi, düşünce ve tecrübenin önemli olduğunu anlamak için önemli potansiyel oldukları düşünülmektedir.

Çalışmada veri toplamak amacıyla, öğretmen adaylarına ilk önce kesirlerde bölmeye ilişkin yazılı sorular sorulmuştur. Ardından, öğretmen adaylarının ilköğretim öğrencilerinin (6. ve 7. sınıf) kesirlerde bölmeye ilişkin sahip olabilecekleri kavram yanılgıları ve karşılaştıkları zorluklar hakkındaki bilgilerini ve bu zorlukları gidermeye yönelik sundukları stratejileri daha derinden incelemek üzere yarı-yapılandırılmış görüşmeler yapılmıştır. Yarı yapılandırılmış görüşme protokolü 3 bölümden oluşmaktadır: (1) Kişisel bilgiler, (2) Bölme işlemine yönelik genel bilgilere (3) Yazılı sorulara verilen cevaplara ilişkin sorular.

Bu çalışmada, öğretmen adaylarının öğrenci olarak tecrübeleri ve öğrencilerin bilişsel düşünme süreçlerine yönelik bilgileri sistematik bir şekilde incelenmiştir. Veri analizi, öğretmen adaylarının öğrencilerinin (6. ve 7. sınıf) kesirlerde bölmeye ilişkin sahip olabilecekleri kavram yanılgıları ve karşılaştıkları zorluklar hakkındaki bilgilerinin 4 ana başlıkta toplanabileceğini göstermiştir. Bunlar, (1) Algoritmik tabanlı hatalar (2) Sezgiye dayalı hatalar (3) Formel bilgiye dayalı hatalar (4) Sorunun yanlış anlaşılmasına yönelik hatalar. Bunlara ek olarak, bu hataların ve kavram yanılgılarının olası kaynakları bu başlıklarla ilişkili olarak tartışılmıştır. Bu başlıklar, yapılan araştırmalar, katılımcıların söylemleri ve veriler kullanılarak oluşturulmuştur.

Bu çalışmanın diğer bir amacı ise, öğretmen adaylarının 6. ve 7. sınıf öğrencilerin kesirlerde bölmeye ilişkin karşılaştıkları zorlukları gidermeye yönelik sundukları stratejileri incelemektir. Yapılan analizler, bu stratejilerin, öğretim yöntemlerine bağlı stratejiler, formel matematik bilgisine bağlı stratejiler, ve psikolojik değerlere bağlı stratejiler olmak üzere 3 ana başlık altında toplanabileceğini göstermiştir.

Matematik öğretmeni yetiştirme programları sürekli olarak öğretmen adaylarının matematik öğrenimi ve öğretimine yönelik kavrayışlarını geliştirmek için neler yapılmalı sorusuyla karşı karşıyadırlar (Crespo & Nicol, 2006). Bu çalışmanın bulguları öğretmen yetiştirme programlarının alan bilgisinin yanı sıra pedagojik alan bilgisine önem vermeleri gerekliliği fikrini desteklemektedir

(Lederman, Gess-Newsome, & Latz, 1994). Öğretmen yetiştirme programları, öğretmen adayların öğrencilerin bilişsel süreçleri ve düşünme yapıları hakkında da bilgiler sunmalıdır (Tirosh, 2000). Diğer bir deyişle, öğretmen yetiştirme kurumlarında öğretmen adaylarının yeterliliklerini artırmak için matematik ve matematik eğitimine yönelik içerik ve pedagoji ağırlıklı derslere yer verilmelidir (Lederman, Gess-Newsome, & Latz, 1994; Crespo & Nicol, 2006). Bu derslerde, öğretmen adaylarının bilgilerini paylaşabilecekleri, iletişim kurabilecekleri, tanım ve kavramlar üzerinde tartışıp yorum yapabilecekleri ve öğrencilerin düşünme şekillerine yönelik bilgiler edinebilecekleri ortamlar sunulmalıdır. Böylelikle, öğretmen adaylarının öğrenim gördükleri öğretmen yetiştirme programından mezun olmadan ve aktif öğretim hayatına geçmeden önce yeterli alan ve pedagojik alan bilgisine sahip olacağı düşünülmektedir.

Sonuçlara dayanarak, ileride yapılacak birçok çalışmaya önerilerde bulunulabilir. Örneğin, öğretim yöntemleri ve uygulama derslerinin öğretmen adaylarının matematikteki birçok konuya ilişkin konu ve pedagojik alan bilgilerini nasıl etkilediklerine yönelik çalışmalar yapılmalıdır. Ayrıca, öğretmen adaylarının alan ve pedagojik alan bilgilerinin öğretmen yetiştirme programındaki eğitimleri süresinde nasıl geliştiğinin incelenmesi onların eksik bilgilerinin tamamlanması bakımından da önem taşımaktadır.