Hacettepe Üniversitesi Eğitim Fakültesi Dergisi 22 : 82-90 [2002]

[J. of
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EFFECTS OF COOPERATIVE AND INDIVIDUALISTIC PROBLEM SOLVING METHODS ON MATHEMATICAL PROBLEM SOLVING PERFORMANCE

İŞBİRLİĞİNE DAYALI VE BİREYSEL PROBLEM ÇÖZME YÖNTEMLERİNİN MATEMATİKSEL PROBLEM ÇÖZME PERFORMANSINA ETKİSİ

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ABSTRACT: The purpose of the present study was to investigate the effects of the cooperative problem solving method (CPSM) and the individualistic problem solving method (IPSM) on seventh grade students' mathematical problem solving performance (MPSP). In this quasi-experimental research study, seventh grade "percents unit" was covered. After analyzing the data by using the multivariate analysis of covariance it was found that CPSM and IPSM groups had statistically significantly greater mean scores than the traditional method (TM) group with respect to MPSP. However, there was no significant mean difference between CPSM and IPSM groups with respect to MPSP.

KEY WORDS: Mathematical problem solving performance, cooperative problem solving method, individualistic problem solving method, percent units

ÖZET: Bu çalışmanın amacı işbirliğine dayalı ve bireysel problem çözme yöntemlerinin etkilerini incelemekti. Yarı-deneysel olan bu araştırmada, 7. sınıf "yüzdeler ünitesi" kapsanmıştır. Veriler çok yönlü kovaryans analizi ile analiz edildikten sonra, matematiksel problem çözme performansları açısından işbirliğine dayalı ve bireysel problem çözme yöntemleri gruplarının ortalamaları, geleneksel yöntem grubunun ortalamasından istatistiksel olarak anlamlı bir şekilde daha yüksek bulunmuştur. Bununla beraber, problem çözme yöntemi kullanılan grupların ortalamaları arasında matematiksel problem çözme performansları açısından anlamlı bir farklılık yoktur.

ANAHTAR SÖZCÜKLER: Matematiksel problem çözme performansı, işbirliğine dayalı problem çözme yöntemi, bireysel problem çözme yöntemi, yüzdeler ünitesi

1. INTRODUCTION:

People should have mathematical problem solving skills and ability to work cooperatively [1]. Many countries have emphasized the development of these skills in school mathematics courses; for example, the United States, The United Kingdom and Japan [2]. In the meta-analysis of Qin, Johnson and Johnson about cooperative and competitive problem solving efforts, it was reported that in every part of life, people working in cooperative groups show better performance in solving complex problems than others working in individualized settings [3]. Most of the international research studies about mathematical problem solving have focused on only students' individualistic performance [4]. They concluded that students' mathematical problem solving performance was increased by using heuristics strategies. Duren and Cherrington investigated the effects of cooperative group work versus individualistic effort on the learning of problem solving strategies [5]. They summarized that students in cooperative groups were more active in problem solving process and more open to solve the problems in different ways. Therefore, problem-solving method can be used with cooperative learning. This is called "cooperative problem solving" that enables students to work together in learning and applying higher level thinking skills and leads to higher achievement in solving mathematical word problems [6]. The positive effects of cooperative learning on mathematical problem solving performance were reported by

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many researchers [7]. The use of a structured problem solving plan might have a positive effect on mathematical problem solving performance[8]. According to results of a couple of studies, there was a consensus on Polya's assumption [9], that is, heuristic teaching method had a positive influence on students' mathematical problem solving performance [4,10]

There is no enough study on the development of mathematical problem solving skills in Turkey [e.g.11,12]. Consequently, the purpose of this study was to investigate the effects of cooperative and individualistic problem solving methods by utilizing Polya's problem solving stages on students' mathematical performance.

2.METHOD

2.1. Problem and Hypothesis

The problem of the present study is "What is the effect of different teaching methods on students' mathematical problem solving performance?". The hypothesis is stated as "$H_0$1: There are no significant differences among the mean scores of the students taught by the cooperative problem solving method and those taught by individualistic problem solving method, and those taught by the traditional method with respect to mathematical problem solving performance".

2.2. Research Design

In this study we utilized the quasi experimental design. There were three groups and two of them were experimental groups: one group was taught by cooperative problem solving method (CPSM); another was taught by individualistic problem solving method (IPSM). The control group was taught by traditional method (TM). Both Mathematical Problem Solving Performance Test (PSPT) and Decimal Number Test (DNT) were administered as pre-tests. The PSPT was also administered as the post-test.

2.3. Subjects of the Study

The subjects of the present study consisted of 79 conveniently sampled seventh grade students in an elementary school in Ankara, Turkey. There were 25 students in the CPSM group, 24 students in the IPSM group and 30 students in the TM group. The study was carried out during the spring semester of 1997-1998 academic year. The experimental groups were taught by the first author; however, the traditional method (TM) group was instructed by the regular teacher.

2.4. Instruments

Mathematical Problem Solving Performance Test (PSPT) was developed by the researchers to determine the students' mathematical problem solving performance (MPSP) on the "percents unit", including percent, profit, commission, discount and interest concepts. It was used as pre and post-tests, but the numbers and the names in the pre-test were changed in the administration of the post-test in order to prevent recalling. The test assesses various types of mathematical problem solving performance, including finding extra information in a problem, finding mathematical mistake in a problem, giving a value to the missing information, writing a problem composed of the given information, filling in the blanks in a mathematical sentence, organizing given data, solving a given problem, interpreting data on a graphic or a table, and making logical interpretations. Type of the items in the test can be listed as follows: 1.Restricted response; 2.Interpretive; 3.Fill in the blank; 4.Interpretive, Alternative-response; 5. Short answer; and 6.Extended response essay.

A pilot study of the PSPT was conducted with twenty-two items, and seven of them were eliminated after the pilot study. In addition, each item had a different weight and was graded by giving partial credits for each significant step [13]. Two mathematics teachers and a mathematics educator examined the test and reported that they collected content-related evidences of validity; for example, they presented that the test covers instructional objectives. The PSPT did not contain objective-type test items, so the rater reliability was investigated to eliminate the subjectivity. In order to reduce subjectivity, an answer key was prepared. Concerning inter-reliability, both the first author and a mathematics educator scored the test administered in the pilot study separately. The correlation coefficient between the two scorings was found as 0.99.

2.4.1. Activity Sheets Based on the Theory of Problem Solving

The major curriculum materials used throughout the treatment were the activity sheets developed by the first author. Those sheets covered problems on "percents unit" consisting of the percent concept, discount, commission, profit and interest. There was one problem on each activity sheet and the problems included the eight steps of the problem-solving plan with space for student work. The steps for "understanding" the problem are; writing the given data, paraphrasing the problem, and listing the unknown facts. Steps for "devising a plan" were drawing the diagram and choosing the operations with their reasons. "Carrying out the plan" involved making the operations. "Looking back and extending" was operationalized as checking the computations and using another way to solve the problem. The activity sheets were prepared so that students could actually pass through Polya's problem solving stages [9] and use heuristic problem solving strategies. The problems were prepared by the first author and two mathematics teachers.

2.4.2. Monitoring Form Based on the Theory of Cooperative Learning

We used the Cooperative Learning Monitoring Form (CLMF) developed by Bulut [14] in order to make sure of the occurrence of cooperative learning. The items of the CLMF were in a five-point Likert-type scale: Always, often, sometimes, rarely and never. Items were coded starting from "always" as "4" to "never" as "0". The form was prepared to evaluate components of cooperative learning stated by Johnson and Johnson [15].

2.5. Treatment of Groups

Three classes were selected at an elementary school in Ankara. They were conveniently assigned to different teaching methods as the CPSM, the IPSM and the TM. However, all the groups were taught the same content to reach exactly the same objectives in cognitive domain. In addition, the same problems were solved in all treatment groups throughout the instruction.

The instruction of the control group was the traditional method (TM) in which students were taught in a teacher-centered way. In this group the problems were solved traditionally; in other words, the teacher was writing the problem on the board and then students attempted to solve it. This group received 320 minutes of instruction during three weeks, and a forty-minute period of this instruction was observed by the first author in order to examine how the lecture was taught.

The students taught by the cooperative problem solving method (CPSM) were heterogeneously divided into groups of 3 or 4 in terms of gender, ability and personality type with the help of their regular teacher. Four of the groups composed of four members, but three groups contained three students. Groups found a name for themselves in order to have them had group feelings. Before the treatment, the regular instructor explained the purpose of the treatment, the procedure to be followed, expected collaborative behavior, and definition of group success to the students. She also informed students about the goals of problem solving and Polya's problem solving stages. When the treatment began, the instructor presented basic concepts and processes covered by the "percents unit" in twenty minutes. Then, she handed out the same problem on an activity sheet to each group. Next, solved problems were collected back; however, students copied group solution to their notebooks. The first group completed the solution was given the right to solve the problem on the blackboard, but a member of that group was chosen randomly to solve the problem on the blackboard to make students understand how to solve the problem. If the problem solved correctly by the group member, that group would get plus (+), otherwise minus (-). If the solution was not correct, a member of another group was asked to present the group's solution respectively until a group presented the correct solution. When the treatment finished, the group with the most plus signs was declared as the champion group whose members were given championship certificates as a reward. Groups were monitored by their group observers, the regular teacher, observer and students own evaluation by using CLMF. The instruction of the CPSM lasted 360 minutes during three weeks.

Like the implementation of the CPSM, the instruction of individualistic problem solving method (IPSM) lasted 360 minutes during three weeks. When the treatment began, the instructor gave some information on "percents unit" in twenty minutes. The instructor handed out the same activity sheet used in the CPSM group to each individual. Each student individually worked on the activity sheets and solved the problems according to Polya's problem solving stages. Teacher encouraged students to pass through all the stages during problem solving. Students were asked to work on their own, to avoid interaction with other students, to seek help and assistance from only the instructor, to work at a self-regulated pace, and to complete as much of the assignment as possible. The instructor also guided each individual during the treatment and monitored the individuals. While the first author was teaching the group, the regular teacher of the classroom also observed the class. She confirmed that the problems were solved according to Polya's problem solving stages. In addition, she declared that students worked individually and did not interact with each other.

3. RESULTS

At the beginning of the treatment, the Mathematical Problem Solving Performance Test (PSPT) and the Decimal Number Test (DNT) were administered as pre-tests. In addition, the subjects' mathematics and Turkish grades in the fall semester were obtained. Then, the equivalences of the treatment groups were tested in terms of these pre-treatment measures by using Multivariate Analysis of Variance (MANOVA).

According to multivariate tests of significance, Hotellings T^2 revealed that there was overall significant difference among the mean scores of students taught by the CPSM, those taught by IPSM and those taught by the TM (Hotellings T^2 = 0.391, $p< 0.05$) with respect to pre-treatment measures. However, the univariate F-test followed in MANOVA procedure indicated no significant effect on pre-treatment measures with respect to treatment groups at significance level 0.05 as indicated in Table 1. For the purpose of understanding which groups created the significant effect joint univariate Bonferroni t-test were obtained i Table 2.

Table 1: Results of Univariate F-tests for Pre-Treatment Measures

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	Sig. of F
PSPT	940.698	13948.340	470.349	183.531	2.563	0.084
DNT	369.884	7419.660	184.942	97.627	1.894	0.157
MATH	5.242	109.973	2.621	1.447	1.811	0.170
TURKISH	8.006	103.792	4.003	1.366	2.931	0.059

Table 2: Results of Joint Univariate Bonferroni t-Test of the Prior MPSP

Ref.	Parameter	Coeff.	Std. Err.	t-Value
TM	CPSM	-4.187	3.669	-1.141
	IPSM	-8.383	3.710	-2.260*
CPSM	IPSM	-4.197	3.871	-1.084
	TM	4.187	3.669	1.141

* $p < 0.05$

As seen in Table 2, only significant mean difference was found between students taught by the IPSM and those taught by the TM with respect to mathematical problem solving performance ($p < 0.05$). Therefore, prior MPSP was taken as a potential covariate in order to remove its variance from the variances of post-PSPT across the experimental and control groups.

As Table 3 indicated, there was no significant difference among the mean score of students taught by CPSM, those taught by the IPSM and those taught by the TM with respect to decimal number achievement (DNA). Although there was no significant mean difference with respect to DNA, it was taken as a potential covariate in order to take its variance from the variances of post-PSPT because decimal numbers were prerequisite knowledge for "percents unit". Hence, some of the research study showed that there were problems in teaching/learning decimal numbers [16]. The means and standard deviations of pre-treatment measures are given in Table 4.

The post-PSPT had significant correlations with MPSP on pre- PSPT ($r=0.535$) and DNA ($r=0.854$) at 0,05 level. Consequently, DNA and MPSP were taken as covariates for post-PSPT.

As seen in Table 5 after testing the hypothesis, it was found that there were significant differences among the mean scores of students taught by the CPSM, those taught by the IPSM and those taught by the TM on post-score of the PSPT after analysis of covariance (ANCOVA) was accomplished with covariates prior MPSP and DNA ($p < 0.05$).

Table 3: Results of Joint Univariate Bonferroni t-Test of DNA

Ref.	Parameter	Coeff.	Std. Err.	t-Value
TM	CPSM	2.393	2.676	0.895
	IPSM	5.267	2.705	1.946
CPSM	IPSM	2.873	2.824	1.018
	TM	-2.393	2.676	-0.895

Table 4: Means and Standard Deviations of Pre-Treatment Measures

	$\overline{X}$			SD		
	CPSM	IPSM	TM	CPSM	IPSM	TM
PSPT	31.280	27.083	35.467	13.655	15.717	11.434
DNA	29.960	32.833	27.567	11.396	10.007	8.303
MATH	2.920	2.583	2.300	1.187	1.349	1.088
TURKISH	3.400	2.625	2.833	1.291	1.135	1.085

Table 5: Analysis of Covariance of the Post-PSPT Scores of Treatment Groups

Source of Variation	SS	MS	df	F
Within+Residual	10983.00	148.42	74	
Regression	6334.44	3167.22	2	21.34*
Group	2389.23	1194.62	2	8.05*
Model	9438.95	2359.74	4	15.90*
Total	20421.95	261.82	78	

* p< 0.05

Table 6 summarizes standard deviations, observed and adjusted mean scores of the post-PSPT across the experimental and control groups. As seen in Table 6, the IPSM group had the highest adjusted mean score of post-PSPT (X_A=46.141). The CPSM group had the second highest adjusted mean score of post-PSPT X_A=43.415). The TM group had the lowest adjusted mean score of post-PSPT (X_A=32.348). The PSPT was out of 112. In order to understand which pairs of treatment groups created the significant mean difference on post-PSPT, Joint univariate Bonferroni t-test results were obtained as in Table 7.

From Table 6 and Table 7, it can be seen that the students taught by the CPSM and the IPSM scored significantly better than the students taught by the TM with respect to MPSP. It can also be seen that there was no significant difference between the mean scores of the students taught by the CPSM and those taught by the IPSM with respect to MPSP.

4. DISCUSSION

In the present study, it was found that there were significant mean differences among the students taught by the CPSM, those taught by the IPSM and those taught by the TM with respect to mathematical problem solving performance. After the data were analyzed to determine which pairs of group had a significant mean difference, it was found that there was no significant mean difference between students taught by the CPSM and those taught by the IPSM with respect to MPSP (p>0.05). The reason of this finding could be that both experimental groups were trained on problem solving stages stated by Polya in 1957. This result was consistent with the findings of

Table 6: Means and Standard Deviations of the Post-PSPT Scores

Variable	Groups	N	$\overline{X}_O$	$\overline{X}_A$	SD
PSPT	CPSM	25	43.320	43.415	14.713
	IPSM	24	46.417	46.141	11.201
	TM	30	32.167	32.348	17.846

Table 7: Joint Univariate Bonferroni t-Test of the Post-PSPT with respect to Treatment Groups.

Ref.	Parameter	Coeff.	Std. Err.	t-Value
TM	CPSM	11.067	3.404	3.251*
	IPSM	13.793	3.767	3.662*
CPSM	IPSM	2.726	3.599	0.758
	TM	-11.067	3.404	-3.251*

* p< 0.05

research studies [12,17]. Georgas revealed that there was no significant difference between cooperative and average individualistic goal structures on mathematical problem solving performance [17]. In another study, Tuncer found that there was no significant mean difference among individual, homogeneous pair and heterogeneous pair groups with respect to mathematics achievement [12]. The result of the present study was not consistent with the result of Duren and Cherrington which revealed that cooperative learning promoted mathematical problem solving performance better than individualistic learning [5]. The reason for this inconsistency could be that Duren and Cherrington investigated only four problem solving strategies: Drawing a picture or diagram, making a table, making it simpler and working backwards, but in the present study Polya's problem solving stages which included more strategies than Duren and Cherrington's strategies were utilized.

According to another finding of the present study, the students taught by the CPSM and those taught by the IPSM had significantly higher mean scores on the PSPT than those taught by the TM with respect to MPSP ($p<0.05$). The explanation of this particular finding could be that unlike the students in the TM group, the students of the CPSM and of the IPSM groups solved the problems by passing through problem solving stages step by step. However, they were not familiar to the questions asked in PSPT. This second result of the present study is consistent with the results of many research studies that reported that teaching problem solving through heuristics improved MPSP better than teaching through traditional learning [4,18]. Heuristics were also found to be positively effective on students' MPSP in cooperative and individualistic settings [19].

Treatment groups of the present study could have low mean scores because the duration of the treatment was short to improve the problem solving skills (i.e. duration was 3 weeks). They could have showed better performance on mathematical problem solving than their current performance if the treatment would last longer because, as Lester stated, problem solving ability develops slowly over a long period of time [2]. In addition, the problems in the PSPT were challenging and new situations for the students because most of them were generally used to solving textbook questions or examples. During the study, students tried to use heuristic strategies such as writing the data and unknown, drawing a suitable table or diagram, restating the problem or checking each step. The first author's observations through the CPSM and the IPSM instruction revealed that students experienced difficulties while using those strategies; for example, most of them could draw tables or graphs.

It was revealed that all statistically significant mean differences on the dependent variable among treatment groups had also medium practical significance because effect size was found as 0,62 (refer to Cohen and Cohen [20] for the formula of effect size). As a result, a medium effect size, f^2, was used in power analysis. The power value was found as 0.85 by using the formula stated by Cohen and Cohen [20]. Thus, the probability of rejecting a false null hypothesis of the present study was 85%, and Beta (b) (probability of failing to reject a false null hypothesis) was 0.15.

Experimental and control groups of the present study were monitored during the instruction. Students taught by the CPSM were seemed happy and being busy with the activities, discussing with each other and teaching each other. The instructor and students together painted a rich picture of cooperative work. The students taught by the IPSM were also interested in their activities. They asked the teacher when they needed help. During the CPSM and IPSM instruction, the regular teacher of these groups observed that the CPSM improved students' active participation more than the IPSM by the observer. On the other hand, the subjects in the control group did not show evidences of such interest and activity during the instruction. Therefore, these observations might be effected by the results of the study. The CPSM group's regular teacher stated that cooperative learning activities increased students' active involvement during the CPSM instruction. Besides, she mentioned that cooperative learning could improve not only low ability students' but also high ability students' mathematical problem solving performance. Passing through Polya's stages was also stated as an effective tool during problem solving process.

V. RECOMMENDATIONS

During problem solving process, students should pass through Polya's stages. Students and teachers should believe that problem solving is more than finding the correct answer. Moreover, students should be active in the teaching/learning process physically as well as mentally. During the CPSM instruction, students were enjoyed learning through cooperative learning. Moreover, cooperative learning increased students' active involvement. Thus, this study showed that active learning methods produced higher mathematical problem solving performance than the TM does.

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