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Research Article

Misconception Analysis of Junior High School Student in Interpreting Fraction

Winda RAMADIANTI[1,2], Nanang PRIATNA[1], KUSNANDI[1]

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Abstract
Interpretation of fraction is the basis of meaningful fraction learning. Students' misconceptions in interpreting fractions can be obstacles for students to understand further mathematical concepts. The purpose of this study is to identify students' misconceptions in interpreting fractions. This research usedcontent analysismethode. The participants of this study were 63 students (32 female, 31 male) in one junior high school in Bengkulu City, Indonesia. This research participant has just studied the topic of fractions. Data collection was carried out by giving tests to 63 participants, and interviews with 6 students to obtain more comprehensive information. The questions asked in interview are based on students' answers on the previous test. The selection of students interviewed was done by purposive sampling. The results showed that students experienced misconceptions in interpreting fractions as part of whole, as quotient, as ratios, as operators, and as measures. Based on the results of the interview, the misconception occurred because students experienced limited context in recognizing fractions.
Keywords:
misconception, fraction, part of whole, quotient, ratio, operator, measures

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[1]Study Program of Mathematics Education, School of Postgraduate Universitas Pendidikan Indonesia, Bandung, Indonesia. [2]Study Program of Mathematics Education, Universitas Muhammadiyah Bengkulu, Indonesia. E-mail: winda.ramadianti@gmail.com, ORCID ID:0000-0001-9840-1192; Corresponding Author: nanangpriatna@upi.edu, ORCID ID:0000-0003-2294-745X; kusnandi@upi.edu,ORCID ID:0000-0002-8900-4563

Introduction

Mathematical knowledge is hierarchical (Nakamura, 2014). Mathematical knowledge is taught in stages which means that a concept can be a prerequisite for other concepts. Therefore, understanding concepts for basic topics must be highly considered in mathematics learning. On the other hand, most mathematics learning in schools focuses on learning about rules, procedures, and formulas to get the correct answer from a problem rather than planting basic concepts for students (Sarwadi & Shahrill, 2014). This can result in misconceptions among students. According to Makonye, misconceptions are false beliefs and principles that underlie a person's mind, causing a series of errors (Makonye & Fakude, 2016). Meanwhile, Ojose believes that misconception is a misunderstanding and misinterpretation (Ojose, 2015). Some researchers looked at the results of mathematics learning as having not been satisfactory (Huda et al., 2019). This can be caused by a misconception in the previous concept. Misconceptions that occur in previous learning can cause obstacles in students in learning concepts that are ongoing, resulting in low student mathematics learning achievement (Mohyuddin & Khalil, 2016). Therefore, misconceptions on basic topics must be overcome.

One of the basic topics in mathematics is fraction. Fraction is not only widely applied in everyday life but is also very important for the development of advanced mathematics and other sciences (Bailey, Siegler, & Geary, 2014; Jannah & Prahmana, 2019; Siegler et al., 2012). Some previous studies indicated that misconceptions occur in fractions. Students experience misconceptions in the adding fractions with different denominators(Ghani & Maat, 2018).Some research results indicate that many students fail to understand the relationship between the numerator and the denominator (Resnick et al., 2016).

The relationship between the numerator and denominator interpret the meaning of a fraction. The meaning of fractions was first proposed by Kieren (1976)(Charalambous & Pitta-Pantazi, 2005; Doyle, 2016; Lazić, Abramovich, Mrđa, & Romano, 2017). Initially Kieran proposed four fractional subcontracts are quotient, ratio, operator, and size, while part of the whole is the basis for the development of other sub-constructs (Charalambous & Pitta-Pantazi, 2005; Lazić et al., 2017). Furthermore, based on Kieran's opinion, in 1983, Behr, et.al recommended that parts of the whole consist of different subcontractions and they also make theoretical models that relate the interpretation of fraction to operations on fraction (Charalambous & Pitta-Pantazi, 2005). The theoretical model is presented as shown in Figure 1 below.

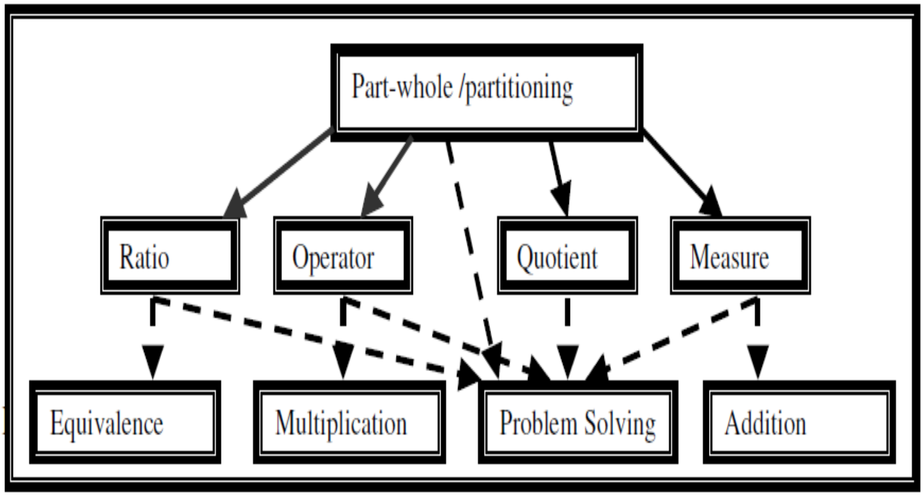


Figure 1.
The Theoretical Model of Fraction Interpretation Developed by Behr, et.al (1983)
Source: (Charalambous & Pitta-Pantazi, 2005)

Part of the whole is the interpretation of fraction that starts learning about fraction (Simon, Placa, Avitzur, & Kara, 2018). Some experts justify that this happens because part of the whole is considered the basis for developing an understanding of the other four subcontracts (Charalambous & Pitta-Pantazi, 2005). Interpretation of a part of the whole refers to a quantity that is partitioned into several equal parts, and fractionis the ratio between the number of the same parts chosen and the sum of all parts (Wijaya, 2017). Fraction as part of a whole does not only mean part of one object but also part of a group of objects (Lamon, 2012). In this interpretation, "the same" does not mean the same shape and size (congruent), but the same is in certain properties such as area, volume, number (Chapin & Johnson, 2006). In this interpretation the numerator must be smaller or equal to the denominator (Charalambous & Pitta-Pantazi, 2007).

Some experts argue that in the interpretation of fraction as quotient there are two meanings, namely the division and the amount received by each recipient (Mamede, Nunes, & Bryant, 2005). For example, 4/3 can mean the distribution of four chocolates to three people, and also a lot of chocolate is obtained by each child. Unlike the part of the whole, in this interpretation the numerator can be smaller, bigger, or the same as the denominator (Charalambous & Pitta-Pantazi, 2007).

Fraction can also be interpreted as ratios, i.e. comparisons between two quantities (Chapin & Johnson, 2006; Doyle, 2016; Lamon, 2012). The type of quantity being compared does not have to be the same. However, comparisons that can be expressed in fractions are comparisons between parts and whole.

Whereas, the comparison between parts and parts is a ratio that cannot be written in the form of $\frac{a}{b}$ fractions (Lamon, 2012). Meanwhile, comparisons between different quantities can be rate (Lamon, 2012). For example, 5 km per hour is a rate that explains the ratio between distance and time.

Fraction as measures can be illustrated on the number line. Fraction is the distance from point 0 to a certain point obtained by partitioning a unit into several subunits (Chapin & Johnson, 2006; Lamon, 2012). In this interpretation, $\frac{1}{n}$ and $\frac{m}{n}$ have meanings that are independent of the whole (Simon et al., 2018). Interpretation of fractions as measures provides an alternative way of introducing fractions. This has been developed in the Japan text series, which focuses on fraction learning not with model areas or discrete models, but with measurements of fluid length or volume (Simon et al., 2018).

Fraction as operators is applied in multiplication operations. The interpretation "$\frac{m}{n}$ of" means the command to multiply by m and divide by n (Doyle, 2016; Lamon, 2012).

Problem of Research

The results of previous studies indicate that there are failure of students in understanding the relationship between the numerator and the denominator, and conducting fraction operations. This indicates the students' misconception in interpreting fractions, because the interpretation of fractions is related to the relation of the numerator and denominator, and fraction operation. Incorrect interpretation can cause students' learning obstacle to fractions and learning fractions that are not meaningful. The main purpose of this study is to analyze the misconceptions of students' interpretation of fraction.

Method

This research is a qualitative descriptive study. Content analysis methods are used to analyze students' misconceptions when interpreting fractions as part of the whole, fraction as quotient, fraction as ratios, fraction as operators, and fraction as measures.

Participants

This research was conducted at a junior high school in Bengkulu City, Indonesia, in 2019. This study involved 63 7th grade students who had just studied the topic of fraction. Participant feature demographics are presented in table 1 below.

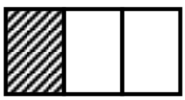
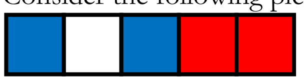
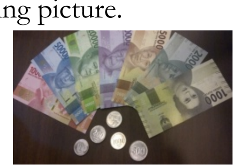
Table 1.
Demographic Features of the Students Participating in the Study

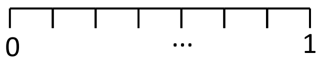
Variable	Demographic feature	f	%
Gender	Female	32	50,8
	Male	31	49,2

Data Collection

Data collection techniques used in this study were the provision of tests to all participants. The questions on the test are based on five interpretations of fractions. Each fraction interpretation is represented by one or two questions. The following tests are given to students.

Table 2.
Test About the Interpretation of Fractions

Fractional Interpretation	Question
Fractions as part of a whole	**Problem 1** Consider the following picture.  The shaded portion is stated as a 1/3 fraction. Make two other examples that can also be expressed as fractions 1/3.
	Problem 2 Consider the following picture.  Does the blue part say 2/5 fraction? Give your reasons.
Fraction as the quotient	**Problem 3** The teacher brings 4 cake pans to be distributed to 3 groups of studentsin order that each group got many of the same cakes. Can many cakes obtained by each group be classified as fractions? Give your reasons.
Fraction as a ratio	**Problem 4** Consider the following picture.  can a comparison between lots of coins and banknotes be expressed as fractions? Explain your answer.

Fraction as operator	**Problem 5** Ibu Rita buys a sack of 5 kg of rice at the store. After she arrives at home, a quarter sack of rice is used to make the diamond. How many kg of rice are used to make rhombus?
	Problem 6 Anto is recovering. The doctor asked him to drink 1/2 liter of water every hour. How much water does Anto drink in 7 hours?
Fraction as a measure	**Problem 7** Consider the following number line.  Find the right fraction to fill the points in the number line above.

Next, the researchers conduct interviews with several students to find out more about the participants' reasons. The selection of participants interviewed is done by purposive sampling with regard to student answers. Based on the scores of students' test results, the researchers chose 6 participants consisting of 2 participants with a high score, 2 participants with a moderate score, and 2 participants with a low score.

Data Analysis

Analysis of research data using content analysis methods.

Table 3.

Step of Data Analysis

Step	Activity
First	All students' answers for each answer are identified and arranged by number of questions.
Second	The researcher classifies the existing misconceptions for each answer.
Third	The researcher does scoring for each student.
Fourth	The researcher chose six students to be interviewed. The selection is based on the scoring results
Fifth	All information submitted by students is recorded and classified according to research objectives
Sixth	Made conclusions based on classification

Results

Based on the results of the classification, there are several variations of students' answers to each question. The following answers appear for each question.

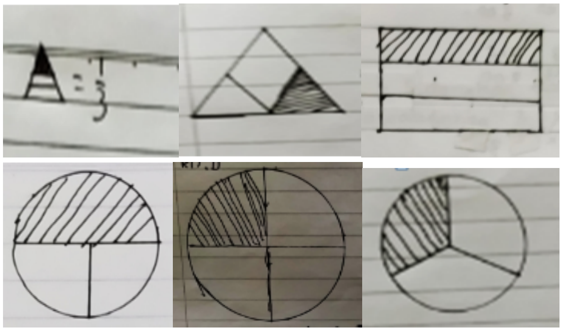


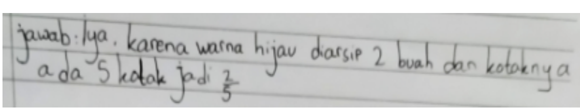
Figure 2.

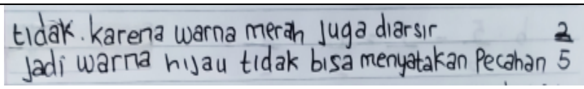
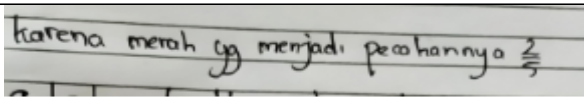
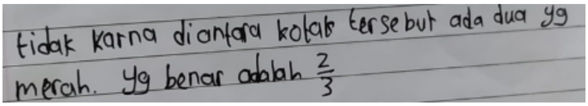
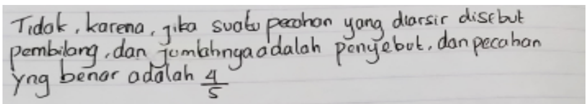
Student' Response to Problem 1

All students' answers to the first question used flat shapes (ie circles, rectangles, and triangles), which were partitioned into several parts and then shaded one part. There were several different ways of partitioning by students, namely (1) based on the number of partitions, there were students who partitione into 3 parts and there were those who partitione into 4 parts; (2) based on partition size, there were those that partitione into parts of the same size, and there were those that partitione into parts of different sizes. Based on the results of the interviews, students with high scores argued that to declare fractions 1/3 was interpreted by a flat figure which was partitioned into three equal parts and then one shaded part. The shaded portion show 1/3 fractions. Meanwhile, students who scored low thought that the size of the partition did not have to be the same. The important thing is that the flat shape must be partitioned into three parts and shaded in one part.Then, the shaded part is declared as a 1/3 fraction. For answers that partition into four equal parts and shaded one part, the researcher cannot dig further information because there were obstacles during the interview process. The researcher suspecte that students thought 1/3 is interpreted as 1 shaded part and 3 unshaded parts.

Table 4.

Student Responses to Problem 2

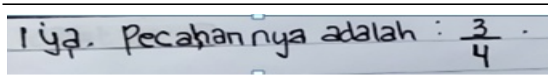
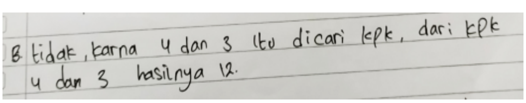
Students' Respones	Translation
 jawab: Iya. karena warna hijau diarsir 2 buah dan kotaknya ada 5 kotak jadi 2/5	Yes, it is because green color is shaded for 2 pieces, and it has 5 boxes. by karen, the box become 2/5
	No, it is because red color is

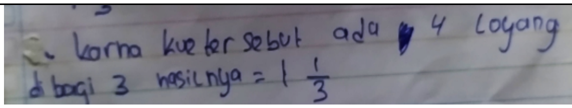
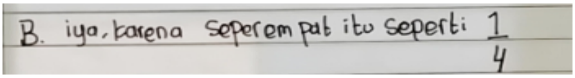
 tidak. karena warna merah juga diarsir jadi warna hijau tidak bisa menyatakan Pecahan 2/5	also shaded to be green color and cannot declare fraction 2/5
 karena merah yg menjadi pecahannya 2/5	It is because red color that becomes fraction 2/5
 tidak karna diantara kotak tersebut ada dua yg merah. yg benar adalah 2/3	No, it is because among the boxes there are two reds. Thereforee, the correct one is 2/3
 Tidak, karena, jika suatu pecahan yang diarsir disebut pembilang, dan jumlahnya adalah penyebut, dan pecahan yng benar adalah 4/5	No, it is because if a shaded fraction is called a numerator, and the number is the denominator, the correct fraction is 4/5

The second problem is a closed problem, but the reasons given for that answer can vary. The correct answer to this problemis "yes" because in the context of the interpretation of fractions as part of the whole, the part that must be considered is the many parts considered (in this case the green part) and the sum of all parts. For "yes" answers, the reasons stated were also true. Meanwhile, for "no" answers, the reasons given is different. Although different, all of these reasons occurred because there were two other parts that were also red. There were students who consider all the parts that were colored. Therefore, answer was 4/5. The researcher also asked students about their experience of learning fractions. According to all students interviewed, this is the first time they encounter a problem like the second problem. According to students, when learning the fraction, illustrations given by the teacher was as problem 1.

Table 5.

Student'Responses to Question 3

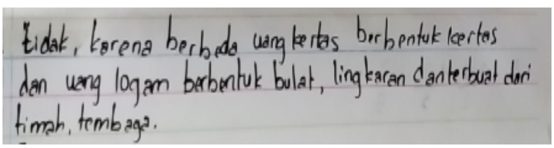
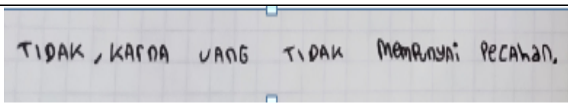
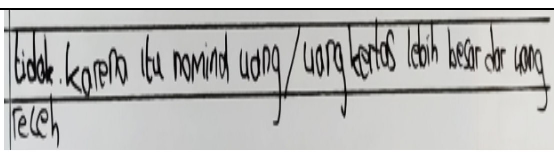
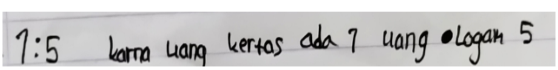
Students' Answer	Translation
 Iya. Pecahannya adalah : 3/4 .	Yes, the fractionis ¾
 B tidak, karna 4 dan 3 itu dicari kpk, dari kpk 4 dan 3 hasilnya 12.	No, it is because 4 and 3 are sought by the Least Common Multiple (LCM). The LCM of 4 and 3 are 12
	Because there are 4 baking pans, the results are

 karna kue tersebut ada 4 loyang dibagi 3 hasilnya = $1\frac{1}{3}$	divided 3 into $1\frac{1}{3}$
 B. iya, karena seperempat itu seperti $\frac{1}{4}$	Yes, because a quarter is like ¼

Similar to the second problem, the third problem from around the student's answer is "yes or no", but the reasons given were quite diverse. This related to the results of students' understanding of fractions as a quotient. The correct answer to this question is "yes" because a lot of cakes obtained by each group can be agreed with $\frac{4}{3}$.

Table 6.

Students' Responses to Problem 4

Students' Responses	Translation
 tidak, karena berbeda uang kertas berbentuk kertas dan uang logam berbentuk bulat, lingkaran dan terbuat dari timah, tembaga.	No, it is different because banknote is in the form of paper, and coins are round, circular and made of lead, copper.
 TIDAK, KARNA UANG TIDAK MEMPUNYAI PECAHAN.	No, it is because money doesnot have fractions
 tidak. karena itu nominal uang / uang kertas lebih besar dari uang receh	No, it is because the nominal banknotes are greater than coins.
 7:5 karna uang kertas ada 7 uang logam 5	7: 5 is because there are 7 banknotes and 5 coins.

Problem 4 aims to see students' understanding of fractions as a comparison. the correct problem for this question is "no". Most students answer "no" and gave several reasons. The reason is related to different forms of money.Therefore, some students consider the two things that can not be compared. Based on the results of the interview, students view coins and banknotes as a different matter. According to them, an example of comparison that can be expressed as a fraction is the comparison of age A and age B.

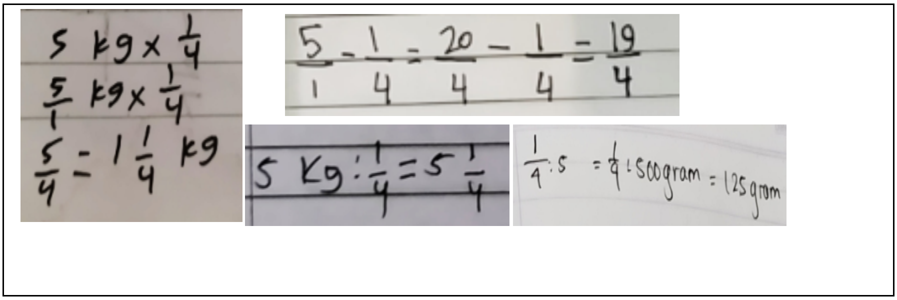


Figure 3.
Student Responses to Problem 5

Problem 5 aims to see students' understanding of fractions as operators. In the given word problem, ¼ is an operator that is run at number 5. From some student answers, many students were wrong in solving the multiplication problem of the fraction. They cannot make mathematical models that show that ¼ is an operator.

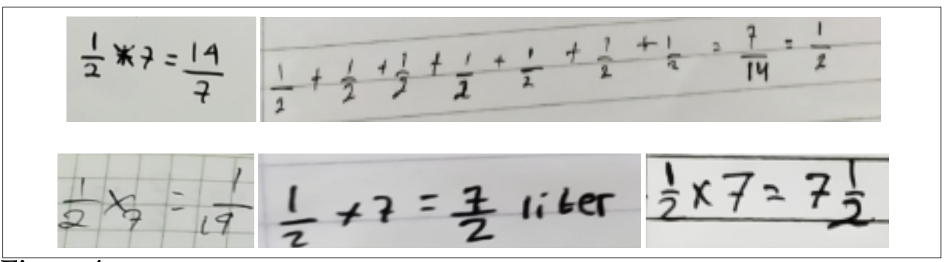


Figure 4.
Student Responses to Problem 6

In contrast to the fifth question, fraction ½ in the seventh question is not an operator, but rather as a measure. Therefore, the right mathematical model is $7 \times \frac{1}{2}$. From students' answers, there are students who are mistaken in writing mathematical models. In addition, there are also students who are wrong in doing multiplication operations.

From the scoring results, the seventh question is the question that was answered at least correctly by students. The correct answer to this question is $\frac{4}{7}$ while the studentis wrong answer is 4; 0.4; 0. At the interview, students who answered correctly can explain their reasons why they answer $\frac{4}{7}$. They say that, from 0 to 1, itis partitioned into seven parts, and the fraction in question is on the fourth partition.Therefore, the fraction in question is $\frac{4}{7}$. Meanwhile, students who gave incorrect answers, at the time of the interview seemed confused in giving reasons. Students who answered "4" said they only counted the number sign from 0, and the part that was asked was on the 4th sign. The same reason was given by students who answered 0.4, but they considered that 0.4 is also a fraction, which is a decimal fraction. While students who answer0 cannot give reasons, because they only guess the answers.

Discussion

Based on the results of the interview, all students have experience in learning fractions while in elementary school, and they actually have knowledge about fractions. However, from students' answers to problems given by researchers, students who do not understand about fraction interpretation are more than students who understand. From the research findings, there are several misconceptions experienced by students in each fraction interpretation.

According to some studies, the interpretation of fractions as part of the whole is the interpretation most mastered by students (Kolar, Hodnik Čadež, & Vula, 2018). Likewise in this study, the number of students who answered correctly for questions related to this interpretation was more than the other interpretations. However, there is still a misconception in this interpretation. In interpreting fractions as part of a whole, misconceptions that occur in students are the size of partitions that do not have to be the same. Students who experience this misconception assume that only many partitions are considered. This is a misconception because in the interpretation of fractions as part of the whole there are two things to consider namely many partitions and partition sizes. In addition, students also experience misconceptions in interpreting fractions as part of a whole with different problem contexts. Students understand this interpretation in the context of something that is partitioned into several partitions, and fractions are shaded or colored parts. Students also consider partitions that can be determined as fractions must be side by side, as students' answers to the second question.

In interpreting fractions as quotient, there are still some students who experience misconceptions.Some students even think that the context of four cakes divided for three people is not fractions. There are also students who understand that the context in the third question is the interpretation of fractions as quotient, but they experience a misconception about the role of the numerator and denominator in a fraction. In the interpretation of fractions as quotient, the numerator is a shared ally, and the denominator is a divisor, but students understand the opposite.

Misconceptions also occur in students when interpreting fractions as ratios. Actually, students can identify that the statement on the matter about the comparison of many coins and banknotes is not a fraction, but students also mention that the comparison of age A and age B can be expressed as fractions. This is a misconception because the comparison between lots of coins and banknotes or the comparison between age A and age B are a comparison between parts of a set and other parts of the set. Such a comparison cannot be written in the form of $\frac{a}{b}$ but $\frac{a\ coins}{b\ banknotes}$ or$a{:}\ b$(Chapin & Johnson, 2006; Lamon, 2012).This misconception occurs because students assume that all comparisons can be expressed in terms of fractions but not always. Comparison that can be expressed as $\frac{a}{b}$fraction is a comparison between parts and whole(Chapin & Johnson, 2006).

In interpreting fractions as operators, students also appear to experience misconceptions. This can be seen from the mathematical model created by students to answer the 5th question. Students write $5 \times \frac{1}{4}$. This shows that students do not understand fractions as operators. For these questions, the correct answer is $\frac{1}{4} \times 5$. Although the results are the same, the interpretation of fractions in the correct model is as an operator. Whereas, in the wrong model, fractions are interpreted more as a measure. The interpretation of fractions as operators is closely related to fraction multiplication. From the answers of students to sixth question, there are many mistakes that occur in multiplication operations. This shows that students experience misconceptions about orders multiplying by 1 and dividing by 4.

Regarding the interpretation of fractions as a measure, most students cannot determine fractions between 0 and 1. Students experience a misconception in looking at the partition of a unit that is the distance from 0 to 1 to 7 subunits to determine the fraction that states the distance from 0 to the specified subunit point. Based on the results of the interview, this misconception occurs because students do not have experience about fractions on the number line.

From the results and discussion, it can be concluded that there was a misconception in students interpreting fractions. This misconception occurs because of the students' lack of experience in interpreting fractions in different contexts. The interpretation that students are most familiar with is part of the whole. This happens because the interpretation of the fraction discussed repeatedly is about the shaded part. Even though some researchers note that there are limitations to the part of whole model (Simon et al., 2018). Therefore, diverse strategies are needed to teach fractions(Naiser, Wright, & Capraro, 2003).

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Biodata of the Author

Winda Ramadianti, M.Pd was born in Bengkulu, Indonesia. She graduated from Magister Mathematics Education Study Program of Universitas Negeri Yogyakarta, Indonesia, in 2012. She is a lecturer in Universitas Muhammadiyah Bengkulu. Since 2017, she has been studying in Doctoral Mathematics Education study Program of Universitas Pendidikan Indonesia, Indonesia.
Affiliation: Universitas Pendidikan Indonesia, West Java, Indonesia
E-mail:winda.ramadianti@gmail.com

Prof. Dr. Nanang Priatna, M.Pd was born in Kuningan, Indonesia. He graduated from Doctoral Mathematics Education Study Program of Universitas Pendidikan Indonesia, Indonesia, in 2003. He has been working as lecturer in Mathematics Education Study Program of Universitas Pendidikan Indonesia, Indonesia.
Affiliation: Universitas Pendidikan Indonesia, West Java, Indonesia
E-mail: nanangpriatna@upi.edu

Dr. Kusnandi, M.Si was born in Garut, Indonesia. He graduated from Doctoral Mathematics Education Study Program of Universitas Pendidikan Indonesia, Indonesia, in 2008. He has been working as lecturer in Mathematics Education Study Program of Universitas Pendidikan Indonesia, Indonesia.
Affiliation: Universitas Pendidikan Indonesia, West Java, Indonesia
E-mail: kusnandi@upi.edu

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