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Suzuki - $F(\psi - \phi) - \alpha$ type fixed point theorem on quasi metric spaces

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Abstract

In this paper, we obtain a α - Suzuki fixed point theorem by using C - class function on quasi metric spaces. Also we give an example which supports our main theorem.

Keywords: Quasi - metric space, Suzuki type contraction, C - Class function, α - admissible mapping.
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1. Introduction

In this paper $\mathbb{N}$ and $\mathbb{R}$ denote the sets of positive integers, respectively the set of real numbers, while $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ and $\mathbb{R}_0^+ := [0, \infty)$.
In 2008, the generalization theorem of Banach contraction principle [2], which was introduced by T.Suzuki [7], later this theorem is also referred as Suzuki type contraction. In 2014, Ansari [1] introduced the concept of C- class functions and proved the unique fixed point theorems for certain contractive mappings with respect to the C - class functions.

The aim of this paper is to prove a α-Suzuki type fixed point theorem by using (C)- class functions in quasi metric spaces.

2. Preliminaries

The aim of Suzuki [7] is to extend the well-known Edelstein's Theorem by using the notion of C-condition. Popescu [5] re-considered this approach to extend Bogin's fixed point theorem:

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Theorem 2.1. *Let a self-mapping T on a complete metric space (X, d) satisfies the following condition:*

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \quad (1)$$

implies

$$d(Tx, Ty) \leq ad(x, y) + b[d(x, Tx) + d(y, Ty)] + c[d(x, Ty) + d(y, Tx)] \quad (2)$$

where $a \geq 0$, $b > 0$, $c > 0$ and $a + 2b + 2c = 1$. Then T has a unique fixed point.

First we recall some basic definitions which play crucial role in the theory of quasi metric spaces.

Definition 2.2. *Let X be a non-empty set X and $q : X \times X \to R^+$ be a function which satisfies: such that for all $x, y, z \in X$:*

(q_1) $q(x, y) = 0$ *if and only if* $x = y$;
(q_2) $q(x, y) \leq q(x, z) + q(z, y)$.

The pair (X, q) is called a quasi- metric space.

Example 2.3. *Let $X = l_1$ be defined by*

$$l_1 = \{\{x_n\}_{n\geq 1} \subset R, \sum_{n=1}^{\infty} |x_n| < \infty\}.$$

Consider $d : X \times X \to [0, \infty)$ such that

$$q(x, y) = \begin{cases} 0 & if x \preceq y, \\ \sum_{n=1}^{\infty} |x_n| & if x \succeq y. \end{cases}$$

q is a quasi - metric. Mention that $x \succeq y$ if $x_n \geq y_n$ for all n, where $x = \{x_n\}$ and $y = \{y_n\}$ are in X.

Definition 2.4. *Let (X, q) be a quasi-metric space.*

- *q(i)* *A sequence $\{x_n\}$ in X is said to be convergent to x if $\lim_{n\to\infty} q(x_n, x) = \lim_{n\to\infty} q(x, x_n) = 0$.*
- *q(ii)* *A sequence $\{x_n\}$ in X is called left-Cauchy if for every $\epsilon > 0$ there exists a positive integer $N = N(\epsilon)$ such that $q(x_n, x_m) < \epsilon$ for all $n \geq m > N$.*
- *q(iii)* *A sequence $\{x_n\}$ in X is called right-Cauchy if for every $\epsilon > 0$ there exists a positive integer $N = N(\epsilon)$ such that $q(x_n, x_m) < \epsilon$ for all $m \geq n > N$.*
- *q(iv)* *A sequence $\{x_n\}$ in X is called Cauchy sequence if for every $\epsilon > 0$ there exists a positive integer $N = N(\epsilon)$ such that $q(x_n, x_m) < \epsilon$ for all $m, n > N$.*

Remark: From definition it is obvious that a sequence $\{x_n\}$ in a quasi-metric space is Cauchy if and only if it is both left-Cauchy and right-Cauchy.

Ansari [1] introduced the concept of C- class functions as the following:

Definition 2.5. *(See [1]) A mapping $F : [0, +\infty)^2 \to R$ is called a C- class function if it is continuous and for all $s, t \in [0, +\infty)$,*
(a) $F(t, s) \leq s$;
(b)$F(s, t) = s$ *implies that either* $s = 0$ *or* $t = 0$.

We denote $\mathcal{C}$ as the family of all C- class functions.

Example 2.6. *(See [1]) The following functions* $F : [0, +\infty)^2 \to R$ *are elements in* C.
(1) $F(s,t) = s - t$ *for all* $s, t \in [0, \infty)$;
(2) $F(s,t) = ms$ *for all* $s, t \in [0, \infty)$ *where* $0 < m < 1$;
(3) $F(s,t) = \frac{s}{(1+t)^r}$ *for all* $s, t \in [0, \infty)$ *where* $r \in (0, \infty)$;
(4) $F(s,t) = (s + l)^{\frac{1}{(1+t)^r}} - l$ *for all* $s, t \in [0, \infty)$ *where* $l > 1$, $r \in (0, \infty)$;
(5) $F(s,t) = s \log_{t+a} a$ *for all* $s, t \in [0, \infty)$ *where* $a > 1$;
(6) $F(s,t) = s - (\frac{1+s}{2+s})(\frac{t}{1+t})$ *for all* $s, t \in [0, \infty)$;
(7) $F(s,t) = s\beta(s)$ *for all* $s, t \in [0, \infty)$ *where* $\beta : [0, \infty) \to [0, 1)$ *and is continuous;*
(8) $F(s,t) = s - \varphi(s)$ *for all* $s, t \in [0, \infty)$ *where* $\varphi : [0, \infty) \to [0, \infty)$ *is a continuous function such that* $\varphi(t) = 0$ *if and only if* $t = 0$;
(9) $F(s,t) = sh(s,t)$ *for all* $s, t \in [0, \infty)$ *where* $h : [0, \infty) \times [0, \infty) \to [0, \infty)$ *is a continuous function such that* $h(t, s) < 1$ *for all* $s, t \in [0, \infty)$;
(10) $F(s,t) = s - (\frac{2+t}{1+t})t$ *for all* $s, t \in [0, \infty)$;
(11) $F(s,t) = \sqrt[n]{\ln(1 + s^n)}$ *for all* $s, t \in [0, \infty)$.

Definition 2.7. *(See [1]) A function* $\psi : [0, \infty) \to [0, \infty)$ *is called an altering distance function if the following properties are satisfied:*
(a) ψ *is nondecreasing and continuous;*
(b) $\psi(t) = 0$ *if and only if* $t = 0$.

We denote Ψ the family of all altering distance function.

Definition 2.8. *(See [1]) A function* $\varphi : [0, \infty) \to [0, \infty)$ *is called an ultra altering distance function if the following properties are satisfied:*
(a) φ *is continuous;*
(b) $\varphi(t) > 0$ *for all* $t > 0$.

We denote Φ the family of all altering distance function.

In 2012, Samet et al. [6] introduced α - admissible mappings as the following:

Definition 2.9. *(See. [6], [3]) A mapping* $f : X \to X$ *is called* α*- admissible if for all* $x, y \in X$ *we have*

$$\alpha(x, y) \geq 1 \Rightarrow \alpha(fx, fy) \geq 1,$$

where $\alpha : X \times X \to [0, \infty)$ *is a given function.*

Definition 2.10. *[4] A mapping* $f : X \to X$ *is called a triangular* α*- admissible if it is* α *- admissible and satisfies*

$$\left.\begin{array}{l}\alpha(x, y) \geq 1 \\ \alpha(y, z) \geq 1\end{array}\right\} \Rightarrow \alpha(x, z) \geq 1,$$

where $x, y, z \in X$ and $\alpha : X \times X \to [0, \infty)$ is a given function.

Definition 2.11. *[4] A mapping* $f : X \to X$ *is said to be weak triangular* α *- admissible if it is* α*-admissible and satisfies*

$$\alpha(x, fx) \geq 1 \Rightarrow \alpha(x, f^2x) \geq 1,$$

where $\alpha : X \times X \to [0, \infty)$ *is a given function.*

Lemma 2.12. *[4] Let* $f : X \to X$ *be a weak triangular* α*-admissible mapping. Assume that there exists* $x_0 \in x$ *such that* $\alpha(x_0, fx_0) \geq 1$. *If* $x_n = f^n x_0$, *then* $\alpha(x_m, fx_n) \geq 1$ *for all* $m, n \in N_0$ *with* $m < n$.

The following auxiliary result is going to be used in the proof of existence theorems.

Lemma 2.13. *Let $f : X \to X$ be a triangular α- admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\alpha(fx_0, x_0) \geq 1$. If $x_n = f^n x_0$, then $\alpha(x_m, x_n) \geq 1$ for all $m, n \in N$.*

Definition 2.14. *Let (X, q) be a quasi metric space and let $f : X \to X$ be a given mapping f is an $F(\psi - \phi) - \alpha$-Suzuki- type rational contraction condition.If there exist two functions $\alpha : X \times X \to [0, \infty)$ such that $\alpha(x, y) \geq 1$ and*

$$\frac{1}{2}q(x, fx) \leq q(x, y)$$

implies that

$$\psi\left(q\left(fx, fy\right)\right) \quad \leq F\left(\psi\left(M\left(x, y\right)\right), \varphi\left(M\left(x, y\right)\right)\right), \tag{3}$$

for all x, y in X, where

$$M\left(x, y\right) \quad = \max\left\{ \; q(x, y), \frac{1+q(x,fx).q(y,fy)}{1+q(x,y)} \; \right\},$$

$\psi \in \Psi$, $\varphi \in \Phi$ and $F \in \mathcal{C}$.

Now we prove our main result.

3. Main Results

Theorem 3.1. *Let (X, q) be a complete quasi metric space and $f : X \to X$ be mappings such that f is $F(\psi - \phi) - \alpha$-Suzuki- type rational contractive suppose that*

(i) $f : X \to X$ is weak triangular α- admissible mapping

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\alpha(fx_0, x_0) \geq 1$

(iii) f is continuous or If $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ and $\alpha(x_{n+1}, x_n) \geq 1$ for all n and as $n \to \infty$, then there exists a subsequence $\{x_n(k)\}$ of x_n such that $\alpha(x_n(k), x) \geq 1$ and $\alpha(x, x_n(k)) \geq 1$ for all k

Then f has fixed point in X.

Proof. By assumption (ii), there exists $x_0 \in X$, such that $\alpha(x_0, fx_0) \geq 1$ and $\alpha(fx_0, x_0) \geq 1$.
Define the sequence $\{x_n\}$ in X as $fx_n = x_{n+1}$, $n = 1, 2, 3, \cdots$
If $x_{n_0} = x_{n_0+1}$ for some $n_0 > 0$, then x_{n_0} is a fixed point of f and the proof is done. Assume that $x_n \neq x_{n+1}$ for all $n \geq 0$. Since f is α- admissible,

$$\alpha(x_0, fx_0) = \alpha(x_0, x_1) \geq 1 \Rightarrow \alpha(fx_0, fx_1) = \alpha(x_1, x_2) \geq 1$$

and continuing we obtain

$$\alpha(x_n, x_{n+1}) \geq 1 \quad forall \quad n \in N.$$

Since

$$\tfrac{1}{2}q(x_n, fx_n) \quad \leq q(x_n, x_{n+1}).$$

From (3),we get

$$\psi\left(q\left(fx_n, fx_{n+1}\right)\right) \quad \leq F\left(\psi\left(M\left(x_n, x_{n+1}\right)\right), \varphi\left(M\left(x_n, x_{n+1}\right)\right)\right).$$

$$\begin{aligned} M(x_n, x_{n+1}) \quad &= \max\left\{ \; q(x_n, x_{n+1}), \frac{1+q(x_n,x_{n+1}).q(x_{n+1},x_{n+2})}{1+q(x_n,x_{n+1})} \; \right\} \\ &= \max\{q(x_n, x_{n+1}), q(x_{n+1}, x_{n+2})\}. \end{aligned}$$

Hence,

$$\psi(q(x_{n+1}, x_{n+2})) \leq F(\psi\left(\max\{q(x_n, x_{n+1}), q(x_{n+1}, x_{n+2})\}\right), \varphi(\max\{q(x_n, x_{n+1}), q(x_{n+1}, x_{n+2})\})) .$$

If $q(x_{n+1}, x_{n+2})$ is maximum then we have

$$\psi(q(x_{n+1}, x_{n+2})) \leq F(\psi(q(x_{n+1}, x_{n+2})), \varphi(q(x_{n+1}, x_{n+2}))) < \psi(q(x_{n+1}, x_{n+2}))$$

, which is a contradiction.
Hence $q(x_n, x_{n+1})$ is maximum. Thus

$$\psi(q(x_{n+1}, x_{n+2})) \leq F(\psi(q(x_n, x_{n+1})), \varphi(q(x_n, x_{n+1})) \tag{4}$$

Since ψ is increasing we have $q(x_{n+1}, x_{n+2}) \leq q(x_n, x_{n+1})$.
Thus $\{q(x_n, x_{n+1})\}$ is a non - increasing sequence of non - negative real numbers and must converge to a real number, say, $r \geq 0$. Suppose $r > 0$.
Letting $n \to \infty$ in (4) , we get
$\psi(r) \leq F(\psi(r), \varphi(r))$. This implies that $\psi(r) = 0$ and $\varphi(r) = 0$ which yields

$$\lim_{n\to\infty} q(x_n, x_{n+1}) = 0. \tag{5}$$

Now we prove that $\{x_n\}$ is a left-Cauchy sequence in (X, q). On contrary suppose that $\{x_n\}$ is not left - Cauchy.
Then there exist an $\epsilon > 0$ and monotone increasing sequences of natural numbers $\{m_k\}$ and $\{n_k\}$ such that $n_k > m_k$,

$$q(x_{m_k}, x_{n_k}) \geq \epsilon \tag{6}$$

and

$$q(x_{m_k}, x_{n_k-1}) < \epsilon. \tag{7}$$

From (6) and (7), we obtain

$$\begin{aligned} \epsilon &\leq q(x_{m_k}, x_{n_k}) \\ &\leq q(x_{m_k}, x_{n_k-1}) + q(x_{n_k-1}, x_{n_k-1}) + q(x_{n_k-1}, x_{n_k}) \\ &< \epsilon + q(x_{n_k-1}, x_{n_k-1}) + q(x_{n_k-1}, x_{n_k}). \end{aligned}$$

Letting $k \to \infty$ and then using (6), we get

$$\lim_{k\to\infty} q(x_{m_k}, x_{n_k}) = \epsilon. \tag{8}$$

Letting $k \to \infty$ and then using (5) and (8) in
$|q(x_{m_k-1}, x_{n_k}) - q(x_{m_k-1}, x_{m_k})| \leq q(x_{m_k}, x_{n_k})$
we obtain

$$\lim_{k\to\infty} q(x_{m_k-1}, x_{n_k}) = \epsilon. \tag{9}$$

Letting $k \to \infty$ and then using (5) and (8) in
$|q(x_{m_k}, x_{n_k+1}) - q(x_{n_k}, x_{n_k+1})| \leq q(x_{m_k}, x_{n_k})$
we obtain

$$\lim_{k\to\infty} q(x_{m_k}, x_{n_k+1}) = \epsilon. \tag{10}$$

Hence, we get

Since f is weak triangular α-admissible. Then, from Lemma2.13 we have

$$\alpha(x_{n_k}, x_{m_k}) \geq 1$$

If $\frac{1}{2}q(x_{m_k-1}, x_{m_k}) > q(x_{m_k-1}, x_{n_k})$ then letting $k \to \infty$,
we get $0 \geq \epsilon$ from 5 and 9.
It is a contradiction. Hence
$\frac{1}{2}q(x_{m_k-1}, x_{m_k}) \leq q(x_{m_k-1}, x_{n_k})$.
From (3), we have

$$\begin{aligned} &\psi\left(q(x_{m_k}, x_{n_k+1})\right) \\ &\quad = \psi\left(q(fx_{m_k-1}, fx_{n_k})\right) \\ &\quad \leq F\left(\psi\left(M(x_{m_k-1}, x_{n_k})\right), \varphi\left(M(x_{m_k-1}, x_{n_k})\right)\right), \end{aligned}$$

where

$$M(x_{m_k-1}, x_{n_k}) = \max\left\{ \; q(x_{m_k-1}, x_{n_k}), \frac{1+q(x_{m_k-1}, x_{m_k}).q(x_{n_k}, x_{n_k+1})}{1+q(x_{m_k-1}, x_{n_k})} \; \right\}.$$

Letting $k \to \infty$ and then using (10) and (5) we have

$$\begin{aligned} \psi(\epsilon) &\leq F\left(\psi\left(\max\{\epsilon, 0\}\right), \varphi\left(\max\{\epsilon, 0\}\right)\right) \\ &\leq F\left(\psi(\epsilon), \varphi(\epsilon)\right). \end{aligned}$$

It follows that $\psi(\epsilon) = 0$ or $\varphi(\epsilon) = 0$. This implies that $\epsilon = 0$ which is a contradiction. Hence $\{x_n\}$ is left - Cauchy in (X, q). Similarly, $\{x_n\}$ is right - Cauchy
Thus $\{x_n\}$ is a Cauchy sequence in (X, q).
Hence,

$$\lim_{n,\ m\to\infty} q(x_n, x_m) = 0. \tag{11}$$

Since $x_{n+1} = fx_n$, it follows $\{x_n\}$ is a Cauchy sequence in the complete quasi - metric space (X, q). Therefore, there exists $u \in X$ such that

$$\lim_{n\to\infty} q(x_n, u) = \lim_{n\to\infty} q(u, x_n) = 0. \tag{12}$$

From continuity of f we get

$$\lim_{n\to\infty} q(x_n, fu) = \lim_{n\to\infty} q(fx_{n-1}, fu) = 0. \tag{13}$$

and

$$\lim_{n\to\infty} q(fu, x_n) = \lim_{n\to\infty} q(fu, fx_{n-1}) = 0. \tag{14}$$

Combining (13) and (14), we deduce

$$\lim_{n\to\infty} q(x_n, fu) = \lim_{n\to\infty} q(fu, fx_n) = 0. \tag{15}$$

From 12 and 15, due to the uniqueness of the limit, we conclude that $u = fu$, that is , u is a fixed point of f.

Now we claim that, for each $n \geq 1$, at least one of the following assertions holds.

$$\frac{1}{2}q(x_{n-1}, x_n) \leq q(x_{n-1}, u) \quad \text{or} \quad \frac{1}{2}q(x_n, x_{n+1}) \leq q(x_n, u).$$

On the contrary suppose that

$$\frac{1}{2}q(x_{n-1}, x_n) > q(x_{n-1}, u) \quad \text{and} \quad \frac{1}{2}q(x_n, x_{n+1}) > q(x_n, u)$$

for some $n \geq 1$.
Then we have

$$\begin{aligned} q(x_{n-1}, x_n) &\leq q(x_{n-1}, u) + q(u, x_n) \\ &< \frac{1}{2}\left[q(x_{n-1}, x_n) + q(x_n, x_{n+1})\right] \\ &\leq q(x_{n-1}, x_n), \end{aligned}$$

which is a contradiction and so the claim holds.
Suppose $\frac{1}{2}q(x_n, x_{n+1}) \le q(x_n, u)$.
Suppose $fu \neq u$.
Since the sequence $\{x_n\}$ converges to $u \in X$,from (iii), there exists a subsequence $\{x_n(k)\}$ of x_n such that $\alpha(x_n(k), u) \ge 1$ and $\alpha(u, x_n(k)) \ge 1$ for all k.
We have

$$\tfrac{1}{2}q(x_{n_k}, x_{n_k+1}) \le \tfrac{1}{2}q(x_{n_k}, u)$$

from (3), we have

$$\psi\left(q(fx_{n_k}, fu)\right) \quad \le \quad F\left(\psi\left(M\left(x_{n_k}, u\right)\right), \varphi\left(M\left(x_{n_k}, u\right)\right)\right),$$

where

$$M\left(x_{n_k}, u\right) \quad = \quad \max\left\{ \; q(x_{n_k}, u), \frac{1+q(x_{n_k}, fx_{n_k}).q(u, fu)}{1+q(x_{n_k}, u)} \; \right\}.$$

Letting $n \to \infty$ and using 14,we get

$$\begin{aligned} &\psi\left(q(u, fu)\right) \\ &\quad \le F\left(\psi\left(\max\left\{ \; q(u,u), \frac{1+q(u,u).q(u,fu)}{1+q(u,u)} \; \right\}\right), \varphi\left(\max\left\{ \; q(u,u), \frac{1+q(u,u).q(u,fu)}{1+q(u,u)} \; \right\}\right)\right), \\ &\quad \le F\left(\psi\left(q(u, fu)\right), \varphi\left(q(u, fu)\right)\right) < \psi\left(q(u, fu)\right), \end{aligned}$$

which is a contradiction.
Thus, $fu = u$.
Hence, u is a fixed point of f .

□

(H) for all $x, y \in Fix(f)$, we have $\alpha(x, y) \ge 1$, where Fix(f) denotes the set of fixed points of f.

Theorem 3.2. *Adding (H) to the hypotheses of Theorem(3.1), f has a unique fixed point.*

Proof. Due to Theorem (3.1), we have u is a fixed point of f. Let w be another fixed point of f .
Suppose $u \neq w$.
From (H), we have

$$\alpha(u, w) \ge 1, \;\; for \; all \; u, w \in Fix(f).$$

Since $\frac{1}{2}q(u, fu)\} \le q(u, w)$, from (3), we obtain

$$\begin{aligned} \psi\left(q(u, w)\right) \; &= \psi\left(q(fu, fw)\right) \\ &\le F\left(\psi\left(M(u, w)\right), \varphi\left(M(u, w)\right)\right), \end{aligned}$$

where

$$\begin{aligned} M(u, w) \; &= \max\left\{ \; q(u, w), \frac{1+q(u,u).q(w,w)}{1+q(u,w)} \; \right\} \\ &= q(u, w). \end{aligned}$$

Thus
$\psi\left(q(u, w)\right) \le F\left(\psi\left(q(u, w)\right), \varphi\left(q(u, w)\right)\right)$.
It follows that $\psi\left(q(u, w)\right) = 0$ or $\varphi\left(q(u, w)\right) = 0$.
This implies that $q(u, w) = 0$ which is a contradiction.
Hence $u = w$.

□

Example 3.3. *Let $X = [0, \infty)$ and q be the quasi metric on X given by*

$$q(x,y) = \begin{cases} |x| & if x \neq y, \\ 0 & if x = y, \end{cases}$$

for all $x, y \in X$. It is obvious that (X, q) be a complete quasi- metric space. Suppose that $f : X \to X$ is defined by

$$fx = \begin{cases} x^3 - 2x & if x > 2, \\ \frac{x}{8} & if x \in [0, 2]. \end{cases}$$

Now, define $\alpha : X \times X \to [0, \infty)$ as

$$\alpha(x,y) = \begin{cases} 1 & if x, y \in [0, 1] \\ 0 & otherwise. \end{cases}$$

Let $F(s,t) = s - t$ for all $s, t \in [0, \infty)$. Let $\psi(t) = t$, $\varphi(t) = \frac{t}{2}$.

$$\begin{aligned} \tfrac{1}{2}q(x, fx) &\leq x \\ &\leq q(x,y) \end{aligned}$$

$$\begin{aligned} \psi\left(q\left(fx, fy\right)\right) &= q\left(fx, fy\right) \\ &= fx, \\ &= \tfrac{x}{8} \\ &= \tfrac{1}{2}q(x,y) \\ &\leq \tfrac{1}{2}M\left(x,y\right) \\ &= M\left(x,y\right) - \tfrac{1}{2}M\left(x,y\right) \\ &= F\left(\psi\left(M\left(x,y\right)\right), \varphi\left(M\left(x,y\right)\right)\right) \end{aligned}$$

Therefore, all of the conditions of Theorem 3.1 are satisfied and 0 is the fixed point of f.

If we let $\alpha(x,y) = 1$ for all $x \in X$, we get the following result.

Corollary 3.4. *Let (X, q) be a complete quasi metric space and let $f : X \to X$ be a given mapping f is an $F(\psi - \phi)$-Suzuki- type rational contraction condition. If there exist functions $\psi \in \Psi$, $\varphi \in \Phi$ and $F \in \mathcal{C}$ such that*

$$\frac{1}{2}q(x, fx) \leq q(x,y)$$

implies that

$$\psi\left(q\left(fx, fy\right)\right) \leq F\left(\psi\left(M\left(x,y\right)\right), \varphi\left(M\left(x,y\right)\right)\right),$$

where

$$M\left(x,y\right) = \max\left\{ q(x,y), \frac{1 + q(x,fx).q(y,fy)}{1 + q(x,y)} \right\},$$

for all x, y in X. Then f has a unique fixed point in X.

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