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Obtaining Critical Buckling Load of a Cantilever Beam by Using Measured Natural Frequencies

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A B S T R A C T

In this study, buckling critical loads are calculated by using natural frequency values of the first 4 modes obtained by performing experimental modal analysis (EMA) of a uniform cantilever beam. The analytical results are presented with the experimental results comparatively. According to the results it can be said that the vibration data can be used to determine the critical buckling loads of the beams. So there is no need any buckling tests that cause deformation in the test structures or analytical and numerical solutions that need mechanical properties of the beam for obtaining buckling loads.

1. Introduction

Columns and beams are widely used in engineering applications. These elements are subjected to the buckling loads because of their shapes, while working under various loads depending on the usage type and the usage conditions. For this reason, the buckling conditions and the maximum buckling critical loads to be carried by these structural elements must be determined. For the determination of buckling loads, buckling tests are commonly applied. However, the test specimens are damaged in these tests. On the other hand, in the numerical methods, some mechanical properties of the elements such as elasticity module have to be known. Moreover, modeling of elements with complex structure and shape is very difficult. In this study, the idendification of the buckling critical loads of a cantilever beam structure with uniform shape is investigated by using the natural frequency values obtained by using the experimental modal analysis (EMA) technique which is a nondestructive test method. Many studies have been made on vibration of beam and buckling for years. For the analysis of beam vibrations some analytical approaches can be used, such as, Euler-Bernoulli Beam Theory (EBBT) which is also known as (Simple Beam Theory), Timoshenko Beam Theory (TBT) in which shear deformations and rotational inertia are taken into account and High Order Beam Theory (HOBT). Euler approach is commonly used for buckling analysis of beams.

In this context, Chakrabatri et al. [1], analyzed the vibration and buckling of composite beams using a numerical model based on the HOBT. In the proposed model, the shear deformation of the beams as well as the partial shear interactions between adjacent parts are accounted for, and there is no need to incorporate any shear correction factors. Mikulas and Lake [2] carried out a numerical study on the determination of vibration and buckling properties of tapered beams under axial load. They investigated the differences of the tapered beams with the positive and negative aspects from uniform beams. Deng et al. [3] presented a rigidity matrix for a double-layered functional graded Timoshenko beam system under axial loading, taking into account the damping of the adhesive layer. Using the Wittrick William algorithm, they obtained the natural frequencies and buckling loads of the system and investigated the effect of axial loading and connection stiffness on the natural frequency and buckling load system. Jasion et al. [4] conducted a study on natural frequency and buckling load calculations, taking into account the properties of each layer of a beam consisting of 3 trapezoid grooves and 4 flat plates. Chakrabarti et al. [5] conducted numerical studies based on the high-order Zigzag Theory on the buckling analysis of sandwich composite beams with soft core.

Numerous studies have been carried out on the calculation of buckling loads of structures using vibration data. Kemiklioglu et al. [6] conducted a numerical and experimental study on buckling and vibration analysis for a sandwich composite beam with a curved shape. Matsugana [7] conducted a numerical study on the natural frequencies and buckling loads of beams with square cross section and axial load. In his work, he examined the beams with different length / width ratios and tested the correctness of his approach by comparing the results of his study with the results obtained with different numerical approaches. Laux [8], investigated the determination of axial loads using the resonance frequencies obtained from vibration tests of a lumber beam in his thesis. It is found that there is a close relationship between axial loads and resonance frequencies.

In this study, the buckling critical loads of a cantilever beam with a uniform shape are determined using the natural frequency values obtained from EMA.

2. Material and Method

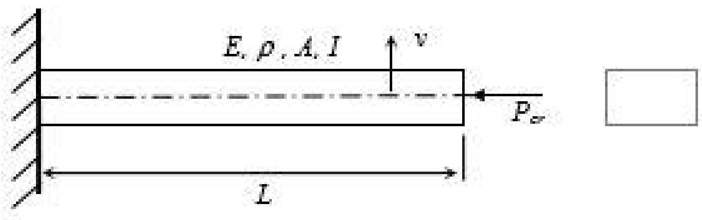


Figure 1. A Euler-Bernoulli cantilever beam

A cantilever Euler-Bernoulli beam with clamped one end and free other end is illustrated in Figure 1. The equation of motion for bending vibrations of this beam, can be expressed by Equation (1).

$$EIv^{IV} \circ A\ddot{v} = 0 \tag{1}$$

Here, *E, I, p, A* and *v* are Young's modulus, moment of inertia, density, cross-section area and displacement, respectively. $(\ddot{\ })$ and $(\ )^{IV}$ represent second time derivative and fourth derivative of the displacement. The natural frequencies of the cantilever beam can be calculated by the formula given as follows.

$$f_n(\mathrm{Hz}) = \frac{\lambda^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \qquad (n:1,2,3,....n) \qquad (2)$$

Here L is the length of the beam, n is the mode number and λ is the dimensionless frequency parameter. The values of λ for the first 4 modes are given as: 1.8751, 4.6941, 7.8547 and 10.9955.
For the same uniform beam, the critical buckling load can be expressed by Equation (3). The first four buckling mode shapes are illustrated in Figure 2.

$$(P_{cr})_n = \frac{n^2\pi^2}{Le^2}EI \qquad (n:1,2,3,....n) \qquad (3)$$

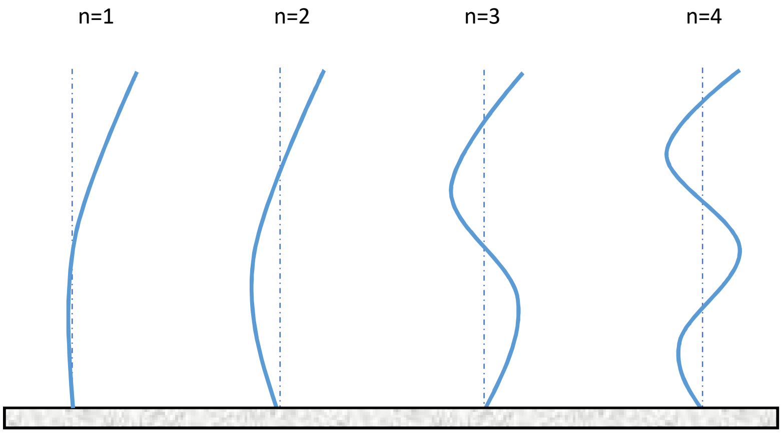


Figure 2. Buckling mode shapes for cantilever beam

Where $(P_{cr})_n$ refers to the buckling critical load for mode n, EI is bending stiffness and Le is the effective length, which is equal to 2 times of the beam length ($Le=2L$).

If the bending stiffness EI is taken from Equation (2) and substituted in Equation (3), the buckling critical load depended on the natural frequency for the beam can be obtained as follows:

$$(P_{cr})_n = \frac{n^2\pi^4 L^2 \rho A}{\lambda^4} f_n^2 \qquad (n:1,2,3,....n) \qquad (4)$$

In this study, the experimental modal analysis method is used to obtain the natural frequencies of the cantilever beam for the first 4 modes, and then the buckling loads of the beam are calculated by using the natural frequencies obtained in Equation (4)

In the experimental modal analysis method, the system is excited with a known force and the response of the system to this force is measured then the frequency response functions (FRFs) of the system are obtained. FRFs specify the dynamic property of the system and express the relationship between the input and output of the linear system [9]. So, the dynamic characteristics of the system (natural frequencies, mode shapes and modal dampings) can be determined by using the measured FRFs. In addition, FRFs can be used for the verification of numerical results, model updating, structural modification and so on.

In the modal test, the structure is excited by using a modal hammer or a shaker. The applied force and the response of the structure to this force is measured at the same moment using a load sensor and accelerometer(s), respectively. In Figure 3. a simple modal test setup is illustrated schematically.

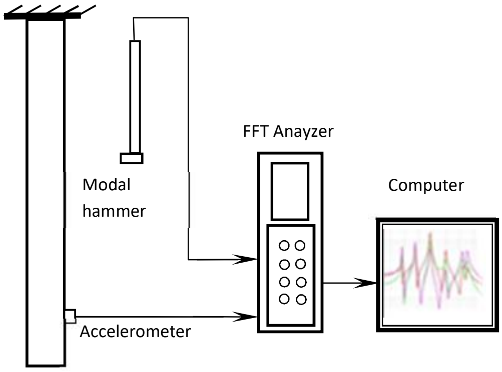


Figure 3. A basic modal test setup

In Figure 4, the cantilever beam used in experimental studies is seen. The mechanical and physical properties of this beam are given in Table 1.

Figure 4. The cantilever beam under test

Table 1. The mechanical and physical properties of the cantilever beam used in experiments

Parameters	Young's modulus (GPa)	Poisson's ratio	Shear modulus (GPa)	Density (kg/m^3)	Length (mm)	Height (mm)	Width (mm)
Values	200	0.3	80	7850	800	12	25

The experimental studies in this study are carried out in Firat University Machine Theory and Dynamic Laboratory. In order to perform the vibration tests of the cantilever beam element, the beam is portioned into 20 segments and 21 points are determined on the beam as in Figure 5.

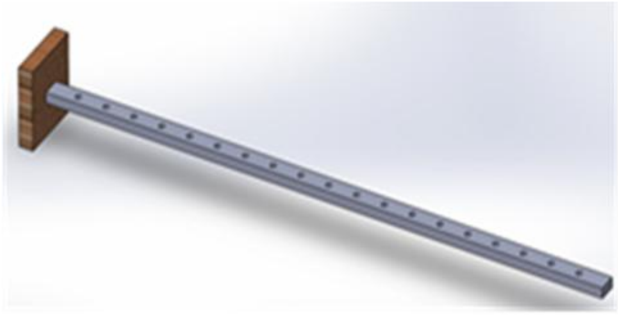


Figure 5. Portioned cantilever beam for the experiment

From these points, the beam element is excited using a modal hammer and the response of the system is measured from these points using a single unidirectional accelerometer. Totally 21 FRFs are measured. To excite the system an impact hammer (Kistler Model: 9724A2000) and to measure the response of the system an ICP accelerometer (DYTRAN, Model: 3097A2) are used. To acquire the vibration data Or36 series vibration analyzer and to determine the modal parameters Oros Modal software are used.

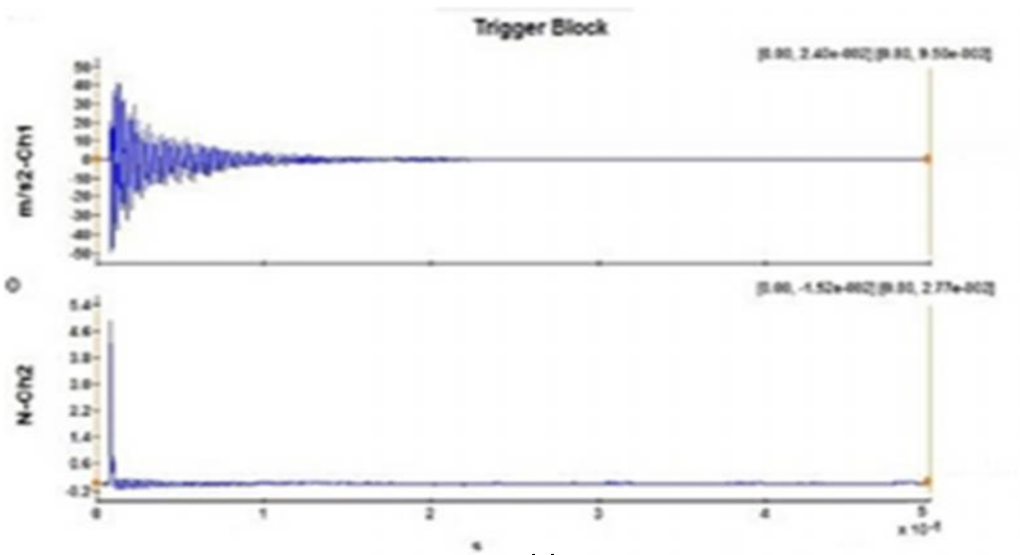
3. Findings

The experimental setup and a measurement operation are shown in Figure 6. The measurement parameters used for the test are given in Table 2.

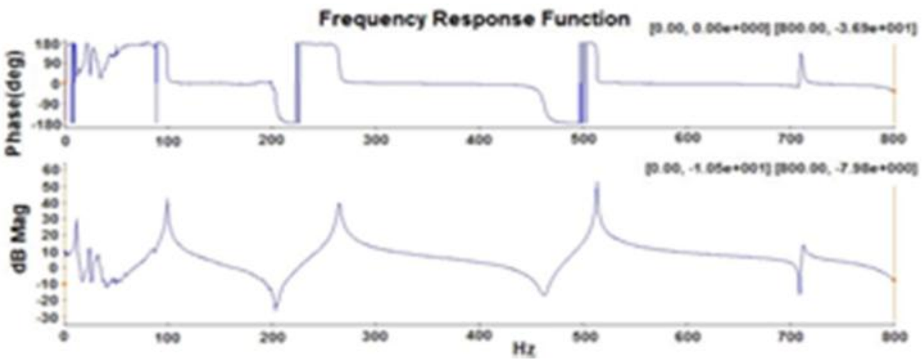
Figure 6. Experimental setup and a measurement

Table 2. The measurement parameters

Parameters	**Frequency bandwidth (Hz)**	**Frequency resolution (Hz)**
Values	0-800	0.5



(a)



(b)

Figure 7. a) A sample of time signals from force transducer and accelerometer b) Magnitude and phase plots of an FRF

For the reliability of test results, some calibration tests such as repeatability and reciprocity checks are performed before the measurements. A sample of an impulse, response and FRF are shown in Figure 7 (a,b). Any windowing function was not applied because the time responses die down in the measurement period.

The experimentally obtained first four natural frequencies of the beam are given in Table 3. The natural frequencies of the beam were also determined by using Equation (2) and were compared in Table 3.

Table 3. The comparison of natural frequencies obtained analytically and experimentally (Hz)

Mode Number	Analytical (Hz)	Experimental (Hz)	Difference (%)
1	15.28	14.30	-6.8
2	93.78	99.33	5.6
3	268.22	264.86	-1.3
4	525.65	513.69	-2.3

The vibration mode shapes obtained by EMA technique for the first 4 modes are given in Figures 8-11.

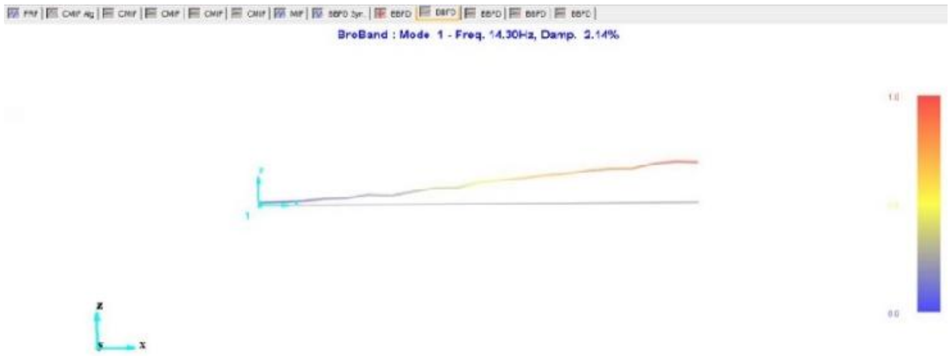


Figure 8. The first vibration bending mode shape

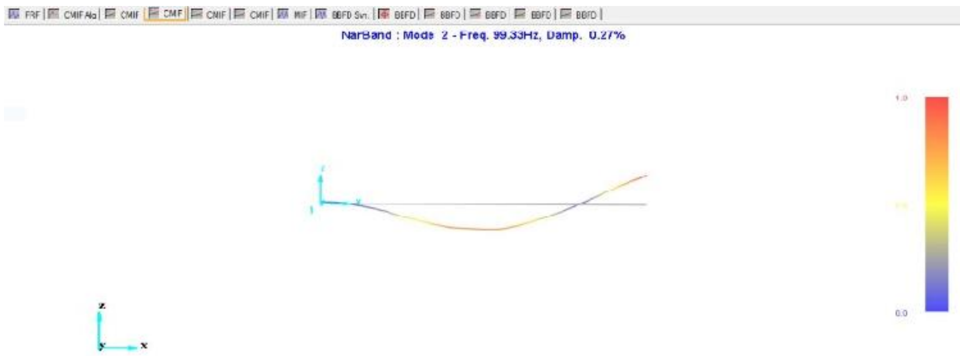


Figure 9. The second vibration bending mode shape

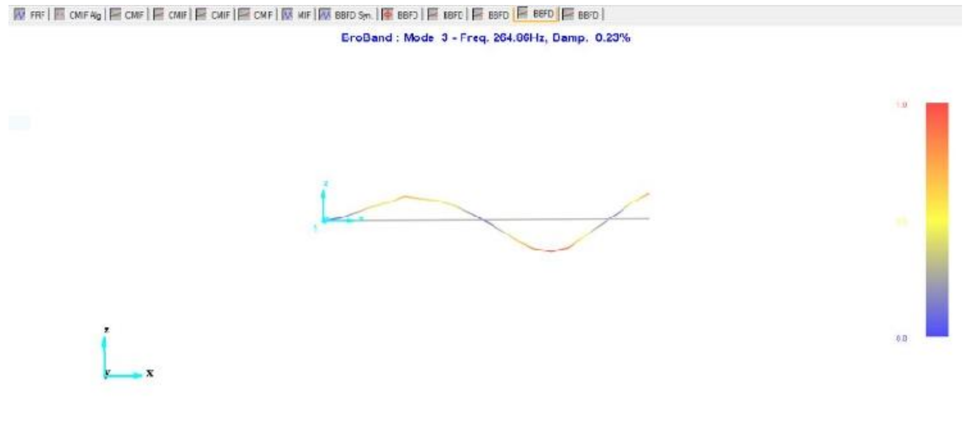


Figure 10. The third vibration mode shape

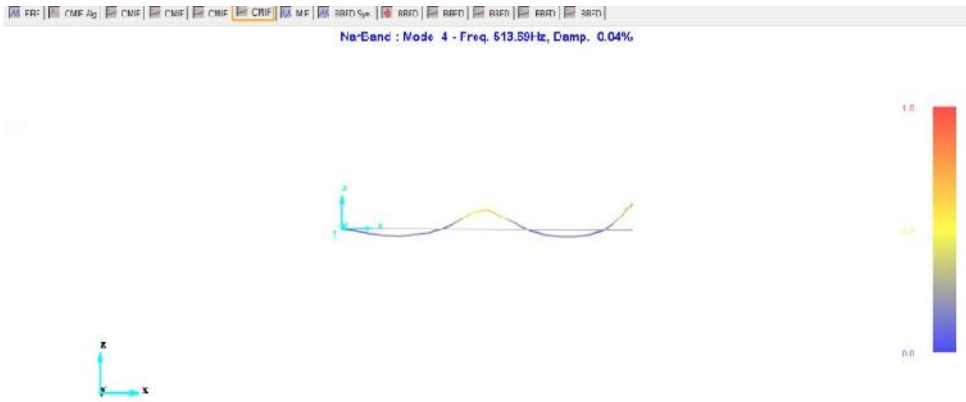


Figure 11. The fourth vibration mode shape

Using the measured natural frequency values, the buckling loads for the first 4 modes of the cantilever beam are calculated via Equation (4). The buckling loads are also calculated by using Equation (3) and compared with those of obtained by using Equation (4) in Table 4. Also the relative errors to the analytical results are given in Table 4.

Table 4. Comparison of the buckling loads calculated from natural frequencies and obtained analytically (N)

Mode Number	Analytical (N)	This Study (N)	Difference (%)
1	2776	2429	-12.5
2	24984	26856	7.49
3	69400	67643	-2.5
4	136024	129870	-4.5

4. Results and Discussion

In this study, the determination of the critical buckling loads from the measured natural frequencies was investigated. It was shown that the critical buckling loads of the beam could be determined by using only the natural frequencies of the beam obtained from the experimental modal analysis without performing the destructive buckling test.

According to the results, the maximum percentage of difference between the buckling loads calculated by using natural frequencies and the analytical solutions is 12.5 when the minimum percentage is 2.5. The experimental setup and the experiment tolerances may cause this difference. The frequency resolution effects the results.

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