Contaminant Diffusion along uniform flow velocity with pulse type input sources in finite porous medium

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Abstract: Solute transport inside pore system occurs due to advection and diffusion which are the important mechanisms of contaminant transport in porous medium. Analytical solutions of one-dimensional advection-diffusion equation (the coefficient of second order space derivative being temporally dependent) are obtained in a finite domain for two sets of pulse type input boundary conditions. Initially the domain is not solute free. It is supposed uniformly distributed at the initial stage. The Laplace transform technique is used with the help of new space and time variables. The solutions are graphically illustrated and compared solute distribution for finite and semi-infinite domain.

1. Introduction

Many examples of porous material are seen in everyday of life and environment. Soil, paper towels, Textiles, leather and tissue paper are highly porous. There are many examples where porous media play an important role in science and technology. The most important area of science and technology that to great extent depend on the properties of porous media is hydrology, which relates to water movement in earth and sand structures such as dame, flow to wells from water bearing formations, intrusion of sea water in coastal area and nutrient transport in soil. Contaminant (e.g. pesticides, chemicals, fertilizers, hygienic substance etc.) transport in subsurface, in soil and its analysis is complicated due to the complex behaviour of porous medium. Knowledge of various physical, chemical, and biological processes, which affect the movement of subsurface contaminants, is necessary for soil, blood, aquifer and groundwater remediation research and practice. From several decades the uncontrolled use of pesticides in agriculture, human and factory activities cause serious damages the environment and affected flora-fauna (forest, animals, human body etc.), soil and groundwater. Contaminant behaviours in the soil/aquifer system are subject to many processes. The study of contaminant transport requires the fundamental knowledge of many of the basic principle of physics and mathematics. A large number of theoretical and mathematical models have been developed and deployed to study the hydrodynamic processes involved in groundwater and surface water.

In previous literatures, the longitudinal dispersion coefficient have been considered either linearly or squarely proportional to the fluid velocity. Banks and Jerasate [1] observed well agreement between concentration distributions and theoretical values except at very low concentrations. Ogata and Banks, Lin, Al-Niami and Rushton [2-4] obtained analytical solutions for dispersion in a porous media. Harleman and Rumer [5] obtained

solution for longitudinal and lateral dispersion in an isotropic porous medium i.e. the permeability does not change with direction. Bruch [6] derived a series of two-dimensional dispersion problems in one and two layered porous medium. The experimental results were compared with theoretical and numerical solutions both of which describe the two-dimensional dispersion of a miscible. Most of such works have been compiled by van Genutchen and Alves [7]. Chen and Liu [8] dealt with solute transport form an injection well into an aquifer. A macroscopic boundary condition of the Cauchy type (the third type) can be formulated at the well-aquifer interface if the mass balance principle is invoked. Tracy [9] first gave some simple one-dimensional solutions. Next, by use of a transformation, the nonlinear partial differential equation is converted to a linear one for a specific form of the moisture content vs. pressure head and relative hydraulic conductivity vs. pressure head curves. This allows both two and three dimensional solutions to be derived.

Aral and Liao [10] examined solutions of two dimensional advection-dispersion equation with time dependent dispersion coefficients and demonstrated the time and space dependent nature of the dispersion coefficient in subsurface contaminant transport problems. They developed instantaneous and continuous point source solutions for constant, linear, asymptotic, and exponentially varying dispersion coefficients. Ataie-Ashtiani et al. [11] studied the influence of tidal fluctuation effects on groundwater dynamics and contaminant transport in unconfined coastal aquifers. Sander and Braddock [12] obtained analytical solutions of advection-dispersion equation in one-dimension with scale and time dependent dispersivities. In order to perform a general analysis of groundwater contaminant transport, Sirin [13] considered (i) non-divergence-free pore flow velocity since non-divergence-free pore flow velocity occurs during density department flows, (ii) unsteady pore flow velocity. Chen and Liu [14], studied an analytical solution for one-dimensional advective-dispersive transport in finite spatial domain with time-dependent inlet conditions including constant, exponentially decaying and sinusoidally periodic input functions and demonstrate the applicability of solution. Yadav et al. and Jaiswal

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et al. [15-16] obtained analytical solutions for solute dispersion in finite porous media. Sharma [17] obtained solution of an advective dispersive transport equation, including equilibrium sorption and first-order degradation coefficient in the fracture and simultaneously a diffusive transport equation for porous media using numerical implicit finite difference method and discussed numerical results of various temporal moments have been predicted to investigate the behaviour of reactive solute in the fracture.

In the present study, advection-diffusion equation is considered one dimensional. The solute dispersion parameter is considered temporally dependent along uniform flow in longitudinal finite domain of length, . The input source condition is assumed to be of pulse type, introduced at the origin of the domain. The second condition is considered at the other end of the domain which is of second type (flux type) of homogeneous nature. The domain is assumed initially not solute free i.e. the domain is supposed to have uniformly distributed solutes at the initial stage. Laplace transformation technique is used to getting the analytical solutions.

2. Mathematical Model of the Problem

For the analyses presented here, the governing equation for a solute transport model represent a mathematical description of the assumed transport mechanisms and processes in ideal case which include the effect of adsorption, in one dimension may be written as,

$$\frac{\partial C}{\partial t} + \frac{1-n_p}{n_p}\frac{\partial F}{\partial t} = \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial C}{\partial x} - u(x,t)C \right) \qquad (1)$$

where C is the solute concentration in the liquid phase and F is the concentration in the solid phase. As is generally known, the mass transport equation uses hydrodynamic dispersion, which is the combination of mechanical dispersion and diffusion, however molecular diffusion is negligible due to very low seepage velocity.

The advection-diffusion Eq. (1) has served as the main theoretical framework for modelling and transport of solute in porous media and for addressing critical environmental issues or waste disposal operations during the last few decades Jury and Fluhler [18]-. In Eq. (1), D and u may be constants or functions of time or space. n_p is porosity. Lapidus and Amundson [19] considered two cases, namely,

$$F = k_1 C^n + k_2 \qquad (2)$$

and

$$\frac{\partial F}{\partial t} = k_1 C^n - k_2 \qquad (3)$$

respectively, equilibrium and non-equilibrium isotherm between the concentrations in the two phases, where k_1 and k_2 are empirical constants of the medium. The isotherm is linear if n=1, and is non-linear if n>1. For simplicity, the former relationship is adopted in the present analysis. This assumption is generally valid when the adsorption process is fast in relation to the ground-water velocity Cherry et al. [20]. Using Eq. (2) in Eq. (1) for n=1we may get linear advection-diffusion equation,

$$\frac{\partial C}{\partial t} + \frac{1-n_p}{n_p}\frac{\partial (k_1 C + k_2)}{\partial t} = \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial C}{\partial x} - u(x,t)C \right) \qquad (4a)$$

or

$$R\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial C}{\partial x} - u(x,t)C \right)$$

where

$$R = 1 + \frac{1-n_p}{n_p}k_1 \qquad (4b)$$

The term on the left side of the equal sign of Eq. (4b) indicate the retardation factor (R) and change of concentration in time, the first two terms on the right side describe hydrodynamic diffusion and flow velocity. The dimensions of each term of Eq. (4b) i.e. dimension of diffusion is $(L^2 T^{-1})$ and of dimension velocity is (LT^{-1}), respectively. If both the parameters are independent to space variable x and time variable t, then these are called constant diffusion and uniform flow velocity respectively.

Let us write $R = 1$, $D(x,t) = D_0 f_1(x,t)$ and $u(x,t) = u_0 f_2(x,t)$, the linear advection-diffusion partial differential equation in one dimension in general form is,

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D_0 f_1(x,t)\frac{\partial C}{\partial x} - u_0 f_2(x,t)C \right) \qquad (5)$$

Introducing a new independent variable

$$X = -\int \frac{dx}{f_1(x,t)} \quad \text{or} \quad \frac{dX}{dx} = -\frac{1}{f_1(x,t)} \qquad (6)$$

which in case of temporally dependent dispersion along a uniform flow, i.e., for

$$f_1(x,t) = f(mt) \qquad \text{and} \qquad f_2(x,t) = 1 \qquad (7)$$

where m is a unsteady coefficient whose dimension is inverse of the dimension of t, i.e. of dimension (T^{-1}). Thus $f(mt)$ is expression in non-dimensional variable. Function is chosen such that $f(mt) = 1$, for $m = 0$ or $t = 0$. Thus Eq. (5) will assume the form

$$f(mt)\frac{\partial C}{\partial t} = D_0 \frac{\partial^2 C}{\partial X^2} + u_0 \frac{\partial C}{\partial X} \qquad (8)$$

where

$$X = -\frac{x}{f(mt)} \qquad (9)$$

Now the time dependent coefficient on left hand side may be got rid of by introducing another new time variable, T_n (Crank [21])

$$T_n = \int_0^t \frac{dt}{f(mt)} \qquad (10)$$

The partial differential equation (8) reduces into that with constant coefficients as

$$\frac{\partial C}{\partial T_n} = D_0 \frac{\partial^2 C}{\partial X^2} + u_0 \frac{\partial C}{\partial X} \qquad (11)$$

Further using transformations

$$Z = -X \qquad (12)$$

and

$$C(Z,T_n) = K(Z,T_n)\exp\left(\frac{u_0}{2D_0}Z - \frac{u_0^2}{4D_0}T_n \right) \qquad (13)$$

The advection-diffusion equation (11) reduces to a diffusion equation in terms of new dependent variable $K(Z, T_n)$, which is

$$\frac{\partial K}{\partial T_n} = D_0 \frac{\partial^2 K}{\partial Z^2} \tag{14}$$

3. Analytical Solutions

To proceed further, let us consider initial and boundary conditions for (5) in a finite longitudinal domain of length L. The analytical solutions are obtained for two cases. The input source is introduced at the origin of the domain. In both cases second boundary condition of flux type and homogeneous nature is imposed at the extreme end $x = L$ of the domain. In case of pulse type input source, the domain is assumed to be not solute free, instead it is assumed to be uniformly polluted by solute particles.

3.1. Case-I Pulse type input concentration of uniform nature

If the source of the input concentration remains uniform up to certain time period and after its elimination forever the input becomes zero. This type of condition is defined by first type or Dirichlet boundary condition. For uniform pulse type input concentration the initial and boundary conditions are,

$$C(x, t) = C_i, \qquad ; 0 \le x \le L, \ t = 0 \tag{15}$$

$$C(x, t) = \begin{cases} C_0 & ; x = 0, 0 < t \le t_0 \\ 0 & ; x = 0, t > t_0 \end{cases} \tag{16}$$

$$\frac{\partial C(x, t)}{\partial x} = 0 \qquad ; x = L, \ t \ge 0 \tag{17}$$

These conditions in terms of new space and time variables may be written as

$$K(Z, T_n) = C_i \exp-\left(\frac{u_0 Z}{2D_0}\right); \ 0 \le Z \le Z_0, \ T_n = 0 \tag{18}$$

$$K(Z, T_n) = \begin{cases} C_0 \exp\left(\frac{u_0^2}{4D_0} T_n\right) & ; Z = 0, 0 < T_n \le T_{n0} \\ 0 & ; Z = 0, T_n > T_{n0} \end{cases} \tag{19}$$

and

$$\frac{\partial K}{\partial Z} = -\frac{u_0}{2D_0} K \ ; \ Z = Z_0, \ T_n \ge 0 \tag{20}$$

respectively. Applying Laplace transform on the diffusion equation (14) and using initial condition (18), we may get

$$\frac{d^2 \bar{K}}{dZ^2} - \frac{p}{D_0} \bar{K} = -\frac{C_i}{D_0} \exp-\left(\frac{u_0 Z}{2D_0}\right) \tag{21}$$

The boundary conditions (19) and (20) become

$$\bar{K}(Z, p) = \frac{C_0}{(p - \alpha^2)} \left[1 - \exp-\left\{(p - \alpha^2) T_{n0}\right\}\right];$$

$$Z = 0 \ ; \alpha^2 = u_0^2 / 4D_0 \tag{22}$$

$$\frac{d\bar{K}}{dZ} = -\frac{u_0}{2D_0} \bar{K} \ ; Z = Z_0 \tag{23}$$

Thus the general solution of equation (21) may be written as

or

$$\bar{K}(Z, p) = c_1 \exp-\left(Z\sqrt{p / D_0}\right) + c_2 \exp\left(Z\sqrt{p / D_0}\right) + \frac{C_i \exp-(u_0 Z / 2D_0)}{\left\{p - D_0(u_0 / 2D_0)^2\right\}} \tag{24}$$

Using conditions (22) and (23) on the above solution, we get values of c_1 and c_2 as

$$c_1 = \frac{\left[C_0\left(1 - \exp-\left\{(p - \alpha^2) T_{n0}\right\}\right) - C_i\right]}{(p - \alpha^2)\left\{1 + \left(\frac{\sqrt{p} - \alpha}{\sqrt{p} + \alpha}\right) \exp-\left(2Z_0\sqrt{p / D_0}\right)\right\}} \tag{25}$$

and

$$c_2 = \frac{\left[C_0\left(1 - \exp-\left\{(p - \alpha^2) T_{n0}\right\}\right) - C_i\right]\left(\frac{\sqrt{p} - \alpha}{\sqrt{p} + \alpha}\right) \exp-(2Z_0\sqrt{p / D_0})}{(p - \alpha^2)\left\{1 + \left(\frac{\sqrt{p} - \alpha}{\sqrt{p} + \alpha}\right) \exp-\left(2Z_0\sqrt{p / D_0}\right)\right\}} \tag{26}$$

Thus the solution of Eq. (24) in the Laplace parameter may be written as

$$\bar{K}(Z, p) = \frac{\left[C_0\left(1 - \exp-\left\{(p - \alpha^2) T_{n0}\right\}\right) - C_i\right]}{(p - \alpha^2)\left[1 + \left(\frac{\sqrt{p} - \alpha}{\sqrt{p} + \alpha}\right) \exp-\left(2Z_0\sqrt{p / D_0}\right)\right]}$$

$$\times \left[\exp-\left(Z\sqrt{p / D_0}\right) + \left(\frac{\sqrt{p} - \alpha}{\sqrt{p} + \alpha}\right) \exp-\left\{(2Z_0 - Z)\sqrt{p / D_0}\right\}\right]$$

$$+ \frac{C_i \exp-(u_0 Z / 2D_0)}{(p - \alpha^2)} \tag{27}$$

Applying inverse Laplace transform on it, we get $K(Z, T_n)$ and using back transformation (13), we may get the desired analytical solution in $C(Z, T_n)$ of the initial and boundary value problem (5), (15), (16) and (17), as follows

$$C(Z, T_n) = \begin{cases} C_i + (C_0 - C_i) E(Z, T_n) \\ C_i + (C_0 - C_i) E(Z, T_n) - C_0 E(Z, T_n - T_{n0}) \end{cases}$$

$$\begin{array}{l} ; 0 < T_n \le T_{n0} \\ ; T_n > T_{n0} \end{array} \tag{28}$$

where

$$E(Z, T_n) = \frac{1}{2} \operatorname{erfc}\left(\frac{Z - u_0 T_n}{2\sqrt{D_0 T_n}}\right) + \frac{1}{2} \exp(u_0 Z / D_0) \operatorname{erfc}\left(\frac{Z + u_0 T_n}{2\sqrt{D_0 T_n}}\right)$$

$$+ \frac{1}{2}\left(2 + \frac{u_0(2Z_0 - Z)}{D_0} + \frac{u_0^2 T_n}{D_0}\right) \exp(u_0 Z_0 / D_0) \operatorname{erfc}\left(\frac{(2Z_0 - Z) + u_0 T_n}{2\sqrt{D_0 T_n}}\right)$$

$$- \left(\frac{u_0^2 T_n}{\pi D_0}\right)^{1/2} \exp\left(\frac{u_0 Z_0}{D_0} - \frac{(2Z_0 - Z + u_0 T_n)^2}{4D_0 T_n}\right) \tag{29}$$

$Z = x / f(mt)$, $Z_0 = L / f(mt)$ and T_n may be obtained from transformation (10).

3.2. Case-II Pulse type input concentration of varying nature

It may happen that if the input concentration continuously uniform up to certain time period and after its elimination forever the input becomes zero. But due to human, industries and some other responsible activities, the source of input concentration increases till certain time period, beyond that it starts decreasing, when source of concentration is eliminated forever. The type of condition defined by third type of boundary condition and is

known as Cauchy or mixed boundary condition. Thus varying pulse type input condition may be written as,

$$-D(x,t)\frac{\partial C}{\partial x}+u(x,t)C=\begin{cases}u_0C_0 & ;x=0 \ ,0<t\le t_0 \\ 0 & ;x=0 \ ,t>t_0\end{cases} \quad (30)$$

Using the expressions i.e., $D(x,t)=D_0f(mt)$ and $u(x,t)=u_0$, the above condition may be written as

$$-D_0f(mt)\frac{\partial C}{\partial x}+u_0C=\begin{cases}u_0C_0 & ;x=0 \ ,0<t\le t_0 \\ 0 & ;x=0 \ ,t>t_0\end{cases} \quad (31)$$

It may be written in terms of $K(Z,T_n)$ as

$$D_0\frac{\partial K}{\partial Z}-\frac{u_0}{2}K=\begin{cases}-u_0C_0\exp(\alpha^2T_n) & ;Z=0 \ ,0<T_n\le T_{n0} \\ 0 & ;Z=0 \ ,T_0>T_{n0}\end{cases} \quad (32)$$

where $\alpha^2=u_0^2/4D_0$.

Applying Laplace transformation on equation (32), the input boundary condition is reduced to

$$D_0\frac{d\bar{K}}{dZ}-\frac{u_0}{2}\bar{K}=\frac{-u_0C_0}{(p-\alpha^2)}\Big[1-\exp-\{(p-\alpha^2)T_{n0}\}\Big] \ ;Z=0 \quad (33)$$

Using conditions (33) and (23), we get value of c_1 and c_2 from equation (24) are

$$c_1=\frac{u_0\Big[C_0\big(1-\exp-\{(p-\alpha^2)T_{n0}\}\big)-C_i\Big]}{\sqrt{D_0}\,(p-\alpha^2)(\sqrt{p}+\alpha)\left\{1-\left(\dfrac{\sqrt{p}-\alpha}{\sqrt{p}+\alpha}\right)^2\exp-\left(2Z_0\sqrt{p/D_0}\right)\right\}} \quad (34)$$

$$c_2=\frac{u_0\Big[C_0\big(1-\exp-\{(p-\alpha^2)T_{n0}\}\big)-C_i\Big]\left(\dfrac{\sqrt{p}-\alpha}{\sqrt{p}+\alpha}\right)\exp-(2Z_0\sqrt{p/D_0})}{\sqrt{D_0}\,(p-\alpha^2)(\sqrt{p}+\alpha)\left\{1-\left(\dfrac{\sqrt{p}-\alpha}{\sqrt{p}+\alpha}\right)^2\exp-\left(2Z_0\sqrt{p/D_0}\right)\right\}} \quad (35)$$

Thus the solution of Eq. (24) in the Laplace parameter may be written as

$$\bar{K}(Z,p)=u_0\Big[C_0\big(1-\exp-\{(p-\alpha^2)T_{n0}\}\big)-C_i\Big]$$

$$\times\frac{\left[\exp-(Z\sqrt{p/D_0})+\left(\dfrac{\sqrt{p}-\alpha}{\sqrt{p}+\alpha}\right)\exp-\{(2Z_0-Z)\sqrt{p/D_0}\}\right]}{\sqrt{D_0}\,(p-\alpha^2)(\sqrt{p}+\alpha)\left\{1-\left(\dfrac{\sqrt{p}-\alpha}{\sqrt{p}+\alpha}\right)^2\exp-\left(2Z_0\sqrt{p/D_0}\right)\right\}}$$

$$+\frac{C_i\exp-(u_0Z/2D_0)}{(p-\alpha^2)} \quad (36)$$

Applying inverse Laplace transform on it, we get $K(Z,T_n)$ and using back transformation (13), we may get the desired analytical solution in $C(Z,T_n)$ of the initial and boundary value problem (5), (15), (16) and (17), as follows

$$C(Z,T_n)=\begin{cases}C_i+(C_0-C_i)F(Z,T_n) \\ C_i+(C_0-C_i)F(Z,T_n)-C_0F(Z,T_n-T_{n0})\end{cases}$$
$$;0<T_n\le T_{n0}$$
$$;T_n>T_{n0} \quad (37)$$

where

$$F(Z,T_n)=\frac{1}{2}\text{erfc}\left(\frac{Z-u_0T_n}{2\sqrt{D_0T_n}}\right)+\left(\frac{u_0^2T_n}{\pi D_0}\right)^{1/2}\exp\left(-\frac{(Z-u_0T_n)^2}{4D_0T_n}\right)$$

$$-\frac{1}{2}\left(1+\frac{u_0Z}{D_0}+\frac{u_0^2T_n}{D_0}\right)\exp\left(\frac{u_0Z}{D_0}\right)\text{erfc}\left(\frac{Z+u_0T_n}{2\sqrt{D_0T_n}}\right)$$

$$+\left(\frac{4u_0^2T_n}{\pi D_0}\right)^{1/2}\left(1+\frac{u_0}{4D_0}(2Z_0-Z+u_0T_n)\right)\exp\left(\frac{u_0Z_0}{D_0}-\frac{(2Z_0-Z+u_0T_n)^2}{4D_0T_n}\right)$$

$$-\frac{u_0}{D_0}\left((2Z_0-Z)+\frac{3u_0T_n}{2}+\frac{u_0}{4D_0}(2Z_0-Z+u_0T_n)^2\right)\times$$

$$\exp\left(\frac{u_0Z_0}{D_0}\right)\text{erfc}\left(\frac{(2Z_0-Z)+u_0T_n}{2\sqrt{D_0T_n}}\right) \quad (38)$$

$Z=x/f(mt)$, $Z_0=L/f(mt)$ and T_n may be obtained from transformation (10).

4. Numerical Results and Discussion

Two analytical solutions (28) and (37) are obtained for the temporally dependent diffusion of uniform pulse type and varying pulse type input sources, respectively, along uniform flow. Concentration values are evaluated from these solutions in the finite domain $0\le x\le1$, (i.e., $L=1.0$ km is chosen) at times t (yr) $=0.1,\ 0.4,\ 0.7$ and 1.0, for input values $C_0=1.0$, $u_0=0.11$ (km/yr), $D_0=0.21$ (km²/yr) and $m(\text{yr})^{-1}=0.1$. Figure (1a) shows the concentration distribution along the finite domain of a pulse type uniform input concentration, in the time domain $t<t_0$. The time of elimination of the source is chosen as $t_0=1.2$ (yr). The four curves represent the solute concentration for $f(mt)=\exp(-mt)$ at $t(\text{yr})=0.1,\ 0.4,\ 0.7,\ 1.0$. The figure (1b) represent the solute concentration for same $f(mt)$, at $t(\text{yr})=1.3$, 1.6, 1.9 and 2.2 in the time domain $t>t_0$. In the former figure input concentration (C/C_0) is 1.0 i.e. in the presence of solute particles, while in the latter figure, it is zero since solute particles are absent. So the input remains uniform in both the time domains.
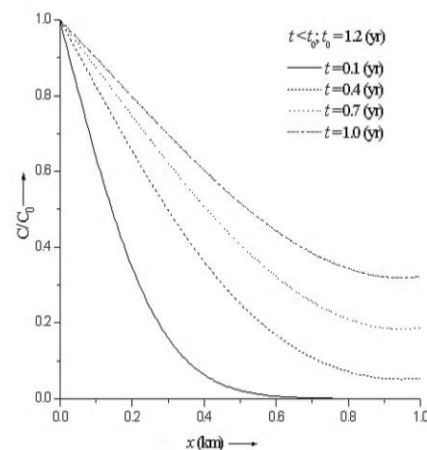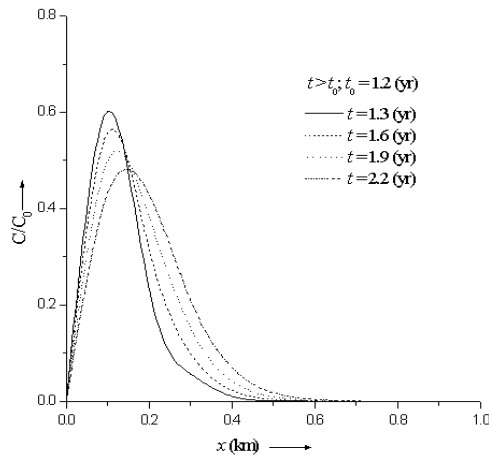

Figure 1a. Concentration values obtained from solution (28), i.e., for uniform pulse type input in the time domain $t<t_0$ in a finite domain of length $L=1.0(\text{km})$. Curves are drawn for $f(mt)=\exp(-mt)$, where $m=0.1(\text{yr})^{-1}$ and $t_0=1.2(\text{yr})$.
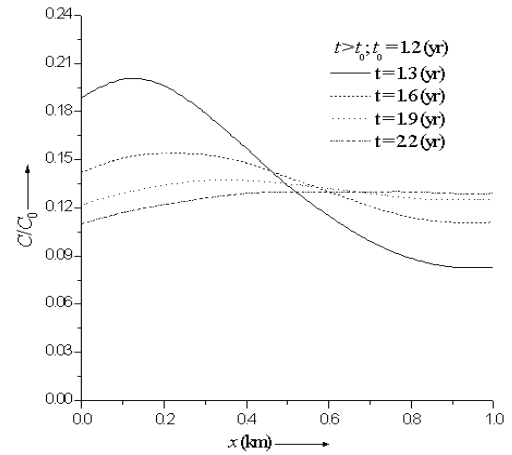
Figure 1b. Concentration values obtained from solution (28), i.e., for uniform pulse type input in the time domain $t > t_0$ in a finite domain of length $L = 1.0(\mathrm{km})$. Curves are drawn for $f(mt) = \exp(-mt)$, where $m = 0.1(\mathrm{yr})^{-1}$ and $t_0 = 1.2(\mathrm{yr})$.

Similarly figures (2a,b) represent the solution (37), for $f(mt) = \exp(-mt)$. It is evident from the figures that the solution (37) represents that the input concentration increases in the time domain $t < t_0$ i.e. in the presence of solute particles and it decreases in the time domain $t > t_0$ since solute particles are absent and decreases to zero.
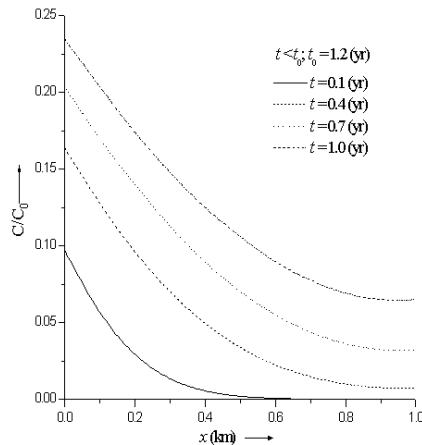

Figure 2a. Concentration values obtained from solution (37), i.e., for varying pulse type input in the time domain $t < t_0$ in a finite domain of length $L = 1.0(\mathrm{km})$. Curves are drawn for $f(mt) = \exp(-mt)$, where $m = 0.1(\mathrm{yr})^{-1}$ and $t_0 = 1.2(\mathrm{yr})$.

Figure 2b. Concentration values obtained from solution (37), i.e., for varying pulse type input in the time domain $t > t_0$ in a finite domain of length $L = 1.0(\mathrm{km})$. Curves are drawn for $f(mt) = \exp(-mt)$, where $m = 0.1(\mathrm{yr})^{-1}$ and $t_0 = 1.2(\mathrm{yr})$.

Figure (3) compares the concentration distribution behaviour in finite and semi-infinite domains. The input conditions for the both the domains are of uniform pulse type. It means the input concentration, (C/C_0) is 1.0 i.e. in the presence of solute particles ($t < t_0$) and is zero i.e. in the absence of solute particles ($t > t_0$) where $t_0 = 1.8$ (yr) is considered. The curves are drawn for exponentially decreasing function, $f(mt) = \exp(-mt)$.

The dotted curves represent the concentration values in the finite domain evaluated from solution (28), while solid curves represent those in the semi-infinite domain obtained when the initial solute concentration is assumed exponentially decreasing function of space variable which tends to zero at infinity i.e., $C(x,t) = C_i \exp(-\gamma x)$ for $x \geq 0$ at $t = 0$.

Thus the desired analytical solution for temporally dependent dispersion along uniform flow in semi-infinite domain for uniform pulse type input concentration is Jaiswal et al. [22]

$$C(x,T_n) = \begin{cases} F_1(x,T_n) - F_2(x,T_n) + F_3(x,T_n) \\ F_1(x,T_n) - F_1(x,T_n - T_{n0}) - F_2(x,T_n) + F_3(x,T_n) \end{cases}$$

$$; 0 < T_n \leq T_{n0}$$
$$; T_n > T_{n0} \qquad (39)$$

where

$$F_1(x,T_n) = \frac{C_0}{2}\left[\mathrm{erfc}\left\{\frac{x/f(mt) - u_0 T_n}{2\sqrt{D_0 T_n}}\right\} + \exp\left\{\frac{u_0 x}{D_0 f(mt)}\right\}\mathrm{erfc}\left\{\frac{x/f(mt) + u_0 T_n}{2\sqrt{D_0 T_n}}\right\}\right] \quad (40)$$

$$F_2(x,T_n) = \frac{1}{2} F_3(x,T_n)\left[\mathrm{erfc}\left\{\frac{x/f(mt) - (2\gamma D_0 + u_0)T_n}{2\sqrt{D_0 T_n}}\right\}\right.$$

$$\left. + \exp\left\{(2\gamma + u_0/D_0)(x/f(mt)\right\}\mathrm{erfc}\left\{\frac{x/f(mt) + (2\gamma D_0 + u_0)T_n}{2\sqrt{D_0 T_n}}\right\}\right] \quad (41)$$

$$F_3(x,T_n) = C_i \exp\left\{\left(\gamma^2 D_0 + \gamma u_0\right)T_n - \gamma x/f(mt)\right\} \qquad (42)$$

and T_n may be written in terms of t using the transformation (10) for an expression $f(mt)$.
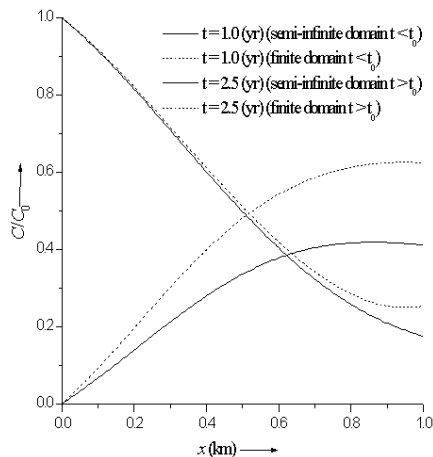
Figure 3. Comparison of concentration values of uniform pulse type, in semi-infinite and finite domains for $f(mt) = \exp(-mt)$ at $t = 1.0$ (yr) when $t < t_0$ and at $t = 2.5$ (yr) when $t > t_0$, respectively, where $t_0 = 1.8$ (yr). The solid curves represent the concentration values in the semi-infinite domain (39), while dotted curves represent those in the finite domain (28), for $u_0 = 0.25$ (km/yr), $D_0 = 0.14$ (km²/yr).

The comparison are done for $\gamma = 0.01 \left(\text{km} \right)^{-1}$ at $t = 1.0$ (yr) when $t < t_0$ and $t = 2.5$ (yr) when $t > t_0$. It may be observed that, a position away from the source gets less polluted in a semi-infinite domain than that in a finite domain, in the presence of source. After its elimination, the rehabilition of finite domain will be slower than that of a semi-infinite domain.

Analytical solutions of temporally dependent diffusion along uniform flow are useful for understanding the transient response of soil and groundwater pollution. However, water level usually varies arbitrarily in time. Even though, this solution may still provide us with valuable insight in the transient response of aquifer to such seasonal forcing fluctuations. The transformation (6) may be used for space-time dependent advection-diffusion equation where functions are considered in degenerate form, with an assumption $f(mt) = 1$, for $m = 0$ or $t = 0$ to.

5. Conclusion

Analytical solutions of one-dimensional advection-diffusion equation are obtained in a finite domain for two sets of boundary conditions. In the both set, initial condition is non-homogeneous. The input condition is pulse type. The second boundary condition in each set is flux type of homogeneous nature . Laplace transformation technique is utilized in order to attain the analytical solutions. From figures (1,2), distribution of solute constration shows the respective boundary conditions. The pulse type input boundary condition helps predicting the rehabilitation process of a degraded system once the source of the solute contamination is eliminated for ever. Such analytical solutions may serve as tools in validating numerical solutions in more realistic dispersion problems. These solutions are facilitating to assess the transport of pollutants solute concentration away from its source along a flow through soil medium, aquifers, and oil reservoirs etc. which has always been difficult because of the inherent complexities.

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