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Implementation of a Vibration Absorbers to Euler-Bernoulli Beam and Dynamic Analysis of Moving Car

*Mehmet Akif Koç
*Sakarya Applied Sciences University, Technology Faculty Mechatronics Engineering

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Abstract

In this study, the dynamic analysis of Euler-Bernoulli bridge beam, single-degree of freedom moving vehicle and vibration absorber is discussed according to vehicle and bridge dynamics. The equations of motion entire system representing the physical model of the system are obtained using the Lagrange function. Then the vibration equation of vehicle and bridge beam which attached vibration absorber has been solved numerically with a special program written in computer software. The effect of the basic parameters such as frequency (f_i), damping (c) coefficient, mass (m), mass ratio (α) and location of the vibration absorber on the bridge are examined in detail in terms of vehicle and bridge dynamics.

Keywords: Vibration absorbers, Moving Car, Mass ratio, Absorber Location.

1. INTRODUCTION

Dynamic analysis of structures effect of moving car are important in science and have been studied over a century by scientist [1,2]. The first studies related to moving load studies are those in which moving load is considered as concentrated load. Since these studies are mainly focused on bridge dynamics rather than vehicle dynamics, this acceptance is not a problem in terms of bridge engineering. However, if the vehicle dynamics are to be analysed in addition to the bridge dynamics, the adoption of the concentrated load is insufficient and the studies in this field are limited [3]. With the passing of vehicles such as heavy tonnage trains on flexible structures such as bridges, this issue was examined by mechanical engineers in terms of vehicle dynamics. If vehicle dynamics is to be investigated in addition to bridge dynamics, then vehicle-bridge interaction (VBI) analysis is required. In this field, some of numerical studies [4–8] are important for the dynamics of the bridge effect of different conditions and boundary cases. Taking into account inertial impact of the moving loads the subject have been studied using analytical and numerical by finite element method (FEM) [9–11]. Dynamic analysis of Euler-Bernoulli beam and control have been studied by In the studies performed, it has been shown that the PTMD placed in the anti-mode part of the mode shapes of the bridge beam is highly effective in damping the vibrations.

The biggest problem of using vibration absorbers in flexible structures, the impact frequency of the moving car is variable and when the car velocity changed. The others, the moving vehicles on the bridge have different dynamic characteristics. In this study, in order to dampen the vibrations of Euler-Bernoulli beam under the influence of moving carriage, the idea of placing the anti-nodes of the beam used in previous studies[12] as a vibration absorber was developed. For this purpose, the idea of placing a vibration absorber at the joint points corresponding to more than one mode shape of the bridge beam was used. Vibrations occurring in any structural system can be considered as the sum of multiple vibration modes that make up this structural system. Therefore, in this study, the vibrations of the first four modes of the bridge beam are considered. For this purpose, a vibration absorber is placed in the antinode section (bridge half distance) corresponding to the first mode of the bridge beam. A second vibration absorber is located in the first quarter and last quarter of the bridge distance, which are the anti-node portions of the second mode of the bridge beam. In addition, the effect of damped and non-damped vibration absorber on the bridge dynamics and the effect of the position of the vibration absorber on the bridge dynamics are also examined. In addition, the effects of the mass ratio parameter defined by the ratio of the mass of the absorber and the bridge beam to the vehicle and the bridge dynamics are shown. In conclusion, this study proved that the use of vibration absorber can be effective in damping vibrations caused by forces caused by moving mass in flexible structures such as bridges.

2. MATHEMATICAL MODELLING

2.1. Governing of equation of motion vehicle-bridge absorber interaction system

In order to damping the disturbing vibrations between the moving vehicle and the flexible structure, the physical model given in Figure 1 is discussed. The parameters m, c, k on this

*Corresponding Author: Sakarya Applied Sciences University, Technology Faculty Mechatronics Engineering

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model are respectively the mass of the moving vehicle, the damping value and the spring coefficient. The parameters and, respectively, are the mass and spring constants of the damper suspended on the bridge beam. Besides, $w_b\ (x,\ t)$ parameter is the transverse deformation of the bridge at point x at time t.

For the system given by Figure 1, the kinetic and potential energies are given by Eq.(1-2) respectively.

$$E_k = \frac{1}{2}\left\{\begin{array}{l} \int_0^L \mu\left[\dot{w}_b^{\,2}(x,t)\right]dx \\ +m\dot{y}^2(t)+m_1\dot{y}_1^2(t) \\ +m_2\dot{y}_2^2(t)+m_3\dot{y}_3^2(t) \end{array}\right\}, \tag{1}$$

$$E_p = \frac{1}{2}\left\{\begin{array}{l} \int_0^L EIw_b''^2(x,t)dx \\ +k\left[y(t)-w_b(\xi(t),t)\right]^2 H(x,\xi(t)) \\ +k_1\left[y_1(t)-w_b(L/4,t)\right]^2 \\ +k_2\left[y_2(t)-w_b(L/2,t)\right]^2 \\ +k_3\left[y_3(t)-w_b(3L/4,t)\right]^2 \end{array}\right\} \tag{2}$$

where μ and EI, refer to the mass of the unit length and flexural rigidity of the bridge beam, respectively. For any point x on the bridge beam and any time t, transverse deformation is written by Galerkin function as follows:

$$\begin{aligned} w_b(x,t) &= \sum_{i=1}^{n}\varphi_i(x)\eta_{bi}(t), \\ \dot{w}_b(x,t) &= \sum_{i=1}^{n}\varphi_i(x)\dot{\eta}_{bi}(t), \\ w_b''(x,t) &= \sum_{i=1}^{n}\varphi_i''(x)\eta_{bi}(t), \\ \varphi_i(x) &= \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi x}{L}\right), \quad i=1,2,\ldots,n. \end{aligned} \tag{3}$$

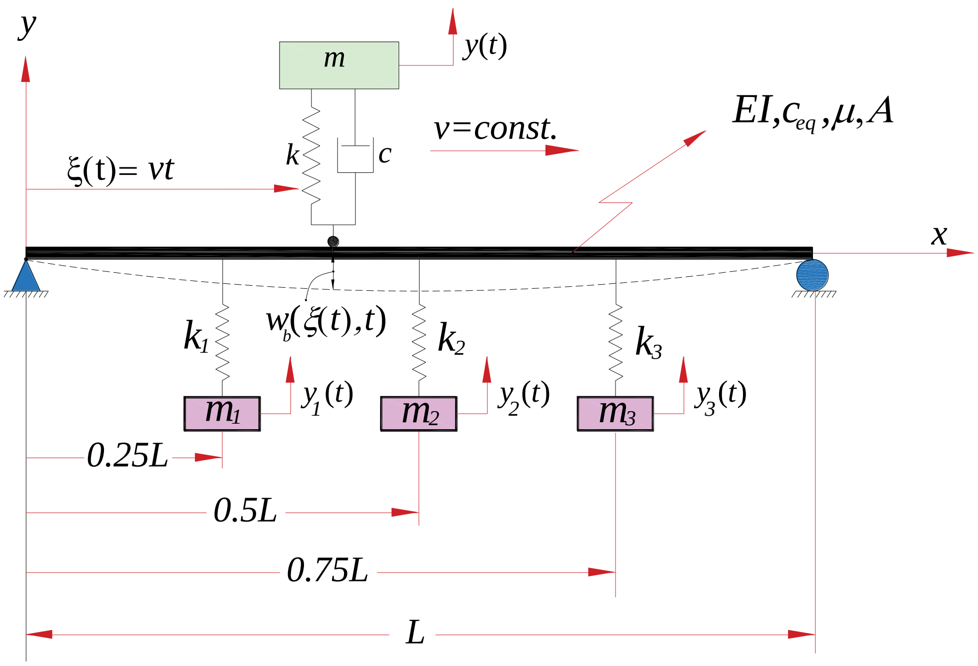

Figure 1. Model of Euler-Bernoulli Bridge beam with attached PTMDs and subjected to moving vehicle.

where η_{bi} is the *i-th* time dependent generalized nodal coordinate, φ_i is *i-th* mode shape function. The orthogonality of the modes can be expressed as:

$$\int_0^L \mu\varphi_i(x)\varphi_j(x)dx = N_i\delta_{ij},$$
$$\int_0^L EI\varphi_i''(x)\varphi_j''(x)dx = \Pi_i\delta_{ij}, \tag{4}$$

In Eq. (4) δ_{ij} (i, j=1,2,…,n) are the Kronecker delta function. For the vehicle- bridge system the Rayleigh's dissipation function can be expressed as below:

$$D = \frac{1}{2}\left\{\begin{array}{l}\int_0^L\left\{c_{eq}\dot{w}_b{}^2(x,t)dx\right.\\ +c[\dot{y}(t)-\dot{w}_b(\xi(t),t)]^2 H(x-\xi(t))\end{array}\right\} \tag{5}$$

In Eq. (5) c_{eq} is the equivalent damping coefficient of bridge girder. By using the Galerkin function given by equation (3), the equations of motion of the VBI system are obtained by Lagrange equation. Accordingly, the equation of motion for vehicle body vertical vibrations can be obtained by:

$$\ddot{y} = \left\{-c[\dot{y}(t)-\dot{w}_b(x,t)]-k[y(t)-w_b(x,t)]\right\}\frac{1}{m} \tag{6a}$$

And first absorber vertical acceleration is governed by:

$$\ddot{y}_1 = \left\{-k_1[y(t)-w_b(L/4,t)]\right\}\frac{1}{m_1} \tag{6b}$$

The equation of motion of second vibration absorber is given by:

$$\ddot{y}_2 = \left\{-k_2[y(t)-w_b(L/2,t)]\right\}\frac{1}{m_2} \tag{6c}$$

And finally the third absorber motion equation:

$$\ddot{y}_3 = \left\{-k_3[y(t)-w_b(3L/4,t)]\right\}\frac{1}{m_3} \tag{6d}$$

the equation of motion representing the bridge dynamics is written as follows:

$$\begin{aligned}&N_i\ddot{\eta}_i(t)+c_{eq}\varphi_i^2(x)\dot{\eta}_i(t)\\ &+\Pi_i\eta_i(t)+\Lambda\varphi_i(\xi_1(t))\left\{\begin{array}{l}f_c+c\left[\dot{w}_b(\xi(t),t)\Lambda_1-\dot{y}(t)\right]\\ +k\left[w(\xi(t),t)\Lambda-y(t)\right]\end{array}\right\}\\ &+k_1\left[w_b(L/4,t)-y_1(t)\right]+k_2\left[w_b(L/2,t)-y_2(t)\right]\\ &+k_3\left[w_b(3L/4,t)-y_3(t)\right]=0 \quad i=1,2,3,4.\end{aligned} \tag{7}$$

where Λ is:

$$\Lambda = \begin{cases}1, & for\ 0\le t<t_1\\ 0 & elsewhere,\end{cases} \tag{8}$$

2.2. Solution of equation of motion entire system with state-space form and using fourth-order Runge-Kutta method

In this study fourth order Runge-Kutta algorithm has been used for the equation of motion entire system given by Eqs.(6-7). The bridge dynamic is represented by second order differential equation and in this study, the first four modes are used for calculate to dynamic response of bridge beam. For this reason, four second order differential equations have been obtained for bridge dynamics. These four second order differential equations are reduced to eight first order differential equations using appropriate state constants. Similarly, vehicle dynamics is expressed by two first order differential equations. Vibration absorbers are represented by six first order differential equations. As a result, a total of sixteen first order differential equations were solved with a special program prepared in computer. Detailed information about the solution algorithm is given by [13–16] studies.

3. NUMERICAL ANALYSIS

Figure 2 shows four different VBI cases. In case 1, the vibration absorber which contain only spring and mass is attached the midpoint of the bridge. The frequency of this vibration absorber is set to the first mode frequency of the bridge beam (f_1 = 0.2108 Hz). In the case 2, a damper was added to the absorber placed at the midpoint of the bridge. The parameters of the damper given by Table 1. In case 3, the first two modes of the bridge are taken into account and placed on three absorber bridges (f_1=0.2108 and f_2=0.8432 Hz). In Case 4, no dampers were used to compare other cases. The vehicle and bridge parameters used in this study given by study [16].

Table 1. Damping properties of the vibration absorber used Case 2.

Parameters	Value
Critical damping	26476.46 Nsm^{-1}
Damping ratio	0.377
Damped natural frequency	0.195 Hz

In Figures 3 and 4 comparisons of the matrices constituted from the vectors of displacements and accelerations depending on the position and the vehicle speed on the bridge are shown, respectively. The Frobenius norm used for comparison of the matrices is expressed as given:

$$\|A\|_F = \sqrt{\sum_{i=1}^{m_A}\left(\sum_{j=1}^{n_A}|A_{ij}|^2\right)} \tag{9}$$

In Eq. (9), A is any matrix with the dimension of $m_A x n_A$, and $|A_{ij}|$ is value of the element at the *ith* row and *jth* column. Figures 3a-d are for the displacements, and Figures 4a-d are for the accelerations of the bridge for the cases of the usage of the cases 1, 2, 3 and 4. A comparison of the Figures 3 and 4 is briefly given in Table 2.

For the four cases, the displacements and accelerations of the vehicle are presented in Figures 5a and b depending on the velocity of the vehicle between 1 and 90 m/s with an increment of 1.2 m/s. In all the cases the absorbers are tuned to the fundamental frequency of the bridge which is

f_1=0.2108 Hz. except for case 3 the absorbers at the locations of 0.25L and 0.75 L are tuned to the frequency of the second mode of the bridge that is f_2=0.8432 Hz. As can be seen from Figure 5a the maximum displacement is occurred at the velocity of 19 m/s as 19.9 mm in the case 4 that there is no absorber, while, the maximum is 16.8 mm at 11 m/s in the case 1.

Moreover, for the case 2, it is 17.5 mm at 14 m/s, and for 3 it is 17.3 mm at 12 m/s. In Figure 4, the maximum acceleration is 0.2248 for case 4, and 0.212 for the case 1, 0.2046 for case 2, and for the case 3 it is 0.1298 m/s^2. Generally, the case 3 is the best design both the bridge and the vehicle. In Figures 6a and b, the displacements and accelerations of the vehicle body for a travelling speed of v=25 m/s are given. In addition, Table 3 shows a comparison of the maximum displacements and accelerations and their locations of the bridge at which they are occurred.
The mass ratio β, is the ratio of the mass of the damper and the mass of the bridge beam, is expressed as below

$$\beta = \frac{m_i}{\mu L} \quad (i=1,2,3) \tag{10}$$

In terms of vehicle and bridge dynamics, the effects of the mass ratios up to 1 percent of the bridge mass are presented in Figures 6a and b. The mass of the unit length of the beam is μ=2x10^4 kg/m and total mass of the bridge beam is M=2x10^6 kg for the given length L=100 m. Taking the mass ratio 0.0001≤ β ≤0.01, with an increment of 2.02x10^{-4} the analysis have been done for a travelling velocity of the vehicle v=25 m/s and Δt = 0.01. The frequencies of the absorbers at 0.5L and 0.25L and 0.75 L are kept constant using stiffness coefficients of the springs $k_i = 2f_i^2\pi m_i (i = 1,2)$. Thus, the frequency of the absorber at midpoint is kept as the first natural frequency of the bridge beam. In addition, for the others at 0.25 and 0.75 L the frequencies are kept as the frequency of the second mode. As can be seen from Figure 6a and b the best performance has been achieved for the vehicle and bridge vibrations using the case 3. However for all the cases, when mass ratio (β) is increased the displacements of the vehicle and bridge are generally decreased. Figure 7 shows the effect of the mass ratio on the vertical displacement of the vehicle body and the maximum deflection of the bridge midpoint. As shown in the figures, the maximum displacement amounts decrease as the mass ratio increases.

In this section, for the effect of the position of the one absorber on the bridge, the vehicle and bridge dynamics were studied. The absorber has been placed at the positions on the bridge starting from 5 to 98 m with a 0.8 m increment from the left to the right end of the bridge. For this analysis, the resonance frequency of the absorber is tuned to the fundamental frequency of the bridge beam. In addition, for all the positions of the absorber, analyses were performed using the time step size and constant speed of the vehicle bridge as Δt = 0.01 s and 25 m/s, respectively. Moreover, for analysing the effect of different absorber stiffness and masses for a particular tuning frequency, the stiffness and the mass are multiplied with a parameter α. With this parameter, the tuning frequencies of the absorber are defined as follows.

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{\alpha k_2}{\alpha m_2}} \tag{11}$$

Table 2. Frobenius norm value of the bridge for vertical displacement and acceleration.

cases	$\left\| y_{\max}(x/l,v) \right\|_F$	Vibration reduction rate (%)	$\left\| a_{\max}(x/l,v) \right\|_F$	Vibration reduction rate (%)
1	0.6195	14.29	2.3717	0.67
2	0.6308	12.72	2.2421	6.1
3	0.5875	18.71	0.9968	58.25
4	0.7228	---	2.3878	---

Changing the value of this parameter will not change the natural frequency of the absorber, however, the increase or decrease of this value will cause the absorber to increase or to decrease the stored potential energy in spring and kinetic energy in mass. The effects of the absorber location points on the beam are presented in Figure 8. The minimum displacements have occurred at the midpoint of the beam when the position of the absorber is about 55-60 percent of the beam length for all the values of α. Moreover, the better performance is achieved for the greatest value of the=1.8. This is because the absorption capacity of the absorber is increased. When a realistic application of the passive vibration absorber is considered, a design engineer should optimize the range of this parameter considering its performance in vibration reduction and adverse effect of additional forces on the bridge beam due to the increased mass of the absorber.

Figures 9a and b show the maximum displacements and accelerations of the vehicle body depending on the position and values of α. The displacement of the bridge forces the vehicle to vibrate due to the interaction between them. When the displacements of the bridge are reduced as in Figure 8, this means that the displacements and the accelerations of the vehicle are reduced at the same time. However, as seen from Figures 9a and b, respectively, the minimum value of vehicle body displacement is obtained for the absorber location at 52 percent of the bridge length while the minimum value of the body accelerations is obtained for the location of the absorber between 60-65 percent of the length. It is also observed that any increase in the parameter α does not affect these positions of the absorber. However when the parameter α is increased from 0.6 to 1.8 the displacements of the vehicle is dropped from 0.0158 to 0.0122 m, while the acceleration is dropped from 0.044 to 0.033 m/s^2.

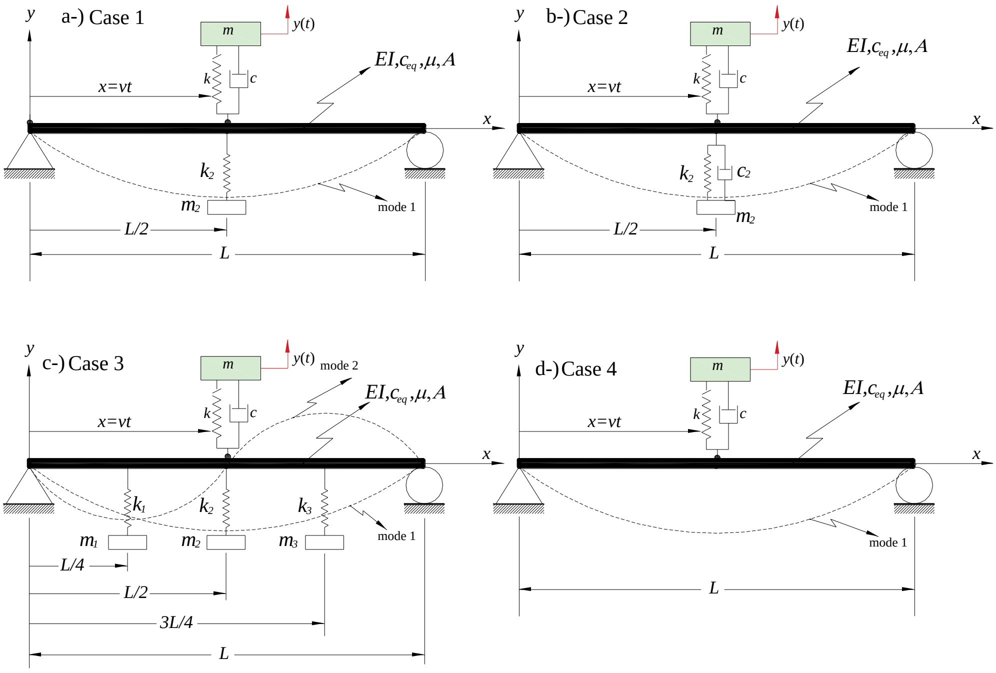

Figure 2. Euler-Bernoulli beam traversed by a moving car (a) Case 1; (b) Case 2; (c) Case 3; (d) Case 4.

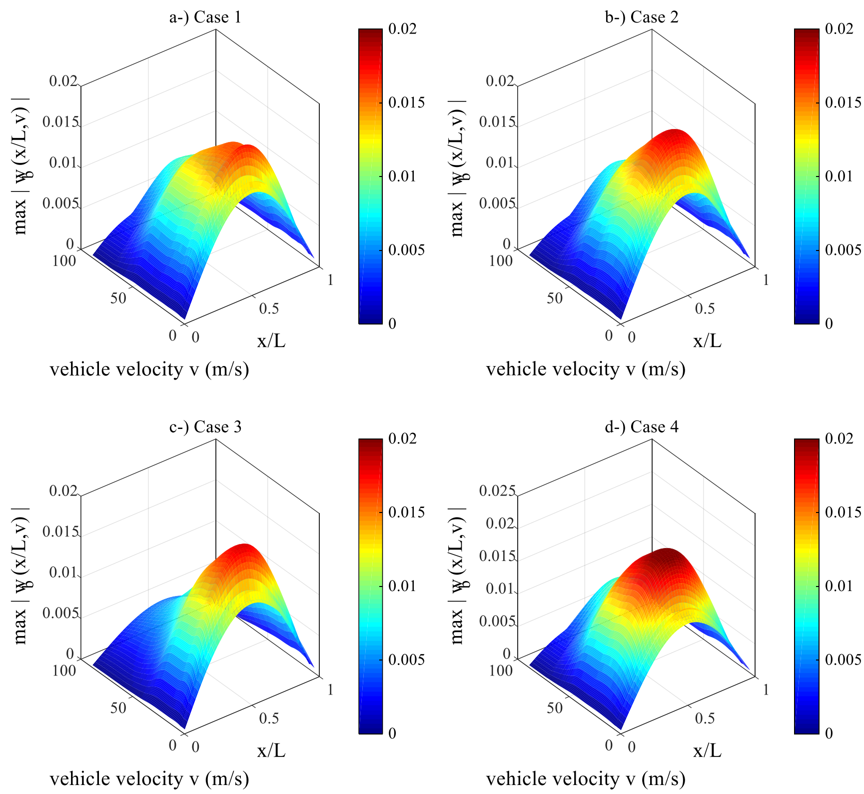

Figure 3. The maximum vertical displacements (m) of the bridge according to vehicle velocity and bridge location (a) case 1; (b) case 2; (c) case 3; (d) case 4.

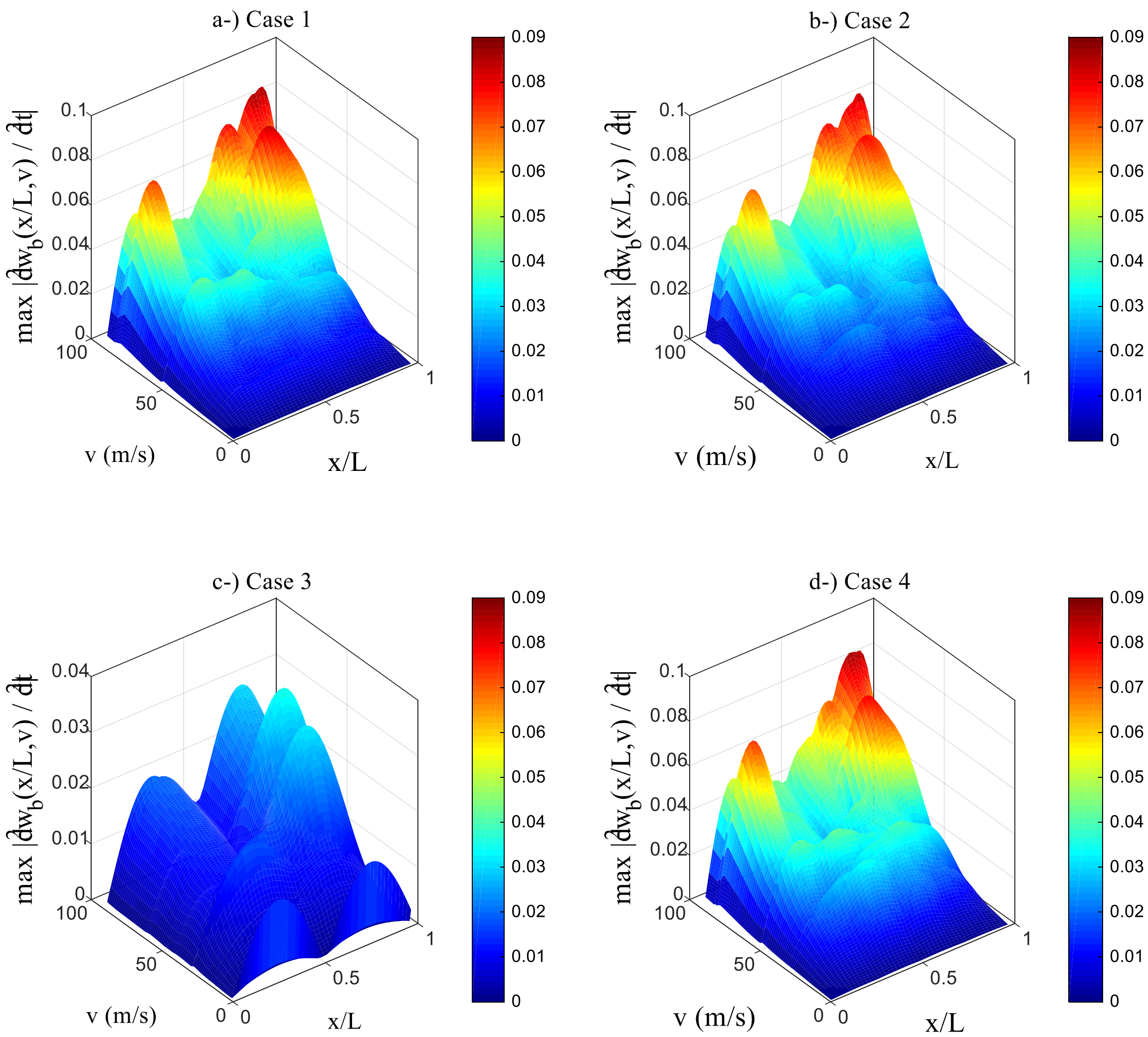

Figure 4. The maximum vertical accelerations (m/s^2) of the bridge according to vehicle velocity and bridge location (a) case 1; (b) case 2; (c) case 3; (d) case 4.

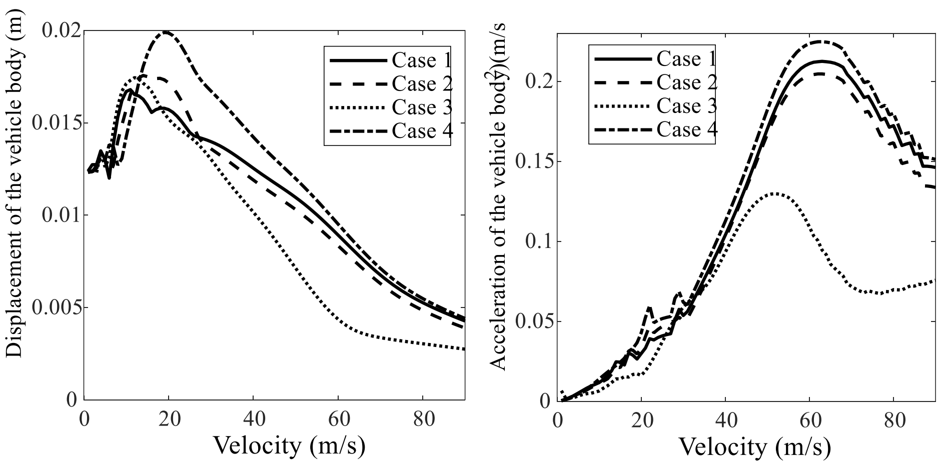

Figure 5. The effect of the absorber upon vehicle dynamic (a) vehicle body displacement (m); (b) vehicle body acceleration (m/s^2)

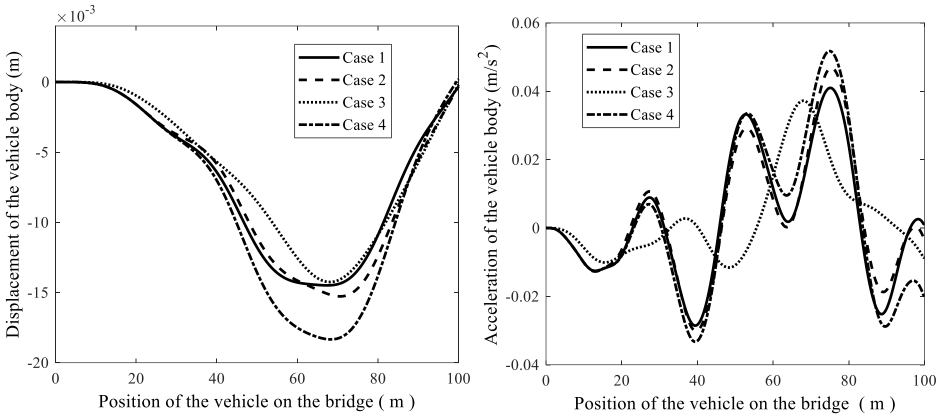

Figure 6. Comparisons of the four different VBA cases for constant vehicle velocity *v*=25 m/s; (a) Vehicle body vertical displacement; (b) vehicle body vertical acceleration (m/s^2).

Table 3. Comparisons of maximum vertical displacements and accelerations for different VBA cases.

cases	Max. disp. (mm)	Location (%)	Vibration reduction rate (%)	Max. acc. (m/s^2)	Location	Vibration reduction rate (%)
1	14.49	67.25	21	0.0407	75.75	21.12
2	15.27	71.5	16.7	0.0468	75.75	9.3
3	14.25	68.25	22.3	0.0372	68	27.9
4	18.34	68.25	---	0.0516	75.25	---

Figures 10a and b show the displacements of the absorber depending of its position on the bridge. For both the displacement and the acceleration of the absorber the maximums are achieved when its location at the 52 percent of the bridge beam length. This is an expected result that where the displacements of the beam are minimum, the displacement, and the acceleration of the absorber mass should be maximum, because the absorber absorbs some portion of the vibration energy of the beam.

Figures 11a-d, respectively, show the maximum displacement of the vehicle body, maximum displacement of the midpoint of the bridge beam, maximum displacements of the absorber and the energies of the absorber, depending on the parameter α and the positions of the absorber on the bridge beam. It is also observed from the Figures that when the absorber possess the maximum energy all the displacements of vehicle and bridge beam are at minimum values for the position of the absorber between 50-55 percent of the bridge beam length.

The total energy of the absorber is defined summing of its kinetic and potential energies$T_a + U_a$ as given below:

$$E_a = T_a + U_a = \frac{1}{2}\begin{pmatrix} k_i(y_i - y(L/2,t))^2 \\ +m_i\dot{y}_i^2 \end{pmatrix} \qquad (12)$$

$(i = 1, 2, 3)$,

In addition, for higher values of the parameter α, the decrease in both displacements of vehicle and bridge beam are more when compared to the lower values. Figures 12a and b show the accelerations of the vehicle body and absorber mass. It is shown that when the vehicle body accelerates in a higher value, the acceleration of the absorber mass is minimum, and vice versa.

4. CONCLUSIONS

In this study, the effect of vibration absorber placed on Euler-Bernoulli beam on vehicle and bridge dynamics was investigated. For this purpose, the equation of motion between Euler-Bernoulli beam and a single degree of freedom moving vehicle is obtained by using Lagrange equation. As a result, it was found that it is sufficient to take into account the first four modes of the bridge beam for the Galerkin function used to calculate the vibrations of any bridge beam.

The results of the study show that the best case among the three cases is the three absorber model, which is placed in consideration of the first two modes of the bridge beam. This is because in other cases, only the vibrations of the first mode are damped, while in cases 3 the vibrations of the first two modes are damped.The effect of the absorber position on the bridge beam has also been analysed and then it is observed that the reduction effect of the absorber was dependent to the position of the absorber and the velocity of the vehicle. For the same tuning frequency, the effect of the parameter α is very significant in terms of the reduction of the vibrations of both the bridge and vehicle. Any increase in α parameter reduces the responses of the VBI system. The more the parameter α is increased the more the vibrations are reduced.

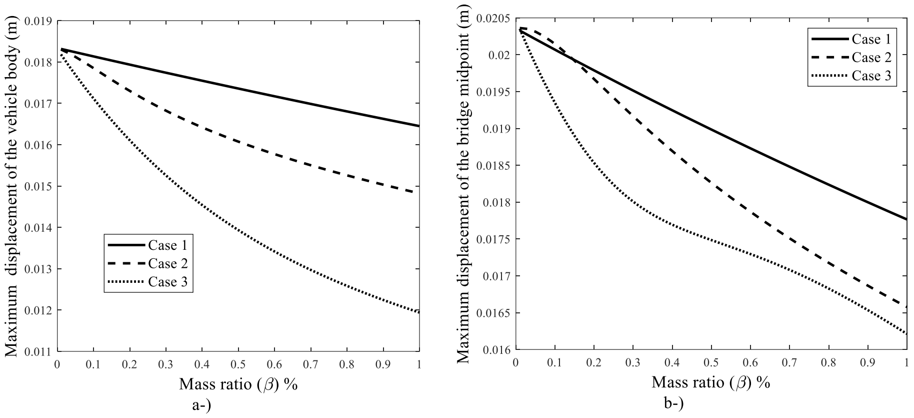

Figure 7. The effect of the mass ratio (β) upon vehicle and bridge dynamics for v=25 m/s. (a) vertical displacements of the vehicle body; (b) vertical displacement of the bridge midpoint.

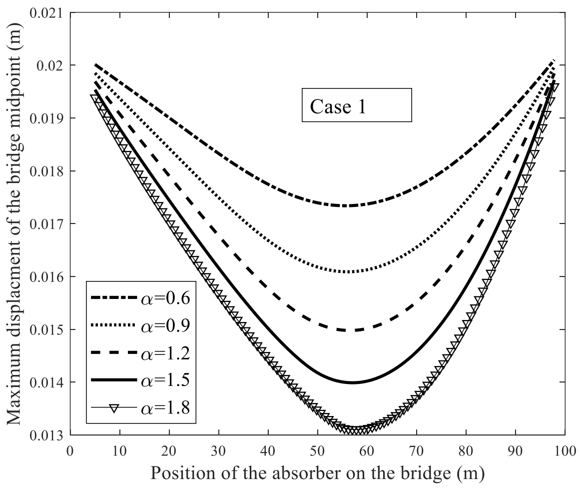

Figure 8. The effect of absorber position on the bridge upon bridge midpoint maximum vertical displacement for different α values and a constant vehicle velocity *v*=25 m/s.

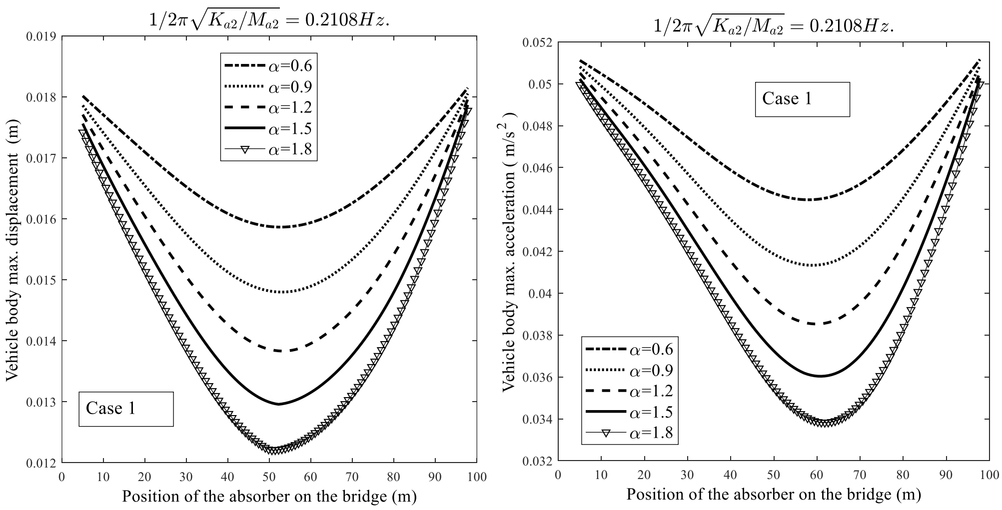

Figure 9. The effect of absorber position on the bridge upon vehicle body dynamics for different α values and *v*=25 m/s, using case 1 (a) vertical maximum displacements of the vehicle body (m); (b) maximum vertical accelerations (m/s^2) of vehicle body.

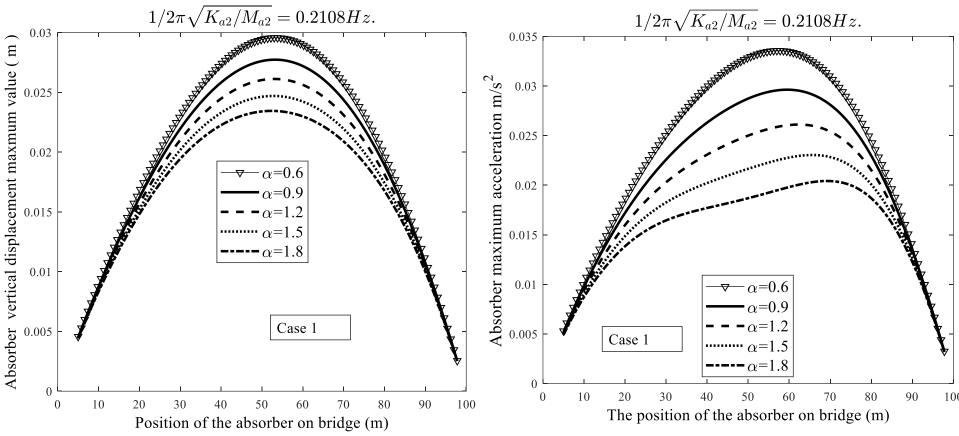

Figure 10. According to absorber position on the bridge for different α values *v*=25 m/s, using case 1. (a) Absorber vertical maximum displacement (m); (b) Absorber vertical acceleration (m/s^2).

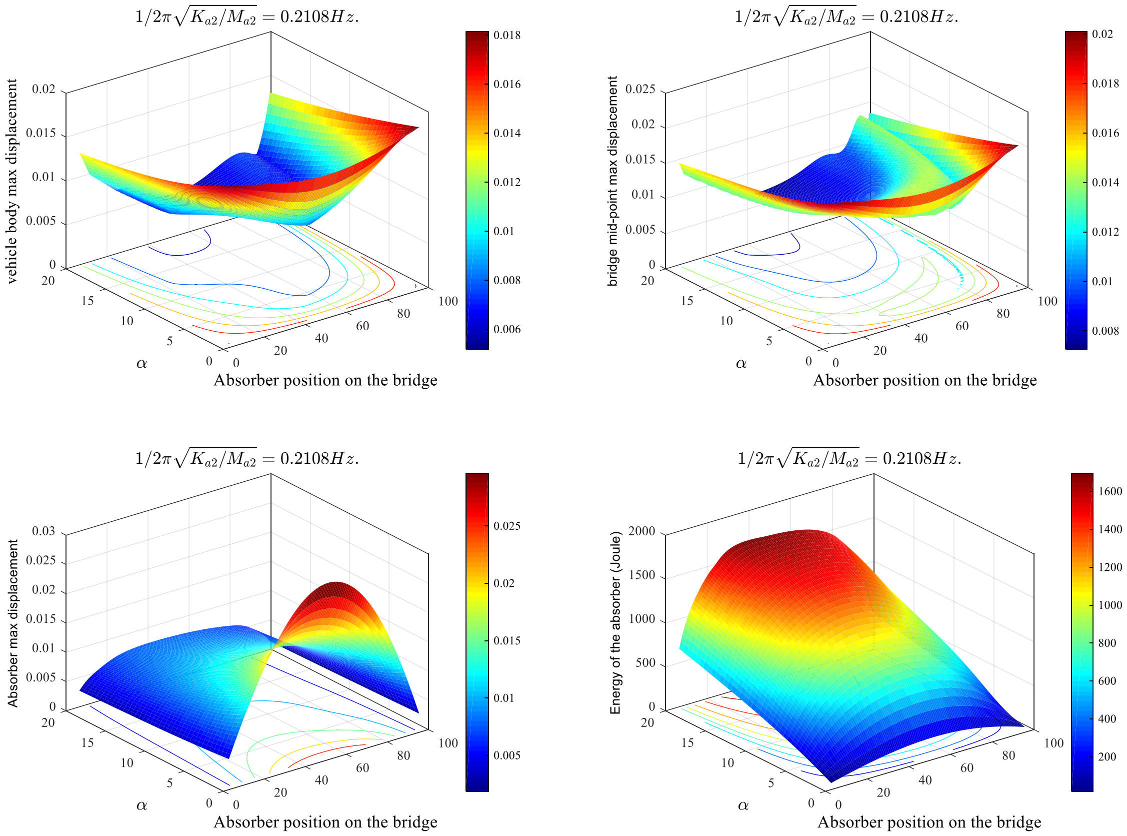

Figure 11. The effect of the parameter α and absorber position on the bridge according to case 1 (a) maximum displacement of the vehicle body (m); (b)Maximum displacement of the bridge midpoint displacement (m); (c) Maximum absorber displacement (m); (d) total energy of the absorber (Joule).

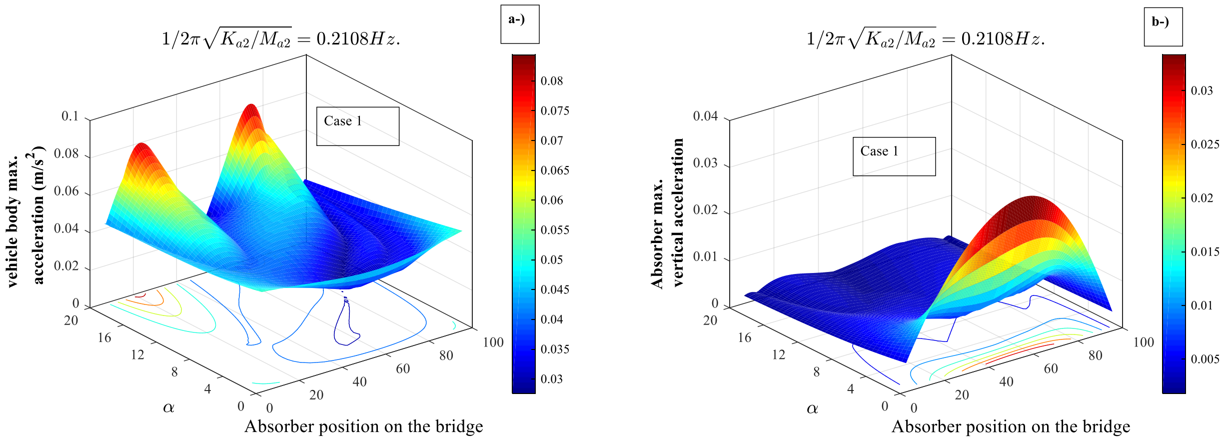

Figure 12. According to parameter α and position of absorber on the bridge for case 1 : (a) maximum acceleration of the vehicle body (m/s^2); (b) maximum absorber acceleration (m/s^2).

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