RESEARCH ARTICLE / ARAŞTIRMA MAKALESİ

A Vehicle Routing Model for Postal Service Operations and an Application

Posta Hizmeti Operasyonları İçin Bir Araç Rotalama Modeli ve Uygulama

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Öz

Günümüzde firmalar arası rekabetin kilit taşını müşteri memnuniyeti oluşturmaktadır. Daha az maliyet, daha çok üretim gibi hedefler müşteri memnuniyetini sağlanmadıkça firmalar için yeterli olmaz. Bu nedenle küçükten büyüğe bütün firmaların amacı müşteriye en iyi, en hızlı şekilde hizmet sunmak ve müşteri memnuniyeti sağlamaktır. Bir ürün veya hizmetin başlangıç noktasından varış noktasına kadar olan süreçte lojistik ağının çok iyi kurulması ve sürecin işlemesi bir firma için önemlidir. Özellikle temel görevi taşımacılık olan kargo firmalarının araç, ürün ve çalışan sayısı gibi kısıtlar ve zaman açısından çok iyi bir sürece sahip olması gerekmektedir. Bu amaçla, bu çalışmada posta hizmetleri için Araç Rotalama Problemi (ARP) üzerine çalışılmıştır. Araçlarını rotalarken önce dağıtım merkezlerine sonrasında kabul merkezlerine gönderen posta hizmetlerinin çok fazla şubesi bulunabilmektedir. Merkez sayısı çoğaldıkça en iyi çözümü veren algoritmalar bu problemi çözemeyeceği için merkezlerin birbirine olan uzaklıkları temel alınarak kümeleme metodu kullanılmıştır. Böylece birbirine yakın olan merkezler kendi içlerinde planlanarak toplam kat edilen uzaklık azaltılmaya çalışılmıştır.

Anahtar Kelimeler: Geri Toplamalı Araç Rotalama Problemi, Kümeleme, Karma Tamsayılı Programlama

Abstract

Customer satisfaction is an important factor for the success of the companies. Efforts for cost reduction and production productivity are not enough for companies unless customer satisfaction is provided. For this reason, the aim of companies from all sizes is to provide the best and fastest service to customers. Therefore, the establishment and operation of a logistics network is important for a company from the original point to the point of arrival of a product or service. In cargo companies, transportation is the main procedure in terms of time and capacity constraints such as the number of vehicles, products and employees. With this aim, Vehicle Routing Problem with Backauls (VRPB) is investigated to reduce the total distance travelled by vehicles for a hypothetical postal service firm. It is assumed that there are more than a hundred centers of the firm that send their vehicles to the reception centers after the distribution centers are visited. The P-Median clustering model is used based on the distances of the centers to each other, since the exact algorithms cannot solve this problem due to a great number of centers. Then, vehicles in each cluster routed by using a modified mathematical model.

Keywords: Vehicle Routing Problem with Backhauls, Clustering, Logistics

I. INTRODUCTION

VRP generally deals with the management of distribution and/or pick-up activities. It is related to decision of how to use the available vehicle fleet and resources. It is expected to meet the requirements by using the resources effectively. Hence, routes for existing vehicles are tried to be defined **[1]**.

In the classic VRP, the route starts and ends at the storage point and that is the main constraint. Other common constraints may be capacity, demand, duration, time-window and priority **[2].** Some of the application areas of VRP are:

- Transportation and logistics sectors
- Waste collection
- Flight charts
- Delivery of online shopping
- Distribution of goods (such as newspaper, mail, bread and drinks)
- Routing of service vehicles

In transportation and logistics applications, the routes of vehicles from factories to warehouses, from warehouses to customers are ensured. In the transportation sector, especially public transportation vehicles are routed [3].

Solution techniques of the Vehicle Routing Problem (VRP) can be grouped into three main categories as exact, heuristics and meta-heuristics solution techniques in general. With exact solution methods it is possible to find the optimal solution to a problem; whereas with heuristic and meta-heuristic approaches optimality is not guaranteed, however, solutions can be found in close-range to optimal solution [4].

In this study, one of the types of VRP, Vehicle Routing Problem with Bachauls (VRPB) is applied to the postal service operations. In literature review part, VRP studies are investigated based on two main topics which are postal service operations and types of VRP. After that, the purpose of the study is explained in Applied Mathematical Models section. Moreover in this part, the models that are used in this paper have shown in a detailed way. In the Application and Results part, a vehicle routing problem for a hypothetical firm is investigated. In order to solve this problem, clusters that contain distribution and reception centers are generated by using p-median model. In the results part of this section, Goetschalckx and Jacobs-Blecha's vehicle routing model [5] modified and applied. Finally, concluding remarks are given.

II. LITERATURE REVIEW

Related literature is reviewed in terms of two main topics which are VRP in postal service operations and types of VRP. VRP types that investigated are VRPB, capacitated VRP, VRP with time windows, and VRP with delivery and pick-up.

Ji and Chen studied on the vehicle routing problem in postal service operations in Hong Kong in 2007. At that time, scheduling of postal vehicles carried out manually. In the problem, there are a General Post Office and a set of District Post Offices. The study aimed to minimize costs and to maximize resource utilization. To find an optimal solution, an integer linear programming model was built [6].

For Vehicle Routing Problem with Backhauls (VRPB), as Jacobs-Blecha and Goetschalckx mentioned, pick-up is not allowed until distribution is finished. Jacobs-Blecha and Goetschalckx [7] introduced a linehaul-backhaul (LHBH) heuristics for VRPB in 1992. For routing, TSP, 2-opt and 3-opt heuristics are used. LHBH produces routes which are adaptable and flexible. This allows the user to interactively make modifications to the solutions needed in real time applications [7]. Another type of VRP is capacitated VRP and as an example, the study prepared by Nazif and Lee is investigated. They solved capacitated vehicle routing problem by using genetic algorithm. The aim of the study is to find the optimal set of delivery routes by minimizing total cost. A comparison with some other heuristic approaches is also presented in that study [8]. VRP with time windows is another type of VRP and Yassen et al. [9] studied on this type and used Solomon's data to test their algorithms. They developed an adaptive hybrid algorithm and carried out some experiments to examine impact of using local search algorithms, effect of performed mechanisms on the search ability and validate efficiency of the adaptive hybrid algorithm. The outputs showed that adaptive hybrid algorithm achieved better results in comparison to the other methods [9]. The paper presented by Righini et al. [10] can be given as an example of VRP with delivery and pick-up. They applied branch-and-price technique for this problem. Two different solution methods compared to solve the pricing subproblem: state space relaxation and exact dynamic programming [10].

In literature survey, firstly the studies applied for postal service operations are investigated. Only one study was found about vehicle routing problem applied in postal service. This study was prepared for Hong Kong Postal Service [6]. However, there are many practices about VRP in cargo firms. Then, a brief literature survey is realized for different types of VRP. Due to the NP-hard nature of the VRP problems, meta-heuristics and heuristics are the mostly used solution algorithms. As exact solution methods, Branch and Bound Algorithm and Branch and Cut Algorithms are the mostly used ones.

III. MATHEMATICAL MODELS

The aim of this study is minimizing the total distance for postal service operations of a hypothetical firm. This firm's post route can be seen in Figure 1.

A vehicle delivers the posts from Processing Center to the Distribution Centers after that same vehicle collects the new posts from Reception Centers for the aim of returning them to the Processing Center. Vehicles departed and arrived to a single depot. If the distribution is not finished, vehicles can

Figure 1. Route of a post in the firm

not visit the Reception Centers. Therefore, this is a type of Vehicle Routing Problem with Backhauls. In this study, to solve the vehicle routing problem with backhauls for postal service operations, a modified model is used. Before solving the model, to decrease the number of centers, p-median model is used and clusters are generated. In the following, brief literature and the applied mathematical model are given.

The first study in this area was conducted by Dantzig and Ramser in 1959 [11] and the fundamental of this problem is based on Travelling Salesman Problems (TSP). Then, Jacobs-Blecha and Goetschalckx in 1989 [5] studied on Vehicle Routing Problem with Backhauls.

Since the VRPB problems are NP-hard type, an optimization software may not be sufficient to find optimal solutions for large problem instances. Therefore, it is determined to use a clustering method. As the first step of the solution, it is decided to cluster the distribution and reception centers by using P-Median model given in [12].

Parameters

P: Number of clusters

D: Number of distribution centers, $d=1, 2, .., D$

R: Number of reception centers, $d=D+1, D+2, ..., D+R$

d_{ij}: Distance of direct travel from center i to center j (i, j =1, ..., D, D+1, ..., D+R)

Variables

$$x_{ij} = \begin{cases} 1, \text{ if the vehicle directly travels from center } i \text{ to } j, \\ \quad\quad i,j = 1,..,D,...,D+R \\ 0, \text{ otherwise} \end{cases}$$

Minimize

$$\sum_{i=1}^{D+R} \sum_{j=1}^{D+R} d_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{j=1}^{D+R} x_{ij} = 1 \qquad i = 1,2,\ldots,D+R \tag{2}$$

$$\sum_{j=1}^{D+R} x_{jj} = P \qquad j = 1,2,\ldots,D+R \tag{3}$$

$$x_{ij} \le x_{jj} \qquad i,j = 1,2,\ ...,\ D+R \tag{4}$$

$$x_{ij} = 0 \text{ or } 1 \qquad i,j = 1,2,\ ...,\ D+R \tag{5}$$

By applying P-Median Model, 11 clusters are found.

Jacobs-Blecha and Goetschalckx's vehicle routing model [5] modified and applied for this study after clustering the centers. There are distribution and reception centers, and the objective function is minimization of the total distance travelled. It is assumed that the firm has heterogeneous fleet, therefore vehicle index is used. To eliminate subtours and loops, a subtour elimination constraint is added. In addition, it is considered in the tenth constraint that each vehicle must service to centers in total working hours everyday.

Afterwards, a mathematical model is built for the problem as follows:

Parameters

K: Number of vehicles

D: Number of distribution centers, $d= 1, 2,..., D$ (index 0 indicates the depot)

R: Number of reception centers, $d= D+1, D+2,...,D+R$

T: Total service time in a day

C_k: Capacity of the vehicles ($k =1,2,..,K$)

a_i: Demand of distribution center i ($i =1, 2,..., D$)

b_i: Supply of reception center i ($i = D+1, D+2,..., D+R$)

d_{ij}: Distance of direct travel from center i to center j (i, j = 0,1,..., D, D+1,..., D+R)

t_{ij}: Travel time between center i and center j (i, j = 0,1,..., D, D+1, ..., D+R)

y_j: Service time in each center ($j =1,...,D+R$)

Variables

$u_{ik=}$ $\begin{cases} 1, \text{ if distribution center } i \text{ is serviced by vehicle } k, i = 0,1,..,D \\ 0, \text{ otherwise} \end{cases}$

$v_{jk=}$ $\begin{cases} 1, \text{ if reception center j is serviced by vehicle } k, j = D+1,..,D+R \text{ and } j=0 \\ 0, \text{ otherwise} \end{cases}$

$x_{ijk=}$ $\begin{cases} 1, \text{ if vehicle } k \text{ travels directly from center } i \text{ to } j, i, j = 0,1,..,D,...,D+R \\ 0, \text{ otherwise} \end{cases}$

s_j= Subtour elimination variable $j=1,....,D+R$

s_i= Subtour elimination variable $i=1,....,D+R$

Objective function

$$Min \sum_{k=1}^{K} \sum_{i=0}^{D+R} \sum_{j=0}^{D+R} d_{ij} x_{ijk} \tag{6}$$

Subject to

$$\sum_{i=1}^{D} a_i u_{ik} \leq C_K \qquad k = 1,2,...,K \tag{7}$$

$$\sum_{i=D+1}^{D+R} b_i v_{ik} \leq C_K \qquad k = 1,2,...,K \tag{8}$$

$$\sum_{k=1}^{K} u_{ik} = 1 \qquad i = 1,2,...,D \tag{9}$$

$$\sum_{k=1}^{K} v_{ik} = 1 \qquad i = D+1, D+2,...,D+R \tag{10}$$

$$u_{0k} = 1 \qquad k = 1,2,...,K \tag{11}$$

$$v_{0k} = 1 \qquad k = 1,2,...,K \tag{12}$$

$$\sum_{\substack{i=0 \\ and \\ j=0}}^{D} \sum_{j=D+1}^{D+R} x_{ijk} = 1 \qquad k = 1,2,...,K \tag{13}$$

$$\sum_{j=0}^{D+R} x_{ijk} = \begin{cases} u_{ik,} \text{ if } i = 0,1,...,D \\ v_{ik,} \text{ if } i = D+1, D+2,..., D+R \, k = 1,2,..., K \end{cases} \tag{14}$$

$$\sum_{i=0}^{D+R} x_{ijk} = \begin{cases} u_{jk,} \text{ if } j = 1,...,D \\ v_{jk,} \text{ if } j = D+1, D+2,..., D+R \text{ and } j=0 \, k =1,2,...,K \end{cases} \tag{15}$$

$$\sum_i \sum_j x_{ijk} t_{ij} + \sum_i \sum_j x_{ijk} y_j \leq T \qquad k=1,2,...,K \tag{16}$$

$$s_i - s_j + (D+R+1)*\sum_{k=1}^{K} x_{ijk} - (D+R) \leq 0 \qquad i{\neq}j, \, i,j=1,...,D+R \tag{17}$$

$$u_{ik} = 0 \text{ or } 1 \qquad i = 1,2,...,D \quad k = 1,2,...,K \tag{18}$$

$$v_{ik} = 0 \text{ or } 1 \qquad i = D+1, D+2,...,D+R \\ k = 1,2,...,K \tag{19}$$

$$x_{ijk} = 0 \text{ or } 1 \qquad i,j = 0,...., D+R \, k = 1,2, ..., K \tag{20}$$

$$s_i \geq 0 \, s_j \geq 0 \qquad i,j = 1,...., D+R \tag{21}$$

Constraints (7) and (8) ensure that the trucks cannot be loaded more than their capacities for distribution and reception centers, respectively.

Constraint (9) ensures that only one vehicle can be assigned to the distribution center's route.

Constraint (10) ensures that only one vehicle can be assigned to the reception center's route and only K, number of vehicles, can leave or turn back to the depot.

Constraints (11) and (12) ensure that each vehicle must start from and turn back to depot.

Constraint (13) ensures that there is only one link travelled by each vehicle from last distribution center to first reception center in one route.

Constraints (14) and (15) ensure that exactly one vehicle leaves from each center and exactly one vehicle enters each center and depot.

Constraint (16) ensures that total service time of each vehicle and process times at the centers can not be more than total working time in a day.

Constraint (17) is the subtour elimination constraint. $D+R+1$ represents total number of centers including depot.

Constraints (18), (19) and (20) ensure that the decision variables must have binary solution values.

Constraint (21) is the nonnegativity constraint for subtour elimination variables.

IV. APPLICATION AND RESULTS

Vehicles that can carry cargo and posts are considered and 4 types of vehicles are decided to be used in the study. Their capacities are assumed as 14 m³, 17 m³, 22 m³ and 40 m³. By assuming 1 m³ carries 26 kg. load, capacities of vehicles

are calculated in terms of grams. Table 1 demonstrates capacities of vehicles.

Table 1. Capacities of the vehicles

14 m³	364,000 gr
17 m³	442,000 gr
22 m³	572,000 gr
40 m³	1,040,000 gr

To determine the routes of vehicles, a distance matrix is needed. Then a matrix that includes one hundred and sixteen centers is created. Total demand is 8,093,680 gr. in distribution centers while total supply amount is 8,598,620 gr. in reception centers.

In this paper, 11 clusters are created according to relative distances of the centers by using p-median model. Then, each cluster is solved in LINGO 17.0 and the routes of the vehicles are determined. The results obtained are given in Table 2.

Table 2. Total travelled distances of Clusters

Clusters	Total Travelled Distances (m)
Cluster 1	168,544
Cluster 2	133,775
Cluster 3	179,731
Cluster 4	487,822
Cluster 5	354,368
Cluster 6	164,492
Cluster 7	54,110
Cluster 8	122,953
Cluster 9	116,936
Cluster 10	97,337
Cluster 11	114,015
Total	**1,994,083**

Total travelled distance and route for cluster 1 is shown in figure 2.

Total Travelled Distance = Route 1 + Route 2 = 168,544 m.

*PC: Processing Center
*D: Distribution Center
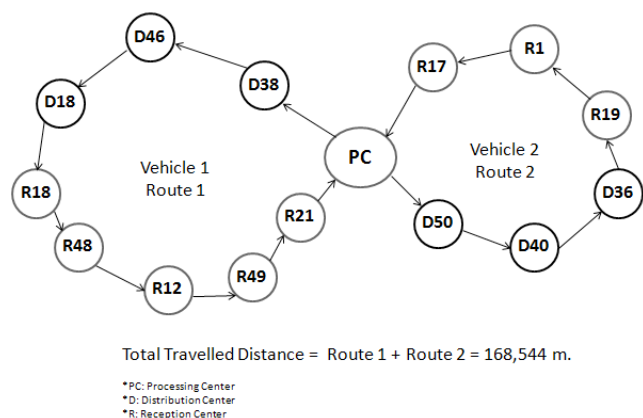*R: Reception Center

Figure 2. Route for cluster 1

Optimal routes for cluster 7, 8 and 9 are given below. For these routes only one vehicle in each cluster is required in the optimal solution. The optimal route of cluster 7 for its vehicle is PC, D7, D8, R56 and PC. The route for cluster 8 is PC, D16, D41, D14, D58, D57, D27, R51, R53, R14, R41 and PC. Lastly, the route is PC, D59, D30, D44, R28, R35, R 55 and PC for cluster 9.

Results are anaylsed based on changes of vehicle speed, process times at each center and demand, supply amounts made a huge difference in the clusters. To observe the variations, cluster 1 is analysed. Initially, it was assumed that vehicle speed is 60 km/h for each vehicle. To analyse the results, it is increased to 90 km/h. For this increase in the speed, optimal route for the cluster did not change. Then, the vehicle speed is decreased to 40 km/h due to traffic congestion at the main roads. As in 90 km/h, the optimal route for the cluster did not change. It was assumed in the paper that unloading the vehicles takes 25 minutes at each center. For analysis this time it is altered to 15 minutes. The optimal route for the cluster remained the same as the previous case. Also, assuming that a vehicle broke down at the first reception center and it took 125 minutes to get repaired. Again, it doesn't effect the optimal route. Finally, supply amount changes are analysed. Supply amounts of the distribution centers were 117020, 27380, 53720, 81260, 149480, 58520, 17180, 19974 gr. and demanded amounts in the reception centers were 14320, 39440, 4260, 251900, 2808400, 29700 gr. To observe the difference amounts at the distribtution centers changed into 100000, 150000, 50000, 40000, 20000, 90000, 20000 and 50000 gr. The amounts in the reception centers became 100000, 140000, 250000, 50000, 30000 and 50000 gr. With this experiment, objective value calculated as 166648 meters and as expected, the optimal routes have changed in the cluster for two vehicles.

V. CONCLUSION

Vehicle routing is one of the important problem areas for logistics management literature. Vehicles routes may have a direct effect on cost effectiveness, on time delivery ratios and as a result, on custumer satisfaction levels. Especially for postal services, this problem becomes the main problem for the companies. Considering this importance level, in this study, vehicle routing problem with backhauls (VRPB) is examined for postal service operations. Considering the possibility of having large number of points for such networks, initially, a clusturing model, p-median model, from literature is applied. Following clustering phase, a modified VRPB model is applied to find proper routes for the vehicles. Finally, results are analysed for some variations

in the data. For the future researches, instead of decreasing the size of the problem by using a clustering approach, a real size problem can be solved by applying a proper heuristic or meta-heuristic algorithm.

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