INTERNATIONAL JOURNAL of ENGINEERING TECHNOLOGIES
Armagan Karamanli, Vol.1, No.3, 2015

Different Implementation Approaches of the Strong Form Meshless Implementation of Taylor Series Method

Armagan Karamanli*[‡]

*Research and Development Department, TIRSAN Treyler Sanayi veTicaret A.S.,Sancaktepe, Istanbul, Turkey
(armagan_k@yahoo.com)

[‡]Corresponding Author; Armagan Karamanli, OsmangaziMah., YıldızhanCad., No:4, 3488, Sancaktepe, Istanbul, Turkey
Tel:+90 216 564 0200, Fax: +90 216 311 7156, armagan_k@yahoo.com

Received: 22.06.2015 Accepted:08.08.2015

Abstract-Based on the Taylor series expansion (TSE) and employing the technique of differential transform method (DTM), three new meshless approaches which are called Meshless Implementation of Taylor Series Methods (MITSM) are presented. In particular, Strong Form Meshless Implementation of Taylor Series Methods (SMITSM) are studied in this paper. Then, the basic functions are used to solve a 1D second-order ordinary differential equation and 2D Laplace equation by using the SMITSM. Comparisons are made with the analytical solutions and results of Symmetric Smoothed Particle Hydrodynamics (SSPH) method. We also compared the effectiveness of the SMITSM and SSPH method by considering various particle distributions, nonhomogeneous terms and number of terms in the basic functions. It is observed that the MITSM has the conventional convergence properties and, at the expense of CPU time, yields smaller L_2 error norms than the SSPH method, especially in the existence of nonsmooth nonhomogeneous problems.

Keywords: Meshless methods, Taylor series, element free method, strong form, heat transfer, differential transform method**.**

1. Introduction

Meshless Smoothed Particle Hydrodynamics (SPH) method, proposed by Lucy [1] to study three-dimensional (3D) astrophysics problems, has been successfully applied to analyze transient fluid and solid mechanics problems. However, it has two shortcomings such as inaccuracy at particles on the boundary and tensile instability. Many techniques have been developed to alleviate these two deficiencies among which are Corrected Smoothed Particle Method (CSPM) [2, 3], Reproducing Kernel Particle Method (RKPM) [4-6] and Modified Smoothed Particle Hydrodynamics (MSPH) method [7-10]. The MSPH method has been successfully applied to study wave propagation in functionally graded materials [9], can capture the stress field near a crack-tip, and simulates the propagation of multiple cracks in a linear elastic body [10]. The SSPH method has been applied to 2D homogeneous elastic problem successfully [11]. On the other hand, the SSPH method [11-13] is more suitable for homogeneous boundary value problems, cannot be easily applicable to nonlinear problems, requires at least fourth order terms in basis functions for the buckling problems which increases the CPU time.

Motivated by the fact that the SSPH method may not yield accurate results for solving nonhomogeneous problems due to its underlying formulation, an alternative approach is investigated especially for nonhomogeneous problems [14]. Three different implementations of MITSM including the approach presented in [14], called Meshless Implementation of Taylor Series Method I, II and III (MITSM) are presented in this paper.

The method presented in [14] requires all derivatives of the kernel function which restricts the choice of the kernel function and only uses all derivatives of the basis function. However, Meshless Implementation of Taylor Series Method I does not require the derivatives of the kernel function and may use any type of kernel function including a constant. On the other hand, Taylor Series Method II uses all derivatives of both basis and kernel functions.

Although the SSPH method and MITSM depend on TSEs, the main difference between these two approaches is as follows: the SSPH method calculates the value of the solution at a node by using the values of the solution at the other nodes and then substitute it into the governing differential equation; thus, nonhomogeneous terms in the governing differential equation are also evaluated pointwise at the nodes. This approach results in approximation errors especially in the existence of nonsmooth nonhomogeneous terms. On the other hand, the proposed MITSM approach substitute the TSEs of the solution and nonhomogeneous term into the governing differential equation and then utilize exact recursive relations between the coefficients of the expansions of the solution and nonhomogeneous term; it yields improvement in accuracy that is verified by solving numerical examples in Section 4. The MITSM can be applied to arbitrary boundary geometries, nonlinear problems, and strong and weak formulations. In particular, Strong Form Meshless Implementation of Taylor Series Methods (SMITSM) are investigated in this paper, whose results are compared with the analytical solutions and solutions of the SSPH method. It is shown that the two of SMITSM has the conventional convergence properties and yields smaller L_2 error norms in numerical examples than the SSPH method, especially in the existence of nonsmooth nonhomogeneous terms.

2. Differential Transform Method

In this study, the DTM technique is employed to develop the MITSM. It is noteworthy that when the DTM is applied to ordinary differential equations, it exactly coincides with the traditional Taylor series method [15] where applications of TSEs and DTM are presented in detail. The 3D differential transform of a function $q(x,y,z)$ is defined as follows

$$Q(k,h,m) = \frac{1}{k!h!m!}\left[\frac{\partial^{k+h+m} q(x,y,z)}{\partial x^k y^h z^m}\right]_{(0,0,0)} \quad (1)$$

where $q(x,y,z)$ is the original function and $Q(k,h,m)$ is the transformed function. The inverse differential transform of $Q(k,h,m)$ is given by

$$q(x,y,z) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\sum_{m=0}^{\infty} Q(k,h,m) x^k y^h z^m \quad (2)$$

Some of the fundamental theorems on differential transform can be found in [16-21].

3. Strong Form Meshless Implementation of Taylor Series Methods (SMITSM)

In this section, three different basis function formulations based on the DTM are given for 1D and 2D dimensional cases. These methods are named as followings;

1. Strong form meshless implementation of Taylor series method I (SMITSM I),
2. Strong form meshless implementation of Taylor series method II (SMITSM II) and
3. Strong form meshless implementation of Taylor series method III (SMITSM III)

3.1 Strong form meshless implementation of Taylor series method I

<u>One Dimensional Case:</u>

For a function $T(x)$ which has continuous derivatives up to the (n+1)th order, the value of the function at a point $\xi = x$ located in the neighborhood of the point $x = x_i$ can be written through the DTM as follows

$$T(x) = \sum_{k=0}^{\infty} U_i(k)(x - x_i)^k \quad (3)$$

By introducing the two matrices P(x) and U_i, equation (3) can be cast into the following form

$$T(x) = \boldsymbol{P}(x,\xi)\boldsymbol{U}_i \quad (4)$$

where

$$\boldsymbol{P}(x,\xi) = [(x-x_i)^0, (x-x_i)^1, \dots, (x-x_i)^k],$$
$$U_i = [U_i(0), U_i(1), U_i(2), \dots, U_i(k)]^T \quad (5)$$

Elements of the matrix $\boldsymbol{U}_i$ are the unknown variables that can be defined as

$$U_i(k) = \frac{1}{k!}\left[\frac{d^k T_i(x)}{dx^k}\right]_{(x_i)} \quad (6)$$

Depending on the number of unknowns of the matrix $\boldsymbol{U}_i$, the derivatives of the $T(x)$ (basis function) are obtained. By neglecting the sixth and higher order terms in the DTM expansions, the formulation of the SMITSM I for a 1D problem can be written as follows

$$T(x) = \boldsymbol{P}(x,\xi)\boldsymbol{U}_i$$
$$\frac{dT(x)}{dx} = \frac{d\boldsymbol{P}(x,\xi)}{dx}\boldsymbol{U}_i$$
$$\frac{d^2T(x)}{dx^2} = \frac{d^2\boldsymbol{P}(x,\xi)}{dx^2}\boldsymbol{U}_i$$
$$\frac{d^3T(x)}{dx^3} = \frac{d^3\boldsymbol{P}(x,\xi)}{dx^3}\boldsymbol{U}_i$$
$$\frac{d^4T(x)}{dx^4} = \frac{d^4\boldsymbol{P}(x,\xi)}{dx^4}\boldsymbol{U}_i$$
$$\frac{d^5T(x)}{dx^5} = \frac{d^5P(x,\xi)}{dx^5}U_i \quad (7)$$

Then multiply both sides of the basis function and its derivatives given above by $W(\xi, x)$

$$W(\xi,x)T(x) = W(\xi,x)\boldsymbol{P}(x,\xi)\boldsymbol{U}_i$$
$$W(\xi,x)\frac{dT(x)}{dx} = W(\xi,x)\frac{d\boldsymbol{P}(x,\xi)}{dx}\boldsymbol{U}_i$$
$$W(\xi,x)\frac{d^2T(x)}{dx^2} = W(\xi,x)\frac{d^2\boldsymbol{P}(x,\xi)}{dx^2}\boldsymbol{U}_i$$
$$W(\xi,x)\frac{d^3T(x)}{dx^3} = W(\xi,x)\frac{d^3\boldsymbol{P}(x,\xi)}{dx^3}\boldsymbol{U}_i$$
$$W(\xi,x)\frac{d^4T(x)}{dx^4} = W(\xi,x)\frac{d^4\boldsymbol{P}(x,\xi)}{dx^4}\boldsymbol{U}_i$$
$$W(\xi,x)\frac{d^5T(x)}{dx^5} = W(\xi,x)\frac{d^5\boldsymbol{P}(x,\xi)}{dx^5}\boldsymbol{U}_i$$

(8)

In the compact support of the kernel function $W(\boldsymbol{\xi},\boldsymbol{x})$associated with the point $\boldsymbol{x}=(x_i,y_i)$ shown in Fig. 1, let there be $N_g$particles.

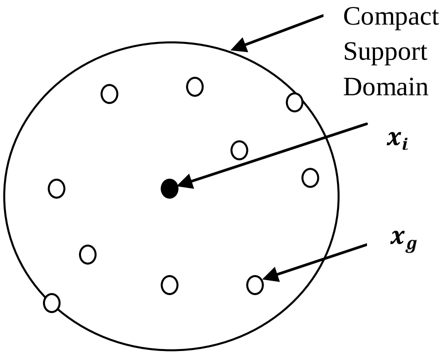

Fig. 1. Distribution of the particles in the compact support of the kernel function W(ξ, x)associated with the point x = (x_i, y_i)

Let'srewrite equation (8) with respect to the compact support domain shown in Fig. 1, evaluate this equation at every particle in the compact support domain of $W(\boldsymbol{\xi},\boldsymbol{x})$and sum each side over these particles, then

$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}(x_g,x_i)\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T_x(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P_x}(x_g,x_i)\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T_{xx}(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}_{xx}(x_g,x_i)\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T_{xxx}(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}_{xxx}(x_g,x_i)\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T_{xxxx}(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}_{xxxx}(x_g,x_i)\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T_{xxxxx}(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}_{xxxxx}(x_g,x_i)\boldsymbol{U}_i$$

(9)

Then, we can solve a set of simultaneous linear algebraic equations given by equation (9) for the unknowns of $\boldsymbol{U}_i$ for all particles.

Two Dimensional Case:

For a function $T(x,y)$which has continuous derivatives up to the (n+1)th order, the value of the function at a point $\boldsymbol{\xi}=(x,y)$ located in the neighborhood of the point $\boldsymbol{x}=(x_i,y_i)$ can be written through the DTM as follows

$$T(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U_i(k,h)(x-x_i)^k(y-y_i)^h \quad (10)$$

With the same approach used for 1D case, the following equation can be written

$$T(x,y)=\boldsymbol{P}(\boldsymbol{x},\boldsymbol{\xi})\boldsymbol{U}_i \quad (11)$$

where

$$\boldsymbol{P}(\boldsymbol{x},\boldsymbol{\xi})=[(x-x_i)^0(y-y_i)^0,(x-x_i)^1(y-y_i)^0,$$
$$(x-x_i)^0(y-y_i)^1,\dots,(x-x_i)^k(y-y_i)^h],$$
$$\boldsymbol{U}_i=[U_i(0,0),U_i(1,0),U_i(0,1),U_i(2,0),U_i(0,2),$$
$$U_i(1,1),\dots,U_i(k,h)]^T \quad (12)$$

Elements of the matrix $\boldsymbol{U}_i$ are unknown that can be defined as

$$U_i(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}T(x,y)}{\partial x^k\partial y^h}\right]_{(x_i,y_i)} \quad (13)$$

By applying the same procedures given for 1D case and neglecting the third and higher order terms in the DTM expansions, the formulation of the SMITSM I for a 2D problem can be written as follows

$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_x(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P_x}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_y(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_y(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_{xx}(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_{xx}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_{yy}(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_{yy}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_{xy}(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_{xy}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$

(14)

The set of simultaneous linear algebraic equations given in equation (14) can be solved for the unknowns of $\boldsymbol{U}_i$ for all particles. The formulation for 3D problems can be obtained in a similar fashion as described above.

3.2 Strong form meshless implementation of Taylor series method II

One Dimensional Case:

If we multiply both sides of equation (4) by W(ξ, x), we obtain

$$W(\xi,x)T(x)=W(\xi,x)\boldsymbol{P}(x)\boldsymbol{U}_i \quad (15)$$

Depending on the number of unknowns of the matrix U_i, the derivatives of Equation 3.13 are obtained. By neglecting the sixth and higher order terms in the DTM expansions, the formulation of the SMITSM II for a 1D problem can be written by evaluating equation (15) and its derivatives at every particle in the compact support domain of $W(\xi,x)$and sum each side over these particles as follows

$$\sum_{g=1}^{N_g} W(x_g,x_i)\,T(x_g)=\sum_{g=1}^{N_g} W(x_g,x_i)\,\boldsymbol{P}(x_g,x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(x_g,x_i)\,T_x(x_g)+W_x(x_g,x_i)T(x_g))=\sum_{g=1}^{N_g}(W(x_g,x_i)\,\boldsymbol{P_x}(x_g,x_i)+W_x(x_g,x_i)\boldsymbol{P}(x_g,x_i))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(x_g,x_i)\,T_{xx}(x_g)+2W_x(x_g,x_i)T_x(x_g)+W_{xx}(x_g,x_i)T(x_g))=\sum_{g=1}^{N_g}(W(x_g,x_i)\,\boldsymbol{P}_{xx}(x_g,x_i)+2W_x(x_g,x_i)\boldsymbol{P}_x(x_g,x_i)+W_{xx}(x_g,x_i)\boldsymbol{P}(x_g,x_i))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(x_g,x_i)\,T_{xxx}(x_g)+3W_x(x_g,x_i)T_{xx}(x_g)+3W_{xx}(x_g,x_i)T_x(x_g)+W_{xxx}(x_g,x_i)T(x_g))=\sum_{g=1}^{N_g}(W(x_g,x_i)\,\boldsymbol{P}_{xxx}(x_g,x_i)+3W_x(x_g,x_i)\boldsymbol{P}_{xx}(x_g,x_i)+3W_{xx}(x_g,x_i)\boldsymbol{P}_x(x_g,x_i)+W_{xxx}(x_g,x_i)\boldsymbol{P}(x_g,x_i))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(x_g,x_i)\,T_{xxxx}(x_g)+4W_x(x_g,x_i)T_{xxx}(x_g)+6W_{xx}(x_g,x_i)T_{xx}(x_g)+4W_{xxx}(x_g,x_i)T_x(x_g)+W_{xxxx}(x_g,x_i)T(x_g))=\sum_{g=1}^{N_g}(W(x_g,x_i)\,\boldsymbol{P}_{xxxx}(x_g,x_i)+4W_x(x_g,x_i)\boldsymbol{P}_{xxx}(x_g,x_i)+6W_{xx}(x_g,x_i)\boldsymbol{P}_{xx}(x_g,x_i)+4W_{xxx}(x_g,x_i)\boldsymbol{P}_x(x_g,x_i)+W_{xxxx}(x_g,x_i)\boldsymbol{P}(x_g,x_i))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(x_g,x_i)\,T_{xxxxx}(x_g)+5W_x(x_g,x_i)T_{xxxx}(x_g)+10W_{xx}(x_g,x_i)T_{xxx}(x_g)+10W_{xxx}(x_g,x_i)T_{xx}(x_g)+5W_{xxxx}(x_g,x_i)T_x(x_g))+W_{xxxxx}(x_g,x_i)T(x_g))=\sum_{g=1}^{N_g}(W(x_g,x_i)\,\boldsymbol{P}_{xxxxx}(x_g,x_i)+5W_x(x_g,x_i)\boldsymbol{P}_{xxxx}(x_g,x_i)+10W_{xx}(x_g,x_i)\boldsymbol{P}_{xxx}(x_g,x_i)+10W_{xxx}(x_g,x_i)\boldsymbol{P}_{xx}(x_g,x_i)+5W_{xxxx}(x_g,x_i)\boldsymbol{P}_x(x_g,x_i)+W_{xxxxx}(x_g,x_i)\boldsymbol{P}(x_g,x_i))\boldsymbol{U}_i \quad (16)$$

The set of simultaneous linear algebraic equations given by equation (16) can be solved for the unknowns of $\boldsymbol{U}_i$ for all particles.

Two Dimensional Case:

If we multiply both sides of equation (11) by $W(\boldsymbol{\xi},\boldsymbol{x})$, we obtain

$$W(\boldsymbol{\xi},\boldsymbol{x})T(x,y)=W(\boldsymbol{\xi},\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})\boldsymbol{U}_i \quad (17)$$

Depending on the number of unknowns of the matrix $\boldsymbol{U}_i$, the derivatives of the equation (17) are obtained. By neglecting the third and higher order terms in the DTM expansions, the formulation of the SMITSM II for a 2D problem can be written by evaluating equation (17) and its derivatives at every particle in the compact support domain of $W(\boldsymbol{\xi},\boldsymbol{x})$and sum each side over these particles as follows

$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T(\boldsymbol{x_g})=\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_x(\boldsymbol{x_g})+W_x(\boldsymbol{\xi_g},\boldsymbol{x_i})T(\boldsymbol{x_g}))=\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_x(\boldsymbol{\xi_g},\boldsymbol{x_i})+W_x(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{P}(\boldsymbol{\xi_g},\boldsymbol{x_i}))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_y(\boldsymbol{x_g})+W_y(\boldsymbol{\xi_g},\boldsymbol{x_i})T(\boldsymbol{x_g}))=\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_y(\boldsymbol{\xi_g},\boldsymbol{x_i})+W_y(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{P}(\boldsymbol{\xi_g},\boldsymbol{x_i}))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_{xx}(\boldsymbol{x_g})+2W_x(\boldsymbol{\xi_g},\boldsymbol{x_i})T_x(\boldsymbol{x_g})+W_{xx}(\boldsymbol{\xi_g},\boldsymbol{x_i})T(\boldsymbol{x_g}))=\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,\boldsymbol{P}_{xx}(\boldsymbol{\xi_g},\boldsymbol{x_i})+2W_x(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{P}_x(\boldsymbol{\xi_g},\boldsymbol{x_i})+W_{xx}(\boldsymbol{\xi_g},\boldsymbol{x_i})\boldsymbol{P}(\boldsymbol{\xi_g},\boldsymbol{x_i}))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g}(W(\boldsymbol{\xi_g},\boldsymbol{x_i})\,T_{yy}(\boldsymbol{x_g})+2W_y(\boldsymbol{\xi_g},\boldsymbol{x_i})T_y(\boldsymbol{x_g})+W_{yy}(\boldsymbol{\xi_g},\boldsymbol{x_i})T(\boldsymbol{x_g}))$$

$$= \sum_{g=1}^{N_g} (W(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}_{yy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) + 2W_y(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{P}_y(\boldsymbol{\xi_g}, \boldsymbol{x_i}) + W_{yy}(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}))\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} (W(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T_{xy}(\boldsymbol{x_g}) + W_y(\boldsymbol{\xi_g}, \boldsymbol{x_i})T_x(\boldsymbol{x_g}) + W_x(\boldsymbol{\xi_g}, \boldsymbol{x_i})T_y(\boldsymbol{x_g}) + W_{xy}(\boldsymbol{\xi_g}, \boldsymbol{x_i})T(\boldsymbol{x_g})) = \sum_{g=1}^{N_g} (W(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}_{xy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) + W_y(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{P}_x(\boldsymbol{\xi_g}, \boldsymbol{x_i}) + W_x(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{P}_y(\boldsymbol{\xi_g}, \boldsymbol{x_i}) + W_{xy}(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}))\boldsymbol{U}_i \quad (18)$$

3.3 Strong form meshless implementation of Taylor series method III

<u>One Dimensional Case:</u>

If we multiply both sides of equation (4) by W(ξ, x), we obtain

$$W(\xi, x)T(x) = W(\xi, x)\boldsymbol{P}(x)\boldsymbol{U}_i \quad (19)$$

Let's rewrite equation (19) with respect to the compact support domain shown in Fig. 1, evaluate this equation at every particle in the compact support domain of $W(\xi, x)$and sum each side over these particles, then

$$\sum_{g=1}^{N_g} W(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i \quad (20)$$

Repeating the above procedure regarding the number of terms included in $\boldsymbol{U}_i$ in equation (5) by replacing W with the following

$$W_x = \partial W/\partial x, \quad W_{xx} = \frac{\partial^2 W}{\partial x^2}, \quad W_{xxx} = \frac{\partial^3 W}{\partial x^3}, \quad W_{xxxx} = \frac{\partial^4 W}{\partial x^4}, \quad W_{xxxxx} = \frac{\partial^5 W}{\partial x^5} \quad (21)$$

and so on. Then, we can solve a set of simultaneous linear algebraic equations for the unknowns of $\boldsymbol{U}_i$ for all particles.

By neglecting the sixth and higher order terms in the DTM expansions, the formulation of the SMITSM III for a 1D problem can be written as follows

$$\sum_{g=1}^{N_g} W(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_x(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W_x(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_{xx}(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W_{xx}(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_{xxx}(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W_{xxx}(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_{xxxx}(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W_{xxxx}(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_{xxxxx}(x_g, x_i)T(x_g) = \sum_{g=1}^{N_g} W_{xxxxx}(x_g, x_i)\, \boldsymbol{P}(x_g, x_i)\boldsymbol{U}_i \quad (22)$$

where

$$\boldsymbol{P}(x) = [1, (x - x_i)^1, (x - x_i)^2, (x - x_i)^3, (x - x_i)^4, (x - x_i)^5]$$

$$\boldsymbol{U}_i = [U_i(0), U_i(1), U_i(2), U_i(3), U_i(4), U_i(5)]^T \quad (23)$$

The set of simultaneous linear algebraic equations given by equation (22) can be solved for the unknowns of $\boldsymbol{U}_i$ for all particles.

<u>Two Dimensional Case:</u>

If we multiply both sides of equation (11) by $W(\boldsymbol{\xi}, \boldsymbol{x})$, we obtain

$$W(\boldsymbol{\xi}, \boldsymbol{x})T(x, y) = W(\boldsymbol{\xi}, \boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})\boldsymbol{U}_i \quad (23)$$

Lets rewrite equation (23) with respect to the compact support domain shown in Figure 1, evaluate this equation at every particle in the compact support domain of $W(\boldsymbol{\xi}, \boldsymbol{x})$and sum each side over these particles, then

$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g}, \boldsymbol{x_i})T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W(\boldsymbol{\xi_g}, \boldsymbol{x_i})\, \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{U}_i \quad (24)$$

Repeating the above procedure regarding the number of terms included in $\boldsymbol{U}_i$ in Equation (12) by replacing W with the following

$$W_x = \partial W/\partial x, \quad W_y = \partial W/\partial y, \quad W_{xx} = \partial^2 W/\partial yx^2 \quad W_{yy} = \partial^2 W/\partial y^2, \quad W_{xy} = \frac{\partial^2 W}{\partial x \partial y} \quad (25)$$

and so on. By neglecting the third and higher order terms in the DTM expansions, the formulation of the SMITSM III for a 2D problem can be written as follows

$$\sum_{g=1}^{N_g} W(\boldsymbol{\xi_g}, \boldsymbol{x_i})T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W(\boldsymbol{\xi_g}, \boldsymbol{x_i})\, \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i})\boldsymbol{U}_i$$

$$\sum_{g=1}^{N_g} W_x(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W_x(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W_y(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W_y(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W_{xx}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W_{xx}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W_{yy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W_{yy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{U}_i$$
$$\sum_{g=1}^{N_g} W_{xy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) T(\boldsymbol{\xi_g}) = \sum_{g=1}^{N_g} W_{xy}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{P}(\boldsymbol{\xi_g}, \boldsymbol{x_i}) \boldsymbol{U}_i \tag{26}$$

where

$$\boldsymbol{P}(x) = [1, (x-x_i)^1, (x-x_i)^2, (x-x_i)^3, (x-x_i)^4, (x-x_i)^5]$$
$$\boldsymbol{U}_i = [U_i(0), U_i(1), U_i(2), U_i(3), U_i(4), U_i(5)]^T \tag{27}$$

The set of simultaneous linear algebraic equations given by equation (26) can be solved for the unknowns of $\boldsymbol{U}_i$ for all particles.

The formulation for 3D problems can be obtained in similar fashions as described above.

4. Numerical Examples

The SMITSM I, II and III are applied to three sample boundary value problems in this section. Since the SMITSM I, II and III and SSPH method depend on TSEs and employ strong form formulations, results of these methods are compared with each other. Although problem types and domains are simple in the following three examples, they are chosen due to the reasons that their analytical solutions can be derived for comparisons and they illustrate the implementation of the SMITSM in a clear way. Nonetheless, the SMITSM I, II and III and SSPH method can be easily applied to any boundary value problem and complex domains in a systematic way. The computer programs that are used to solve the numerical problems are developed by using Matlab.

4.1 1D Nonhomogeneous Boundary Value Problem

Consider the following 1D nonhomogeneous ordinary differential equation

$$\frac{d^2v}{dx^2} = x^3, \qquad 0 \le x \le 2 \tag{28}$$

The boundary conditions are given by $v(0) = 1$and $v(2) = 6.6$. The analytical solution of this boundary value problem is given by

$$v(x) = \frac{1}{20}x^5 + 2x + 1 \tag{29}$$

The above boundary value problem is solved by using the SMITSM I, II and III and SSPH method for the particle distributions of 5, 20 and 100 equally spaced particles in the domain $x \in [0,2]$. The following Revised Super Gauss Function in [11] is used as the kernel function since it resulted in the least L_2 error norms in numerical solutions presented in [13]

$$W(x,\xi) = \frac{G}{(h\sqrt{\pi})^\lambda} \begin{Bmatrix} (4-d^2)e^{-d^2} & 0 \le d \le 2 \\ 0 & d > 2 \end{Bmatrix} \tag{30}$$

where$d = |x-\xi|/h$ is the radius of the support domain which is set to 2, h is the smoothing length, λ is equal to the dimensionality of the space (i.e., λ=1, 2 or 3) and G is the normalization parameter having the values 1.04823, 1.10081 and 1.18516 for λ = 1, 2 and 3, respectively. It is chosen that the smoothing length h equals to the minimum distance Δ between two adjacent particles.

Numerical results obtained by using the SMITSM I, II and III and SSPH method are compared with the analytical solutions, and their convergence and accuracy features are evaluated by using the following global L_2 error norm

$$\|Error\|_2 = \frac{\left[\sum_{j=1}^{m}(v_{num}^j - v_{exact}^j)^2\right]^{1/2}}{\left[\sum_{j=1}^{m}(v_{exact}^j)^2\right]^{1/2}} \tag{31}$$

wherev_{num}^j is the value of numerical solution v at the j^{th} node and v_{exact}^j is the value of analytical solution at the j^{th} node. Considering equation (28), we can obtain the following equation by using the DTM technique

$$(k+1)(k+2)V(k+2) = F(k),$$
$$F(k) = \frac{1}{k!}\left[\frac{d^k x^3}{dx^k}\right]_{x=x_j} \tag{32}$$

By using equation (32), one can solve for the coefficients $V(2), V(3), V(4)$ and $V(5)$ in terms of $F(0), F(1), F(2)$ and $F(3)$ for all particles located in the compact support domain of a particle. The sixth and higher order terms are neglected in derivations since they are equal to zero for this problem.

Following, the expressions for $V(2), V(3), V(4)$ and $V(5)$ for each particle are assembled to obtain global equations, boundary conditions are imposed and then the resulting equation system is solved. Note that $V(0)$ and $V(1)$ are already defined by boundary conditions for particle number 1; thus, there is no unknown for particle number 1 located at$x$=0.

The global L_2 error norms of the solutions of the SMITSM I, II, and III and SSPH method are given in Table 1.toTable 4.where different numbers of particles and terms in expansions are considered. The results in Table 1.to Table 4.are obtained for the parameter values of d and h giving the best accuracy for each method.

In Table 1., it is observed that the SMITSM II, and III and SSPH method give the lowest error for the numerical solution obtained by using 3 terms. The SMITSM I always give the highest error norm when it is compared to other methods.

The SMITSM I cannot provide satisfactory result for the compact support domain radius of 2 by using 5 nodes.,

Table 1. Global L_2 error norm for different number of nodes – 3 term

Meshless Method	Number of Nodes		
	5 Nodes	20 Nodes	100 Nodes
SMITSM I	*	1.4129277	0.15680171
SMITSM II	1.0455434	0.0542322	0.0020706
SMITSM III	1.0455434	0.0542322	0.0020706
SSPH	1.0454434	0.0542322	0.0020706

*There is no solution for the compact support domain radius d=2.

Table 2. Global L_2 error norm for different number of nodes – 4 term

Meshless Method	Number of Nodes		
	5 Nodes	20 Nodes	100 Nodes
SMITSM I	*	0.05299339	0.0020771
SMITSM II	1.0455434	0.0542322	0.0020706
SMITSM III	1.0455434	0.0542322	0.0020706
SSPH	1.0455434	0.0542322	0.0020706

*There is no solution for the compact support domain radius d=2.

In Table 2., it is found that there is no difference between the methods in terms of global L_2 error norm for different number of nodes by using 4 term in the TSE expansion. The SMITSM I cannot provide satisfactory results for the compact support domain radius of 2 by using 5 nodes.

It is observed in Table 3. that the SMITSM I and II always give the lowest global L_2 error norm for different number of nodes by using 5 term in the TSE expansion. The SMITSM I cannot provide satisfactory results for the compact support domain radius of 2 by using 5 nodes.

Table 3. Global L_2 error norm for different number of nodes – 5 term

Meshless Method	Number of Nodes		
	5 Nodes	20 Nodes	100 Nodes
SMITSM I	*	0.0019065	$1.6x10^{-6}$
SMITSM II	$4.5x10^{-14}$	$1.2x10^{-11}$	$2.3x10^{-9}$
SMITSM III	$3.1x10^{-14}$	$4.9x10^{-13}$	$3.6x10^{-12}$
SSPH	0.1258686 **	0.0001205 **	$3.6x10^{-8}$

*There is no solution for the compact support domain radius d=2

**The compact support domain radius d is chosen as 4, because d=2 results in large L_2 error norms or no solution with the current smoothing length assumption.It is clear that, even with the same number of terms, solutions of the SMITSM II and III agree very well with the analytical solution; however, those obtained by using the SSPH method and SMITSM I differ noticeably from the analytical solution especially for 5 nodes and 5 terms in the TSEs.

It is observed in Table 4 that the SMITSM II and III agree very well with the analytical solution. The SSPH method cannot provide solution by using 5 nodes in the problem domain when it uses 6 terms in TSE. The SMITSM I cannot provide satisfactory result for the compact support domain radius of 2 by using 5 nodes.

Table 4. Global L_2 error norm for different number of nodes – 6 term

Meshless Method	Number of Nodes		
	5 Nodes	20 Nodes	100 Nodes
SMITSM I	*	$2.4x10^{-13}$	$3.9x10^{-12}$
SMITSM II	$7.9x10^{-14}$	$1.4x10^{-11}$	$3.6x10^{-9}$
SMITSM III	$7.8x10^{-14}$	$3.3x10^{-13}$	$3.6x10^{-12}$
SSPH	**	$1.3x10^{-9}$ ***	$2.6x10^{-9}$ ***

*There is no solution for the compact support$d$=2

** At least 6 nodes are needed to solve the problem.

*** The compact support domain radius d is used as 5 because d=2, 3 and 4 result in large L_2 error norms with the current smoothing length assumption.Regarding to the results obtained by using 6 terms in the TSEs, the SMITSM I, II and III give the lowest L_2 error norms.

4.2 Homogeneous Laplace Equation in 2D

The Laplace equation in 2D is solved by using the SMITSM I, II and III and SSPH method in the domain shown in Fig. 2. The governing differential equation and essential boundary conditions are given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,\ T_1 = T_3 = T_4 = 0\ ℃, T_2 = 100\ ℃ \quad (33)$$

where$T$ is the temperature and $T_i$denote the prescribed boundary temperatures.

The analytical solution of the above boundary value problem is given by

$$\frac{T(x,y)}{T_2} = \sum_{n=1}^{\infty} \frac{2[1-(-1)^n]}{n\pi} \sin\left(\frac{n\pi x}{a}\right) \frac{\sinh\left(\frac{n\pi y}{a}\right)}{\sinh\left(\frac{n\pi b}{a}\right)} \quad (34)$$

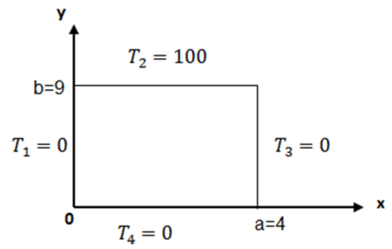

Fig. 2. Problem domain and boundary conditions

When solving this problem, equally spaced 50, 171 and 629 particles are considered in the domain. The smoothing length h is equal to the minimum distance between two adjacent particles (i.e., $h = \Delta$). The following Revised Super Gauss Function in [11] is used as the kernel function

$$W(d) = \frac{G}{(h\sqrt{\pi})^\lambda} \begin{Bmatrix} (16 - d^2)e^{-d^2} & 0 \le d \le 4 \\ 0 & d > 4 \end{Bmatrix} \quad (35)$$

where$d = |\boldsymbol{x} - \boldsymbol{\xi}|/h$ is set to 4, $\lambda = 2$ and G has the same value as in Section 4.1.

Convergence and accuracy of the SMITSM I, II and III and SSPH method are calculated by using the following global L_2 error norm

$$\|Error\|_2 = \frac{\left[\sum_{j=1}^{m}\left\{(u_{num}^j - u_{exact}^j)^2 + (v_{num}^j - v_{exact}^j)^2\right\}\right]^{1/2}}{\left[\sum_{j=1}^{m}\left\{(u_{exact}^j)^2 + (v_{exact}^j)^2\right\}\right]^{1/2}} \quad (36)$$

where u_{num}^j and v_{num}^j are respectively the values of numerical solutions of u and v at the j^{th} node, and u_{exact}^j and v_{exact}^j are respectively the values of analytical solutions of u and v at the j^{th} node.

From equation (33), we can obtain the following recursive equation by using the DTM technique

$$(k+1)(k+2)T(k+2,m) + (m+1)(m+2)T(k,m+2) = 0 \quad (37)$$

The vectors $\boldsymbol{P}$ and $\boldsymbol{U_i}$ can be rearranged as follows

$$\boldsymbol{P}(\boldsymbol{x},\boldsymbol{\xi}) = [1, (x-x_i)^1, (y-y_i)^1, (x-x_i)^2 - (y-y_i)^2, (x-x_i)^1(y-y_i)^1,$$
$$(x-x_i)^3 - 3.(x-x_i)^1(y-y_i)^2, (x-x_i)^2(y-y_i)^1 - (1/3)((y-y_i)^2,$$
$$(x-x_i)^3(y-y_i)^1 - (x-x_i)^1(y-y_i)^3, (x-x_i)^4 - 6.(x-x_i)^2(y-y_i)^2 + (y-y_i)^4$$
$$(x-x_i)^5 - 10.(x-x_i)^3(y-y_i)^2 + 5.(x-x_i)^1(y-y_i)^4$$
$$(x-x_i)^4(y-y_i)^1 - 2.(x-x_i)^2(y-y_i)^3 + (1/5).(y-y_i)^5]$$
$$\boldsymbol{U_i} = [U_i(0,0), U_i(1,0), U_i(0,1), U_i(2,0), U_i(1,1), U_i(3,0),$$
$$U_i(3,1), U_i(4,0), U_i(5,0), U_i(4,1)]^T \quad (38)$$

Following, above nodal equations are assembled to obtain the global equations; then, boundary conditions are imposed and the resulting equation system is solved. To evaluate the performance, numerical solutions are obtained for 6 terms for the SMITSM I, II and III and SSPH method. Numerical solutions obtained by using 6 terms in the associated expansions and 50, 171 and 629 nodes are presented in Fig. 3, 4 and 5, respectively.

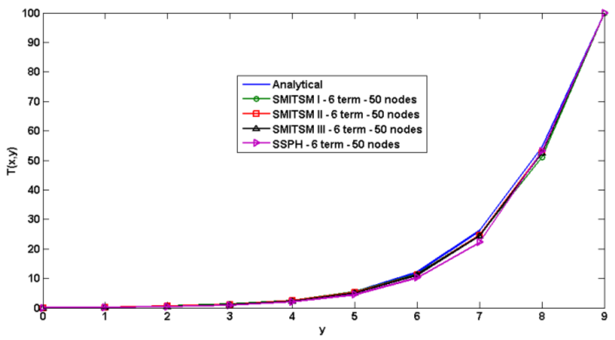

Fig. 3. Temperatures along the *y*-axis (*x*=2) computed by the SMITSM, SSPH method and analytical solution where equally spaced 50 nodes are used

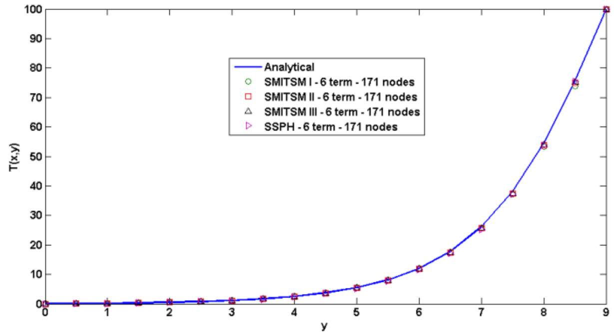

Fig. 4. Temperatures along the *y*-axis (*x*=2) computed by the SMITSM, SSPH method and analytical solution where equally spaced 171 nodes are used

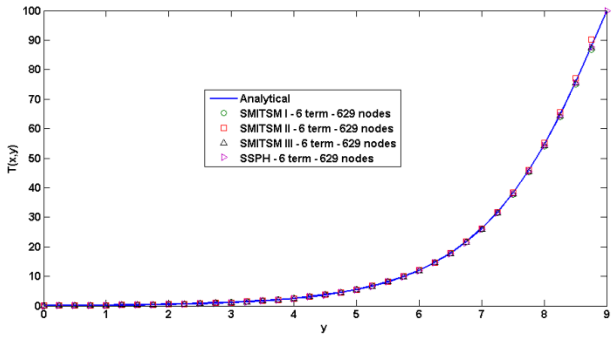

Fig. 5. Temperatures along the *y*-axis (*x*=2) computed by the SMITSM, SSPH method and analytical solution where equally spaced 629 nodes are used

It is observed in Fig. 3, 4 and 5 that accuracy of the SMITSM I, II and III are better than that of the SSPH method and all studied methods show convergence as the number of nodes is increased.

The global L_2 error norms obtained by the SMITSM I, II and III and SSPH method are given in Table 5. It is clear that the L_2 error norms of the results of the SMITSM I, II and III are much lower than those of the SSPH method provided that the same number of terms in the associated expansions are employed for both methods.

By using the same number of terms, the SMITSM II always gives the lowest global L_2 error norm when comparing with the other methods. The SSPH method always gives the highest L_2 error norms for different number of nodes in the problem domain. Numerical results also show that lower L_2 error norms can be obtained for all methods as the number of particles distributed in the problem domain is increased.

Table 5. Global L_2 error norm for different number of nodes

Meshless Method	Number of Nodes		
	50 Nodes	171 Nodes	629 Nodes
SMITSM I	3.7853	2.1886	1.2718
SMITSM II	3.2134	1.6750	0.9927
SMITSM III	3.7313	1.9813	1.0541
SSPH	8.4205	4.3004	2.3956

4.3 Nonhomogeneous Laplace Equation in 2D

Nonhomogeneous Laplace equation in 2D is solved by using the SMITSM I, II and III and SSPH method in the domain shown in Fig. 6. The governing differential equation and essential boundary conditions are given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -2SinxCosy,\ T_1 = 0\ ℃, T_2 = \overline{T}_2\ ℃,$$

$$T_3 = \overline{T}_3\ ℃,\quad \frac{\partial T_4(x,0)}{\partial y} = 0 \quad (38)$$

where T is the temperature and T_i denote the prescribed boundary temperatures.

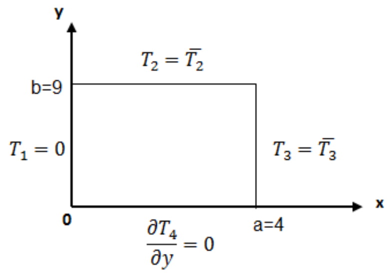

Fig. 6. Problem domain and boundary conditions

The analytical solution of the above boundary value problem is given by

$$T(x,y) = SinxCosy \quad (39)$$

The solution of this problem is obtained by using the same node distributions, kernel function and kernel function parameters as given in Section 4.2. Convergence and accuracy properties of the SMITSM I, II and III and SSPH method are examined by using the global L_2 error norm given by equation (36).

From equation (38), we can obtain the following recursive equation by using the DTM technique

$$(k+1)(k+2)T(k+2,m) + (m+1)(m+2)T(k,m+2) = -2\frac{1}{k!m!}\left[\frac{\partial^{k+m} SinxCosy}{\partial x^k y^m}\right]_{(x,y)} \quad (40)$$

Then, the vectors $\boldsymbol{P}$ and $\boldsymbol{U}_i$ can be written as follows

$$\boldsymbol{P}(\boldsymbol{x},\boldsymbol{\xi}) = \left[1, (x-x_i)^1, (y-y_i)^1, (x-x_i)^2, (y-y_i)^2, (x-x_i)^1(y-y_i)^1\right]$$

$$\boldsymbol{U}_i = [U_i(0,0), U_i(1,0), U_i(0,1), U_i(2,0), U_i(0,2), U_i(1,1)]^T \quad (41)$$

The numerical solutions obtained by using 6 terms in the associated expansions and 50, 171 and 629 nodes are presented in Fig. 7 to Fig. 12.

In Fig. 7 to Fig. 9, it is observed that the L_2 error norms of the SMITSM II and III with the variation of the radius of the support domain (where h=Δ) are much lower than those the SMITSM I and the SSPH method provided that the same number of terms are employed in the associated TSEs for both methods.

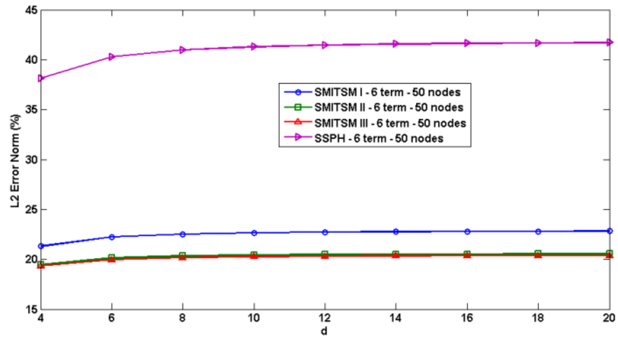

Fig. 7. The global L_2 error norms as the radius of the support domain (h=Δ) varies, where equally spaced 50 nodes are used

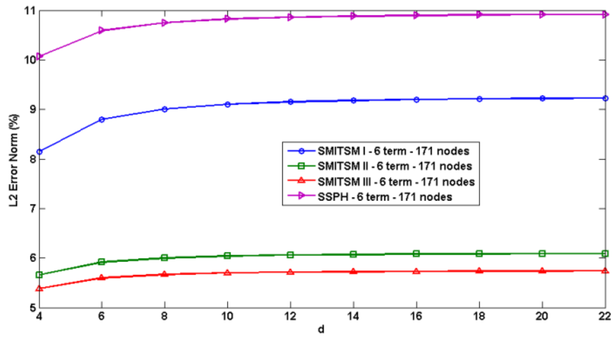

Fig. 8. The global L_2 error norms as the radius of the support domain (h=Δ) varies, where equally spaced 171 nodes are used

It is observed in Fig. 10 to Fig. 12 that accuracy of the SMITSM II and III is better than that of the SMITSM I and SSPH method as the smoothing length parameter varies provided that the same number of terms are employed in the associated TSEs for both methods.

Numerical results imply that the global L_2 error norm of numerical solutions increase as smoothing length parameter increases for all methods. It is observed that the SSPH method is stable for h=1.8Δ and node distribution of 171 nodes; however, the SMITSM I, II and III are stable even for h=2Δ as can be seen in Fig. 10.

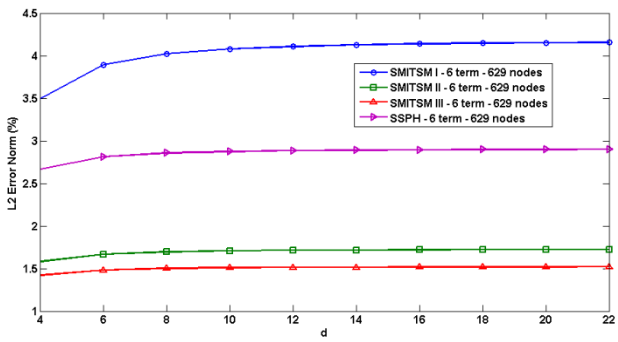

Fig. 9. The global L_2 error norms as the radius of the support domain (h=Δ) varies, where equally spaced 629 nodes are used

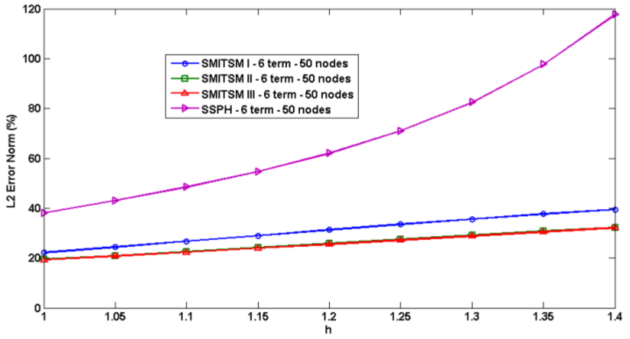

Fig. 10. The global L_2 error norm as the smoothing length varies, where equally spaced 50 nodes are used.

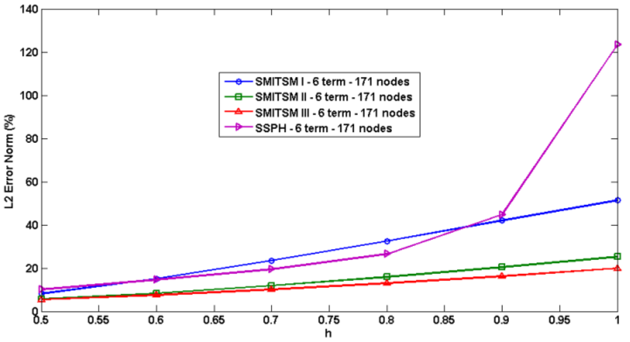

Fig. 11. The global L_2 error norms as the smoothing length varies, where equally spaced 171 nodes are used

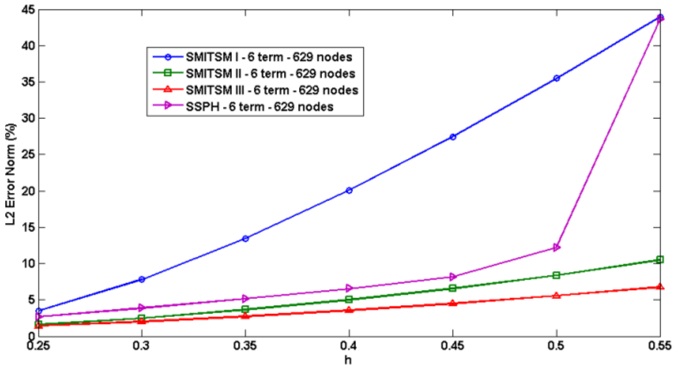

Fig. 12. The global L_2 error norms as the smoothing length varies, where equally spaced 629 nodes are used

It is also observed that the SSPH method is stable for h=2Δ and node distribution of 629 nodes; however, the SMITSM II and III are stable even for h=2.2Δ as can be seen in Fig. 12

Except for 629 nodes in the problem domain, the SSPH method always gives the highest global L_2 error norm; on the other hand, for 629 nodes, the SMITSM I gives the highest global L_2 error norm.

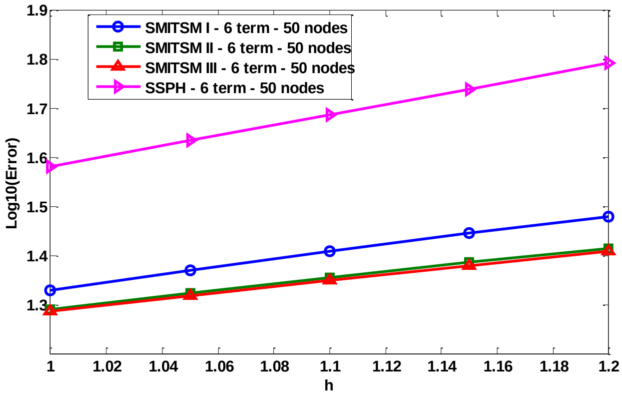

Fig. 13. The convergence rate of the error norm, where equally spaced 50 nodes are used

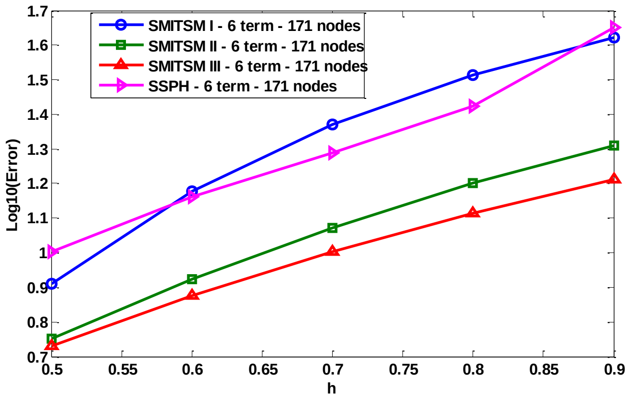

Fig. 14. The convergence rate of error norm, where equally spaced 171 nodes are used

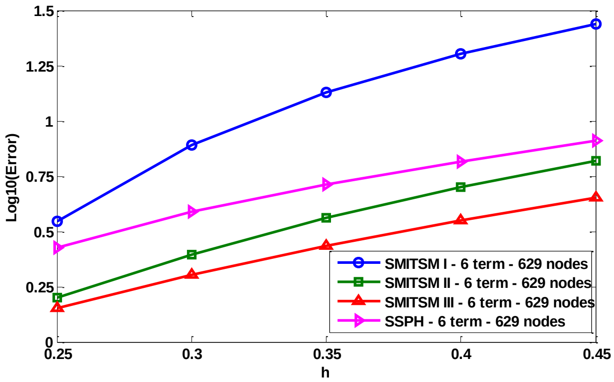

Fig. 15. The convergence rate of the error norm, where equally spaced 629 nodes are used

To find the rate of convergence of numerical solutions with respect to the distance between adjacent particles,the global L_2 error norm is used. The convergence rates of the error norm are presented in Fig. 13 to Fig. 15.

It is observed that the convergence rate of the SSPH method is higher than the other methods for 50 nodes. And also SMITSM I, II and III has nearly the same convergence rate of error norm for 50 nodes in the problem domain.

For 171 and 629 nodes, SMITSM I has the highest convergence rate or error norm. The converge rate of SMITSM II, III and SSPH methods are nearly the same.

5. Conclusion

We presented a new meshless approach called the SMITSM I, II and III by using the TSEs and utilizing the DTM technique. It is observed that the SMITSM II and III yields more accurate results than the SSPH method especially in the existence of nonsmooth nonhomogenous terms. The SMITSM I, II and III does not involve any approximation and its formulations are exact except for the truncations in the TSEs. In addition, as the number of terms in the TSEs and/or nodes in numerical examples are increased, its L_2 error norm decreases that is the evidence of the convergence of the SMITSM I, II and III.

Note that CPU times of the SMITSM I, II and III in solving numerical examples are much larger than those of the SSPH method. Nonetheless, the CPU time and memory requirement of the SMITSM I, II and III can be reduced by utilizing the block form of the associated equation systems, which is not investigated in this paper and will be the subject of future studies.

Even though strong form of the MITSM is considered in this paper, the same approach can easily be applied to weak formulations that leads to Weak Form Meshless Implementation of Taylor Series Method (WMITSM), that will be the subject of future studies as well.

Acknowledgements

The author thanks to anonymous reviewers for their helpful suggestions. The author also thanks to PhD advisor Ata Mugan for his endless support and encouragement.

References

[1] LB Lucy, A numerical approach to the testing of the fission hypothesis, Astron J, Vol. 82, pp. 1013–1024, 1987.

[2] JK Chen, JE Beraun, CJ Jin, An improvement for tensile instability in smoothed particle hydrodynamics,Comput Mech., Vol. 23,pp. 279–287, 1999.

[3] JK Chen, JE Beraun, CJ Jin, Completeness of corrective smoothed particlemethod for linear elastodynamics, Comput Mech. Vol. 24, pp. 273–285, 1999.

[4] WK Liu, S Jun ,YF Zhang, Reproducing kernel particle methods, Int J Num Meth Fluids, Vol. 20, pp. 1081–1106, 1996.

[5] WK Liu, S Jun, S Li, J Adee, T Belytschko, Reproducing kernel particle methods for structural dynamics, Int J Num Meth Eng., Vol. 38 pp. 1655–1679, 1995.

[6] JS Chen, C Pan, CT Wu, WK Liu, Reproducing kernel particlemethods for large deformation analysis of non-linear structures, Comput Method ApplMech Eng.,Vol. 139, pp. 195–227, 1996.

[7] GM Zhang, RC Batra, Modified smoothed particle hydrodynamics method and its application to transient problems, Comput Mech., Vol. 34, pp. 137–146, 2004.

[8] RC Batra, GM Zhang, Analysis of adiabatic shear bands inelasto-thermo- viscoplastic materials by modified smoothedparticle hydrodynamics (MSPH) method, J Comput Phys., Vol. 20, pp. 172–190, 2004.

[9] GM Zhang, RC Batra, Wave propagation in functionally graded materials by modified smoothed particle hydrodynamics (MSPH) method, J Comput Phys., Vol. 222, pp. 374–390, 2007.

[10]RC Batra, GM Zhang, Search algorithm, and simulation of elastodynamic crack propagation by modified smoothed particle hydrodynamics (MSPH) method,Comput Mech., Vol.40, pp. 531–546, 2007.

[11]GM Zhang, RC Batra, Symmetric smoothed particle hydrodynamics (SSPH) method and its application to elastic problems, Comput Mech.,Vol. 43 ,pp.321-340, 2009.

[12]RC Batra, GM Zhang, SSPH basis functions for meshless methods, and comparison of solutions with strong and weak formulations, Comput Mech. Vol. 41,pp. 527-545, 2008.

[13]A Karamanli, A Mugan, Solutions of two–dimensional heat transfer problems by using symmetric smoothed particle hydrodynamics method, Journal of Applied and Computational Mathematics, Vol. 1, pp. 1-6, 2012.

[14]A Karamanli, A Mugan, Strong from meshless implementation of Taylor series method, Appl. Math. Comput., Vol.219, pp. 9069-9080, 2013.

[15]C Bervillier, Status of the differential transformation method, Appl. Math. Comput., Vol. 218, pp. 10158-10170, 2012.

[16]H Liu, Y Song, Differential transform method applied to high index differential–algebraic equations, App Math Comput., Vol. 184, pp. 748-753, 2007.

[17]F Ayaz, Solutions of the system of differential equations by differential transform method, App Math Comput. Vol. 147, pp. 547-567,2004.

[18]HS Yalcin, A Arikoglu, I Ozkol, Free vibration analysis of circular plates by differential transform method, App Math Comput., Vol. 212, pp. 377-386, 2009.

[19]A Arikoglu, I Ozkol, Solution of differential-difference equations by using differential transform method, App Math Comput., Vol. 181, pp. 153-162, 2006.

[20]A Arikoglu, I Ozkol, Solution of difference equations by using differential transform method, App Math Comput., Vol. 174, pp. 1216-1228, 2006.

[21]A Arikoglu, I Ozkol, Solution of boundary value problems for integro differential equationsby using differential transform method, App Math Comput. Vol.168, pp. 1145-1158, 2005.