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SITEM for the Conformable Space-Time fractional (2+1)-Dimensional Asymmetric Nizhnik-Novikov-Veselov Equations

Handan Çerdik Yaslan[1*] and Ayse Girgin[1]

[1]*Department of Mathematics, Pamukkale University, Denizli, 20070, Turkey*
[*]*Corresponding author E-mail: hcerdik@pau.edu.tr*

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Abstract

In the present paper, new analytical solutions for the space-time fractional (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov (ANNV) equations are obtained by using the simplified $\tan(\frac{\phi(\xi)}{2})$-expansion method (SITEM).

1. Introduction

Nonlinear model arising from the field of mathematical physics is a popular topic since it is widely applied in many natural science such as chemistry, biology, mathematics, communication and particularly in almost all branches of physics like the fluid dynamics, plasma physics, field theory, nonlinear optics and condensed matter physics. Exact solutions of nonlinear models have extensively been investigated by different methods. For example, solutions of the $(1+1)$-dimensional KdV-type model by means of the modified tanh-function method with three different ansatz has been obtained [1]. Non-linear differential-difference sine-Gordon equation has been solved by using Jacobian elliptic function method [2]. Hierarchies of Peregrine solution and breather solution have been derived in a (2+1)-dimensional variable-coefficient nonlinear Schrodinger equation with partial nonlocality [3]. Extended tanh-function method based on the mapping method has been applied to the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov system [4].

Nizhnik-Novikov-Veselov (NNV) equations have an important place in many fields of physics including condense matter physics, optics, fluid mechanics and plasma physics [5]-[7]. Solutions of the NNV equations have been investigated by many researchers. Extended tanh-function method, exp-function method, generalized auxiliary equation method have been applied to (2+1)-dimensional ANNV equations [8]-[10]. Generalized Nizhnik-Novikov-Veselov (GNNV) equations have been solved by using exp-function method, the extended hyperbolic function method, the tanh method, generalized F-expansion method and auxiliary ordinary differential equation method [11]-[15]. Combining the generalized direct method with the classical Lie method, solutions of the GNNV equations have been investigated [16]. The generalized, asymmetric and the modified NNV equations have been studied by using Hirota's bilinear method [17].

Fractional NNV equations have been studied in [18]-[22]. In these works, fractional derivatives are described in modified Riemann-Liouville sense (see, for example, [18]-[20]) and conformable sense (see, for example, [21, 22]). Generalized exp-function method has been applied to the space-time fractional ANNV equations [18]. Solitary-wave ansatz method, the (G'/G) expansion method and sub equation method have been used to obtain exact solutions of the space-time fractional GNNV equations [19, 20]. Exp-function method, (G'/G) expansion method and homotopy analysis method have been applied to the time fractional GNNV [21, 22].

Recently, the improved $\tan(\frac{\phi(\xi)}{2})$-expansion method (ITEM) has been applied by many authors [23]-[25]. In [26], ITEM has been simplified and called simplified ITEM (SITEM). SITEM has been applied to the Kundu-Eckhaus equation and Konopelchenko- Dubrovsky equations

Email addresses and ORCID numbers**: hcerdik@pau.edu.tr, https://orcid.org/0000-0002-3243-3703 (H. Çerdik Yaslan), aysegirgin20@gmail.com, http://orcid.org/0000-0002-2972-7583 (A. Girgin)**

in [26, 27], respectively. In this paper, we obtain new analytical solutions of the space-time fractional (2+1)-dimensional ANNV equations by using SITEM.

2. Description of the conformable fractional derivative and its properties

For a function $f:(0,\infty)\to R$, the conformable fractional derivative of f of order $0<\alpha<1$ is defined as (see, for example, [28])

$$T_t^{\alpha}f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}.$$

Some important properties of the the conformable fractional derivative are as given follows:

$$\begin{aligned} T_t^{\alpha}(af+bg)(t) &= aT_t^{\alpha}f(t)+bT_t^{\alpha}g(t),\ \forall a,b\in R,\\ T_t^{\alpha}(t^{\mu}) &= \mu t^{\mu-\alpha},\\ T_t^{\alpha}(f(g(t))) &= t^{1-\alpha}g'(t)f'(g(t)).\end{aligned}$$

3. Analytic solutions to the conformable space-time fractional ANNV equations

Conformable space-time fractional ANNV equations are given in the following form [8, 9]

$$T_t^{\alpha}u-T_x^{\beta}T_x^{\beta}T_x^{\beta}u-3T_x^{\beta}(uv)=0, \tag{3.1}$$
$$T_x^{\beta}u=T_y^{\theta}v,\ 0<\alpha\le 1,\ 0<\beta\le 1,\ 0<\theta\le 1. \tag{3.2}$$

Eqs.(3.1)-(3.2) were first derived by Boiti et al. [29] which may be considered as a model for an incompressible fluid.
Let us consider the following transformation

$$u(x,y,t)=U(\xi),\ v(x,y,t)=V(\xi),\ \xi=k\frac{t^{\alpha}}{\alpha}+m\frac{x^{\beta}}{\beta}+n\frac{y^{\theta}}{\theta}, \tag{3.3}$$

where k, m, n are constants. Substituting (3.3) into Eqs.(3.1)-(3.2) we obtain the following differential equations

$$kU'-m^3U'''-3m(UV)'=0, \tag{3.4}$$
$$mU'=nV' \tag{3.5}$$

Integrating of Eqs.(3.4)-(3.5) with zero constant of integration and eliminating V, we have

$$kU-m^3U''-\frac{3m^2}{n}U^2=0. \tag{3.6}$$

Let us suppose that the solution of Eq.(3.6) can be expressed in the form

$$U(\xi)=\sum_{k=0}^{N}A_k\left[p+\tan\left(\frac{\phi(\xi)}{2}\right)\right]^k+\sum_{k=1}^{N}B_k\left[p+\tan\left(\frac{\phi(\xi)}{2}\right)\right]^{-k}. \tag{3.7}$$

Here $\phi(\xi)$ satisfies the following ordinary differential equation

$$\phi'(\xi)=a\sin(\phi(\xi))+b\cos(\phi(\xi))+c, \tag{3.8}$$

$a,\ b,\ c, A_k(0\le k\le N)$ and $B_k(1\le k\le N)$ are constants to be determined. The solution of Eq. (3.8) has been given in[27].
Substituting Eq.(3.7) into Eq.(3.6) for $p=0$ and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 2. Therefore, Eq.(3.7) reduces to

$$\begin{aligned} U(\xi) &= A_0+A_1\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]+A_2\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]^2+B_1\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]^{-1}\\ &+ B_2\left[\tan\left(\frac{\phi(\xi)}{2}\right)\right]^{-2}.\end{aligned} \tag{3.9}$$

Substituting Eq.(3.9) into Eq.(3.6) and collecting all the terms with the same power of $\tan(\frac{\phi}{2})$, we can obtain a set of algebraic equations for the unknowns $A_0, A_1, A_2, B_1, B_2, k, m, n$:

$$\begin{aligned}
&-6A_2^2m^2-3nA_2b^2m^3+6nA_2bcm^3-3nA_2c^2m^3=0,\\
&-A_1nb^2m^3+2A_1nbcm^3+10aA_2nbm^3-A_1nc^2m^3-10aA_2ncm^3-12A_1A_2m^2=0,\\
&-8A_2na^2m^3+3naA_1bm^3-3naA_1cm^3-6A_1^2m^2+4A_2nb^2m^3-4A_2nc^2m^3-12A_0A_2m^2+2A_2kn=0,\\
&2A_1kn-12A_0A_1m^2-12A_2B_1m^2-2a^2A_1m^3n+A_1b^2m^3n-A_1c^2m^3n-6aA_2bm^3n-6aA_2cm^3n=0,\\
&2A_0kn-6A_0^2m^2-12A_1B_1m^2-12A_2B_2m^2-A_2b^2m^3n-A_2c^2m^3n-b^2B_2m^3n-B_2c^2m^3n-aA_1bm^3n-aA_1cm^3n+abB_1m^3n-aB_1cm^3n\\
&-2A_2bcm^3n+2bB_2cm^3n=0,\\
&2B_1kn-12A_0B_1m^2-12A_1B_2m^2-2a^2B_1m^3n+b^2B_1m^3n-B_1c^2m^3n+6abB_2m^3n-6aB_2cm^3n=0,\\
&-8B_2na^2m^3-3nabB_1m^3-3naB_1cm^3+4B_2nb^2m^3-6B_1^2m^2-4B_2nc^2m^3-12A_0B_2m^2+2B_2kn=0,\\
&-B_1nb^2m^3-2B_1nbcm^3-10aB_2nbm^3-B_1nc^2m^3-10aB_2ncm^3-12B_1B_2m^2=0,\\
&-3nb^2B_2m^3-6nbB_2cm^3-6B_2^2m^2-3nB_2c^2m^3=0.
\end{aligned}$$

Solving the algebraic equations in the Mathematica, we obtain the following set of solutions:
Case 1: $A_0=\frac{1}{2}(b^2-c^2)mn, A_1=0, A_2=0, B_1=-amn(b+c), B_2=-\frac{1}{2}(b+c)^2mn, k=\Delta m^3$:
For $b=c$ and $a=0$, we have

$$U_1(\xi)=-2b^2mn\Big[b\xi+c_1\Big]^{-2}.$$

For $b=c$ and $a\neq 0$, we have

$$U_2(\xi)=-amn2b\Big[c_1\exp(a\xi)-\frac{b}{a}\Big]^{-1}-2b^2mn\Big[c_1\exp(a\xi)-\frac{b}{a}\Big]^{-2}.$$

For $\Delta>0$ and $b\neq c$, we obtain

$$\begin{aligned} U_3(\xi) &= \frac{1}{2}(b^2-c^2)mn \\ - & amn(b+c)\Big[\frac{2}{b-c}\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^{-1} \\ - & \frac{1}{2}(b+c)^2mn\Big[\frac{2}{b-c}\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^{-2}. \end{aligned} \tag{3.10}$$

For $\Delta<0$ and $b\neq c$, we have

$$\begin{aligned} U_4(\xi) &= \frac{1}{2}(b^2-c^2)mn \\ - & amn(b+c)\Big[\frac{a}{b-c}+\frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^{-1} \\ & -\frac{1}{2}(b+c)^2mn\Big[\frac{a}{b-c}+\frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^{-2}. \end{aligned}$$

Case 2: $A_0=\frac{1}{2}(b^2-c^2)mn, A_1=a(b-c)mn, A_2=-\frac{1}{2}(b-c)^2mn, B_1=0, B_2=0, k=\Delta m^3$:
For $\Delta>0$ and $b\neq c$, we have

$$\begin{aligned} U_5(\xi) &= \frac{1}{2}(b^2-c^2)mn+2amn\Big[\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big] \\ & -2mn\Big[\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^2. \end{aligned}$$

For $\Delta<0$ and $b\neq c$, we have

$$\begin{aligned} U_6(\xi) &= \frac{1}{2}(b^2-c^2)mn \\ + & amn\Big[a+\sqrt{-\Delta}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big)\Big] \\ - & \frac{1}{2}mn\Big[a+\sqrt{-\Delta}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^2. \end{aligned} \tag{3.11}$$

Case 3: $A_0=\frac{1}{6}(-2a^2mn+b^2mn-c^2mn), A_1=0, A_2=0, B_1=-amn(b+c), B_2=-\frac{1}{2}(b+c)^2mn, k=-\Delta m^3$:
For $b=c$ and $a=0$, we obtain

$$U_7(\xi)=-2b^2mn\Big[b\xi+c_1\Big]^{-2}.$$

For $b=c$ and $a\neq 0$, we have

$$\begin{aligned} U_8(\xi) &= -\frac{1}{3}(a^2mn)-amn(2b)\Big[c_1\exp(a\xi)-\frac{b}{a}\Big]^{-1} \\ - & 2b^2mn\Big[c_1\exp(a\xi)-\frac{b}{a}\Big]^{-2}. \end{aligned}$$

For $\Delta>0$ and $b\neq c$, we have

$$\begin{aligned} U_9(\xi) &= \frac{1}{6}(-2a^2mn+b^2mn-c^2mn) \\ - & amn(b+c)\Big[\frac{2}{b-c}\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^{-1} \\ - & \frac{1}{2}(b+c)^2mn\Big[\frac{2}{b-c}\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^{-2}. \end{aligned}$$

For $\Delta < 0$ and $b \neq c$, we have

$$\begin{aligned} U_{10}(\xi) &= \frac{1}{6}(-2a^2mn + b^2mn - c^2mn) \\ - \quad & amn(b+c)\Big[\frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^{-1} \\ - \quad & \frac{1}{2}(b+c)^2mn\Big[\frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^{-2}. \end{aligned} \tag{3.12}$$

Case 4: $A_0 = \frac{1}{6}(-2a^2mn + b^2mn - c^2mn), A_1 = a(b-c)mn, A_2 = -\frac{1}{2}(b-c)^2mn, B_1 = 0, B_2 = 0, k = -\Delta m^3$:
For $\Delta > 0$ and $b \neq c$, we have

$$\begin{aligned} U_{11}(\xi) &= \frac{1}{6}(-2a^2mn + b^2mn - c^2mn) \\ + \quad & 2amn\Big[\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big] \\ - \quad & 2mn\Big[\frac{c_1r_1\exp(r_1\xi)+c_2r_2\exp(r_2\xi)}{c_1\exp(r_1\xi)+c_2\exp(r_2\xi)}\Big]^2. \end{aligned}$$

For $\Delta < 0$ and $b \neq c$, we have

$$\begin{aligned} U_{12}(\xi) &= \frac{1}{6}(-2a^2mn + b^2mn - c^2mn) \\ + \quad & amn\Big[a + \sqrt{-\Delta}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big] \\ - \quad & \frac{1}{2}mn\Big[a + \sqrt{-\Delta}\frac{-c_1\sin(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1\cos(\frac{\sqrt{-\Delta}}{2}\xi)+c_2\sin(\frac{\sqrt{-\Delta}}{2}\xi)}\Big]^2, \end{aligned}$$

where $\xi = -\Delta m^3\frac{t^\alpha}{\alpha} + m\frac{x^\beta}{\beta} + n\frac{y^\theta}{\theta}$, $\Delta = a^2 + b^2 - c^2$. From the formula $V(\xi) = \frac{m}{n}U(\xi)$, $v(x,y,t)$ can be computed.

The solutions $u_2(x,y,t)$, $u_5(x,y,t)$, $u_6(x,y,t)$ and $u_{10}(x,y,t)$ of the Eqs.(3.1)-(3.2) are simulated as traveling wave solutions for various values of the physical parameters in Fig.3.1-Fig.3.8. Figs.3.1, 3.2 show kink waves solutions, Figs.3.3 and 3.4 show solitary waves solutions, Figs.3.5, 3.6, 3.7 and 3.8 show periodic waves solutions of Eqs.(3.1)-(3.2). Figs.3.1 and 3.2 are 3D and 2D plots of the traveling wave solution $u_2(x,1,t)$ and $u_2(x,1,1)$ in Eq.(3.10). 3D plot of the obtained solution $u_2(x,1,t)$ is given for parameters $\alpha = 0.5$, $\beta = 1$, $\theta = 0.75$, $m = 0.25$, $n = -0.5$, $a = 0.5$, $b = 0.25$, $c = 0.25$, $c_1 = 1$, $c_2 = 1$ in Fig.3.1. Fig.3.2 demonstrate the same solution with 2D plot for $-40 \leq x \leq 40$ at $t = 1$. Figs.3.3 and 3.4 are 3D and 2D plots of the traveling wave solution $u_5(x,1,t)$ and $u_5(x,1,1)$ in Eq.(3.11) for $\alpha = 0.5$, $\beta = 1$, $\theta = 0.75$, $m = 1$, $n = -0.5$, $a = 0.02$, $b = 0.2$, $c = 0.01$, $c_1 = 2$, $c_2 = 1$, respectively. Figs.3.5 and 3.6 are 3D and 2D plots of the traveling wave solution $u_6(x,1,t)$ and $u_6(x,1,1)$ in Eq.(3.11) for $\alpha = 0.5$, $\beta = 1$, $\theta = 0.75$, $m = 0.1$, $n = -0.5$, $a = 1$, $b = 2$, $c = 5$, $c_1 = 2$, $c_2 = 2$, respectively. Figs.3.7 and 3.8 show 3D and 2D plots of the traveling wave solution $u_{10}(x,1,t)$ and $u_{10}(x,1,1)$ in Eq.(3.12) for $\alpha = 0.75$, $\beta = 1$, $\theta = 0.5$, $m = 0.25$, $n = 0.05$, $a = 1$, $b = 2$, $c = 3$, $c_1 = 1$, $c_2 = 1$, respectively. Note that the 3D graphs describe the behavior of u in space x and time t at fixed $y = 1$, which represents the change of amplitude and shape for each obtained solitary wave solutions. 2D graphs describe the behavior of u in space x at fixed time $t = 1$ and fixed $y = 1$. All graphics in figures are drawn by the aid of Mathematica 10.

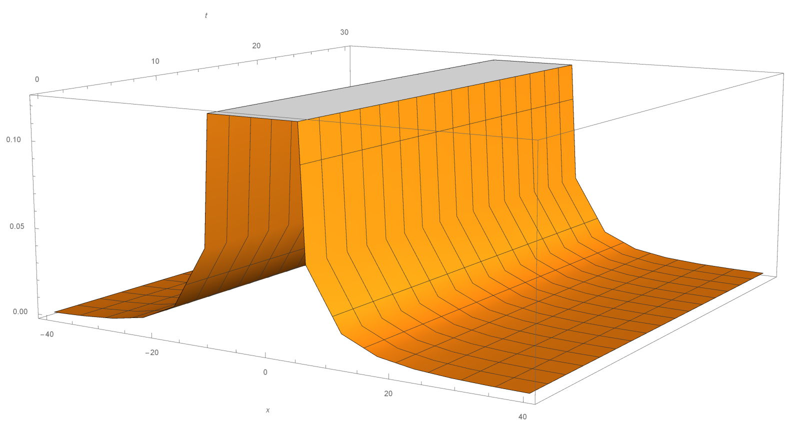

Figure 3.1: King wave solution $u_2(x,1,t)$ of Eq.(3.10).

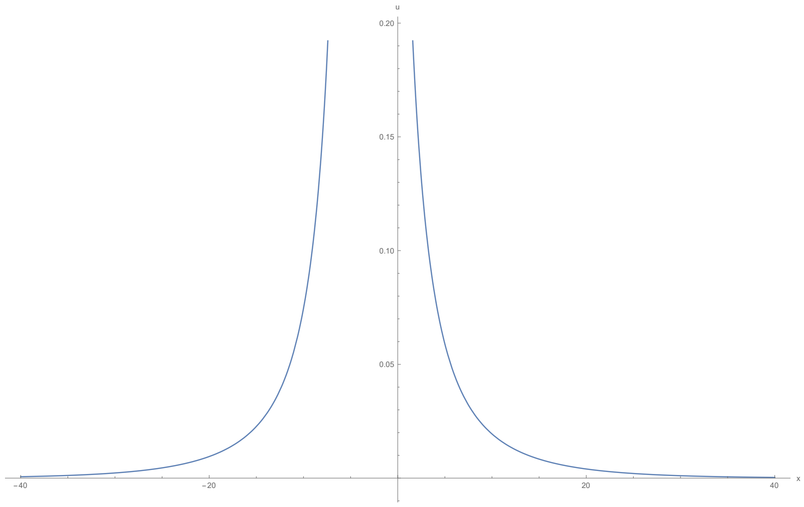

Figure 3.2: King wave solution $u_2(x,1,1)$ of Eq.(3.10).

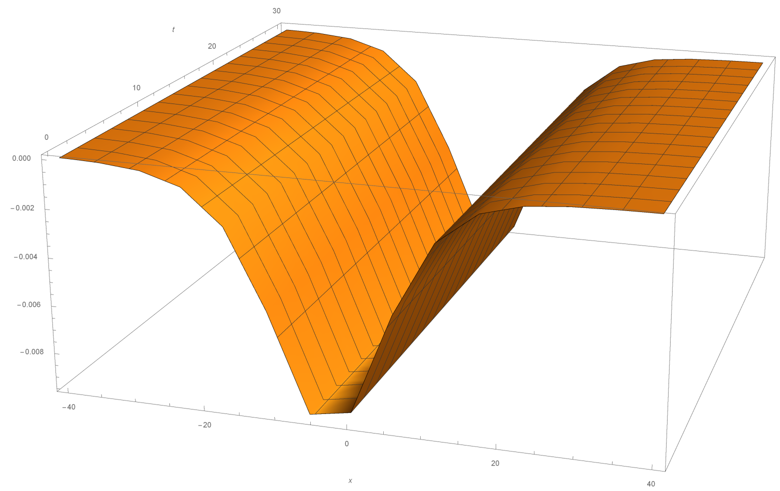

Figure 3.3: Solitary wave solution $u_5(x,1,t)$ of Eq.(3.11).

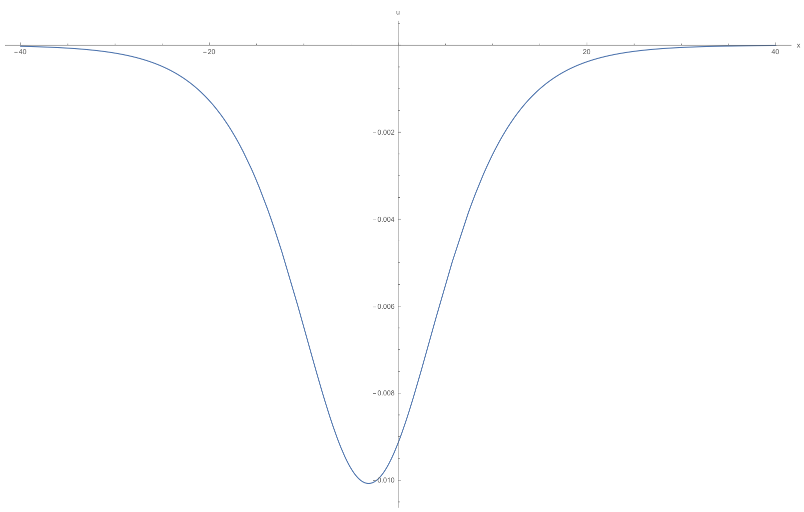

Figure 3.4: Solitary wave solution $u_5(x,1,1)$ of Eq.(3.11).

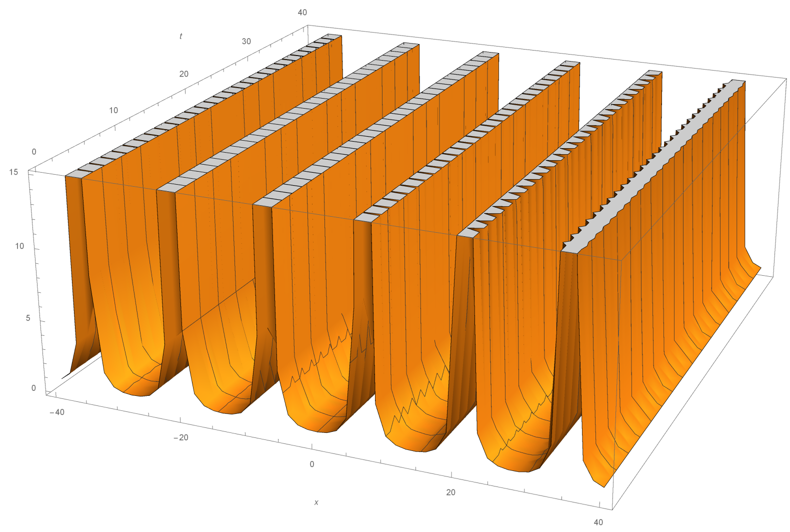

Figure 3.5: Periodic wave solution $u_6(x,1,t)$ of Eq.(3.11).

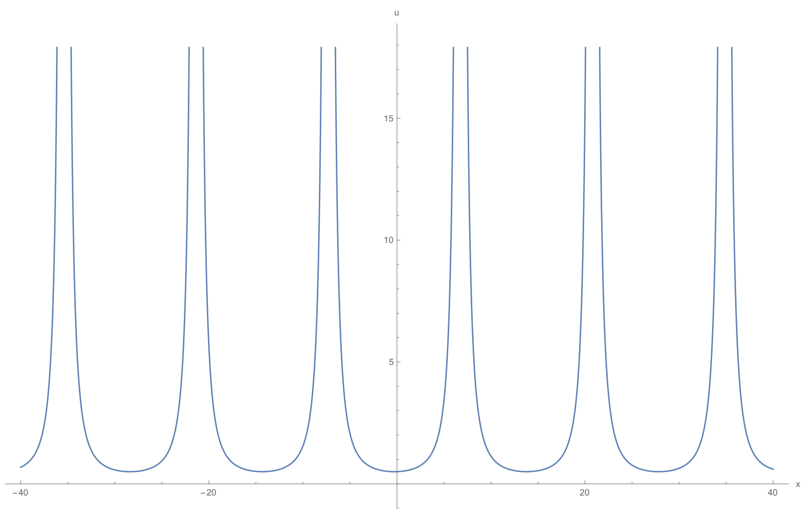

Figure 3.6: Periodic wave solution $u_6(x,1,1)$ of Eq.(3.11).

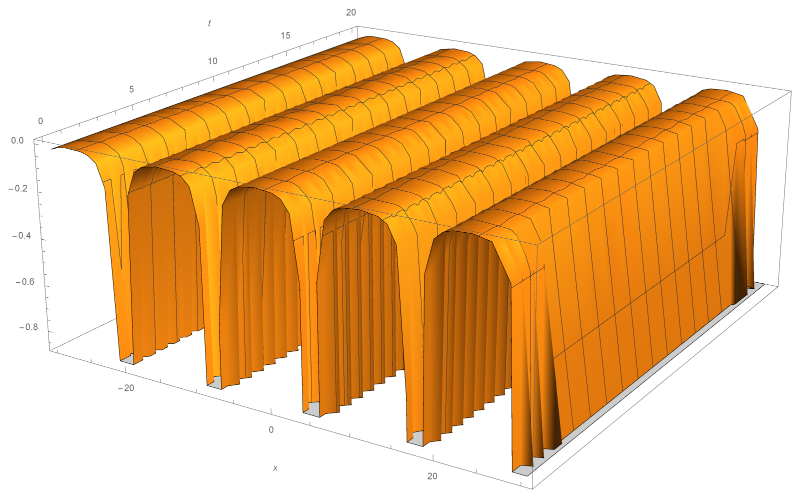

Figure 3.7: Periodic wave solution $u_{10}(x,1,t)$ of Eq.(3.12).

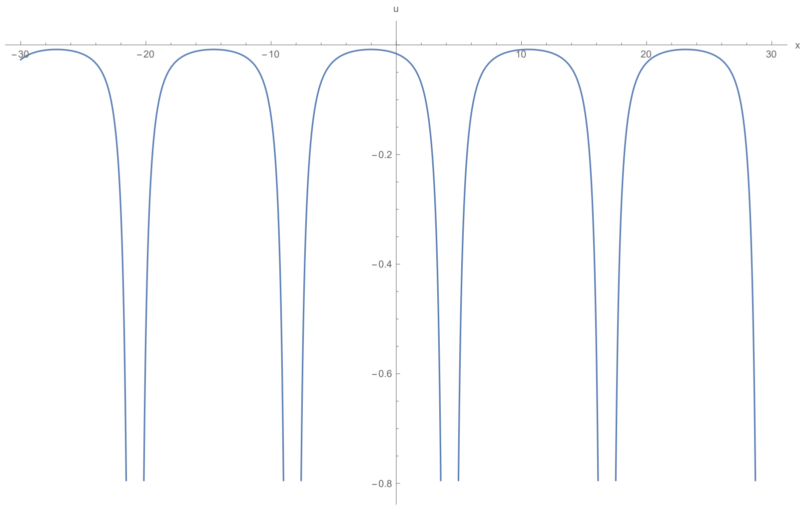

Figure 3.8: Periodic wave solution $u_{10}(x,1,1)$ of Eq.(3.12).

4. Conclusion

In this paper, the conformable space-time fractional ANNV equations have been solved by using the simplified $\tan(\frac{\phi(\xi)}{2})$-expansion method (SITEM). Simulations of the kink wave, solitary wave and periodic wave solutions of the conformable space-time fractional ANNV equations have been obtained. Note that SITEM has been applied to the Kundu-Eckhaus equation for the parameter $p = 0$ in [26] and Konopelchenko-Dubrovsky equations for the nonzero parameter p in [27]. To our knowledge, conformable fractional ANNV equations have been solved for only time fractional case. In our work, SITEM has been applied to both space and time fractional ANNV equations.

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