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2-Absorbing Semiprimary Fuzzy Ideal of Commutative Rings

Deniz Sönmez[1], Gürsel Yeşilot[1], Serkan Onar[2*] and Bayram Ali Ersoy[1]

[1]*Department of Mathematics, Faculty of Science and Arts, Yildiz Technical University, Turkey*
[2] *Management Information Systems, Gelişim University, Turkey*
[*]*Corresponding author E-mail: serkan10ar@gmail.com*

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Abstract

In this work, we introduce the notion of 2-absorbing semiprimary fuzzy ideal which is a generalization of semiprimary fuzzy ideal. Let R be a ring. Then the nonconstant fuzzy ideal μ is called a 2-absorbing semiprimary fuzzy ideal if $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of R. Furthermore, we give some fundamental results concerning these notions.

1. Introduction

Zadeh in 1965 introduced the fundamental concept of fuzzy set [1]. Focusing on the structure of ring, the paper of Liu [2], defining fuzzy ideals, initiated the investigation of rings by means of expanding the class of ideals with these fuzzy objects. Mukherjee and Sen have continued the study of fuzzy ideals by introducing the notion of prime fuzzy ideals [3]. Nowadays, fuzzy algebraic structures were developed and many interesting results were obtained. The concept of 2-absorbing ideals, which is a generalization of prime ideals [4] and 2-absorbing primary ideals, which is a generalization of primary ideals [5] were introduced. Although the prime fuzzy ideals and primary fuzzy ideals have been investigated [3, 6], the concept of 2-absorbing semiprimary fuzzy ideals have not been studied yet. In this study, we characterize the 2-absorbing semiprimary fuzzy ideals, some generalizations of 2-absorbing semiprimary fuzzy ideals and described some their properties. Recall from [4, 5] that a proper ideal I of R is called a 2-absorbing ideal if whenever $a,b,c \in R$ and $abc \in I$ then either $ab \in I$ or $ac \in I$ or $bc \in I$ and a proper ideal I of R is called a 2-absorbing primary ideal if whenever $a,b,c \in R$ and $abc \in I$ then either $ab \in I$ or $ac \in \sqrt{I}$ or $bc \in \sqrt{I}$. Recall also from [7] that a nonconstant fuzzy ideal μ of R is called a 2-absorbing fuzzy ideal of R if for any fuzzy points x_r, y_s, z_t of R, $x_r y_s z_t \in \mu$ implies that either $x_r y_s \in \mu$ or $x_r z_t \in \mu$ or $y_s z_t \in \mu$ and a nonconstant fuzzy ideal μ of R is called a 2-absorbing primary fuzzy ideal of R if for any fuzzy points x_r, y_s, z_t of R, $x_r y_s z_t \in \mu$ implies that either $x_r y_s \in \mu$ or $x_r z_t \in \sqrt{\mu}$ or $y_s z_t \in \sqrt{\mu}$. Based on these definitions, a nonconstant fuzzy ideal μ is called a *2-absorbing semiprimary fuzzy ideal* if $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of R.

2. Preliminaries

We assume throughout that all rings are commutative with $1 \neq 0$. Unless stated otherwise $L = [0,1]$ stands for a complete lattice. $\mathbb{Z}$ denotes the ring of integers, $L(R)$ denotes the set of fuzzy sets of R and $LI(R)$ denotes the set of fuzzy ideals of R. For $\mu, \xi \in L(R)$, we say $\mu \subseteq \xi$ if and only if $\mu(x) \leq \xi(x)$ for all $x \in R$. When $r \in L$, $x, y \in R$ we define $x_r \in L(R)$ as follows :

$$x_r(y) = \begin{cases} x & if\ x = y, \\ 0 & otherwise, \end{cases}$$

and x_r is referred to as fuzzy point of R. Let I be an ideal of R. Then

$$\lambda_I = \begin{cases} 1 & if\ x \in I, \\ 0 & otherwise, \end{cases}$$

Definition 2.1. *[2] A fuzzy subset μ of a ring R is called a fuzzy ideal of R if for all $x, y \in R$ the following conditions are satisfied :*

***Email addresses and ORCID numbers*: dnzguel@hotmail.com, https://orcid.org/0000-0002-7574-4245 (D. Sönmez), gyesilot@yildiz.edu.tr, https://orcid.org/0000-0002-5205-5842 (G. Yeşilot),serkan10ar@gmail.com, https://orcid.org/0000-0003-3084-7694 (S. Onar),ersoya@gmail.com, https://orcid.org/0000-0002-8307-9644 (B. A. Ersoy)**

- $\mu(x-y) \geq \mu(x) \wedge \mu(y)$, $\forall x,y \in R$
- $\mu(xy) \geq \mu(x) \vee \mu(y)$, $\forall x,y \in R$

Let μ be any fuzzy ideal of R; $x,y \in R$, and 0 be the additive identity of R. Then it is easy to verify the following:
(i) *$\mu(0) \geq \mu(x)$, $\mu(x) = \mu(-x)$ and $\mu_t \subset \mu_s$ where $s,t \in Im(\mu)$ and $t > s$.*
(ii) *If $\mu(0) = \mu(x-y)$, then $\mu(x) = \mu(y)$, $\mu(x) = s$ iff $x \in \mu_s$ and $x \notin \mu_t, \forall t > s$.*

Definition 2.2. *[8] Let μ be any fuzzy ideal of R. The ideals μ_t, $(\mu(0) \geq t)$ are called level ideals of μ.*

Definition 2.3. *[3] A fuzzy ideal μ of R is called prime fuzzy ideal if for any two fuzzy points x_r, y_s of R , $x_r y_s \in \mu$ implies either $x_r \in \mu$ or $y_s \in \mu$.*

Definition 2.4. *[6] Let μ be a fuzzy ideal of R. Then $\sqrt{\mu}$, called the radical of μ, is defined by $\sqrt{\mu}(x) = \bigvee_{n \geq 1} \mu(x^n)$.*

Definition 2.5. *[6] A fuzzy ideal μ of R is called primary fuzzy ideal if for any two fuzzy points x_r, y_s of R , $x_r y_s \in \mu$ implies either $x_r \in \mu$ or $y_s \in \sqrt{\mu}$.*

Theorem 2.6. *[6] Let μ be fuzzy ideal of a ring R. Then $\sqrt{\mu}$ is a fuzzy ideal of R.*

Definition 2.7. *[3] Let R be a ring. Then a nonconstant fuzzy ideal μ is said to be weakly completely prime fuzzy ideal iff for $x,y \in R$, $\mu(xy) = max\{\mu(x), \mu(y)\}$.*

Theorem 2.8. *[9] Let $f : R \to S$ be a ring homomorphism and let μ be a fuzzy ideal of R such that μ is constant on $Kerf$ and ξ be a fuzzy ideal of S. Then ,*

- $\sqrt{f(\mu)} = f(\sqrt{\mu})$,
- $\sqrt{f^{-1}(\xi)} = f^{-1}(\sqrt{\xi})$.

Definition 2.9. *[4] A nonzero proper ideal I of a commutative ring R with $1 \neq 0$ is called a 2-absorbing ideal if whenever $a,b,c \in R$ with $abc \in I$, then either $ab \in I$ or $ac \in I$ or $bc \in I$.*

Definition 2.10. *[5] A proper ideal I of R is called a 2-absorbing primary ideal of R if whenever $a,b,c \in R$ with $abc \in I$, then either $ab \in I$ or $ac \in \sqrt{I}$ or $bc \in \sqrt{I}$.*

Definition 2.11. *[10] A proper ideal I of R is called a 2-absorbing quasi primary ideal of R if whenever $a,b,c \in R$ with $abc \in I$, then either $ab \in \sqrt{I}$ or $ac \in \sqrt{I}$ or $bc \in \sqrt{I}$.*

Theorem 2.12. *[5] If I is a 2-absorbing primary ideal of R, then $\sqrt{I}$ is a 2-absorbing ideal of R.*

Definition 2.13. *[11] An element $1 > \alpha \in L$ is called a 2-absorbing element if for any $x,y,z \in L$, $x \wedge y \wedge z < \alpha$ implies either $x \wedge y < \alpha$ or $x \wedge z < \alpha$ or $y \wedge z < \alpha$.*

Lemma 2.14. *[9] Let μ be a fuzzy ideal of R. Then for any positive integer n, $\sqrt{\mu^n} = \sqrt{\mu}$.*

Lemma 2.15. *[9] Let μ and λ be fuzzy ideals of R. If $\mu \subseteq \lambda$ then $\sqrt{\mu} \subseteq \sqrt{\lambda}$.*

Theorem 2.16. *[9] If μ and ξ are two fuzzy ideals of R, then $\sqrt{\mu \cap \xi} = \sqrt{\mu} \cap \sqrt{\xi} = \sqrt{\mu\xi}$*

Theorem 2.17. *[7] $f : R \to S$ be a ring homomorphism . If ξ is a 2-absorbing fuzzy ideal of S then $f^{-1}(\xi)$ is a 2-absorbing fuzzy ideal of R.*

Theorem 2.18. *[7] Let $f : R \to S$ be a surjective ring homomorphism. If μ is a 2-absorbing fuzzy ideal of R which is constant on $Kerf$ then $f(\mu)$ is a 2-absorbing fuzzy ideal of S.*

3. 2-absorbing semiprimary fuzzy ideals

Before we investigate 2-absorbing semiprimary fuzzy ideals, we will give the characterization of cartesian product of some fuzzy ideals which will be used in next parts.

Definition 3.1. *Let μ and α be two fuzzy ideals of R. The cartesian product of μ and α is defined by $\mu \times \alpha$ such that $(\mu \times \alpha)(x,y) = \mu(x) \wedge \alpha(y)$ [12]. In addition to this definition, if $(x_r, y_s) \in \mu \times \alpha$ for any fuzzy points x_r, y_s of R then $x_r \in \mu$ and $y_s \in \alpha$ so $r \wedge s \leq \mu \times \alpha(x,y) = \mu(x) \wedge \alpha(y)$.*

Recall that if μ and α are fuzzy ideals of R then $\mu \times \alpha$ is a fuzzy ideal of $R \times R$.

Lemma 3.2. *Let μ and α be two fuzzy ideals of R. Then $\sqrt{\mu \times \alpha} = \sqrt{\mu} \times \sqrt{\alpha}$*

Proof. $\sqrt{\mu \times \alpha}(x,y) = \bigvee_{n \geq 1} \{(\mu \times \alpha)(x^n, y^n)\} = \bigvee_{n \geq 1} \{\mu(x^n) \wedge \alpha(y^n)\}$
$= \bigvee_{n \geq 1} \{\mu(x^n)\} \wedge \bigvee_{n \geq 1} \{\alpha(y^n)\} = \sqrt{\mu}(x) \wedge \sqrt{\alpha}(y) = \sqrt{\mu} \times \sqrt{\alpha}(x,y)$ □

Lemma 3.3. *Let $R = R_1 \times R_2$, where R_1 and R_2 are rings and μ be a nonconstant fuzzy ideal of R. If μ is a prime fuzzy ideal then either $\mu = \mu_1 \times \lambda_{R_2}$ for some prime fuzzy ideal μ_1 of R_1 or $\mu = \lambda_{R_1} \times \mu_2$ for some prime fuzzy ideal μ_2 of R_2 .*

Proof. Assume that μ be a prime fuzzy ideal of R. Then there exist α and β fuzzy ideals of R_1, R_2 respectively such that $\mu = \alpha \times \beta$. Then for any fuzzy points x_r, y_s of R $(x_r, y_s) = (x_r, 1_1)(1_1, y_s) \in \mu = \alpha \times \beta$. So $(x_r, 1_1) \in \alpha \times \beta$ or $(1_1, y_s) \in \alpha \times \beta$ since μ is a prime fuzzy ideal. Thus we conclude that $\beta = \lambda_{R_2}$ and α is a prime fuzzy ideal or $\alpha = \lambda_{R_1}$ and β is a prime fuzzy ideal. □

Theorem 3.4. *Let $R = R_1 \times R_2$, where R_1 and R_2 are commutative rings with nonzero identity. Let μ be a nonconstant fuzzy ideal of R. Then the following statements are equivalent:*
(1) *μ is a semiprimary fuzzy ideal of R.*
(2) *Either $\mu = \mu_1 \times \lambda_{R_2}$ for some semiprimary fuzzy ideal μ_1 of R_1 or*
$\mu = \lambda_{R_1} \times \mu_2$ for some semiprimary fuzzy ideal μ_2 of R_2 .

Proof. $(1) \Rightarrow (2)$ Assume that μ is a semiprimary fuzzy ideal of R. Then $\mu = \mu_1 \times \mu_2$ for some fuzzy ideal μ_1 of R_1 and some fuzzy ideal μ_2 of R_2. Since $\sqrt{\mu} = \sqrt{\mu_1} \times \sqrt{\mu_2}$ is a prime fuzzy ideal then by the previous lemma either $\sqrt{\mu_1} = \lambda_{R_1}$ so $\mu_1 = \lambda_{R_1}$ and $\sqrt{\mu_2}$ is a prime fuzzy ideal or $\sqrt{\mu_2} = \lambda_{R_2}$ so $\mu_2 = \lambda_{R_2}$ and $\sqrt{\mu_1}$ is a prime fuzzy ideal. Hence $\mu = \mu_1 \times \lambda_{R_2}$ for some semiprimary fuzzy ideal μ_1 or $\mu = \lambda_{R_1} \times \mu_2$ for some semiprimary fuzzy ideal μ_2.
$(2) \Rightarrow (1)$ It is clear that since $\sqrt{\mu} = \sqrt{\mu_1 \times \lambda_{R_2}} = \sqrt{\mu_1} \times \lambda_{R_2}$ is a prime fuzzy ideal of R if μ_1 is any semiprimary fuzzy ideal of R_1. □

Theorem 3.5. *Let μ be a fuzzy ideal of R. If μ is a 2-absorbing then $\mu \times \lambda_R$ $(\lambda_R \times \mu)$ is a 2-absorbing fuzzy ideal of $R \times R$.*

Proof. Assume that $(x_r, a_k)(y_s, b_p)(z_t, c_h) \in \mu \times \lambda_R$ for any fuzzy points $x_r, y_s, z_t, a_k, b_p, c_h$ of R. Then $(x_r y_s z_t, a_k b_p c_h) \in \mu \times \lambda_R$ so $x_r y_s z_t \in \mu$ and $a_k b_p c_h \in \lambda_R$. Since μ is a 2-absorbing fuzzy ideal then $x_r y_s \in \mu$ or $x_r z_t \in \mu$ or $y_s z_t \in \mu$. Thus we get that $(x_r, a_k)(y_s, b_p) \in \mu \times \lambda_R$ or $(x_r, a_k)(z_t, c_h) \in \mu \times \lambda_R$ or $(y_s, b_p)(z_t, c_h) \in \mu \times \lambda_R$. Hence $\mu \times \lambda_R$ is a 2-absorbing fuzzy ideal.
By the similar way it can be seen that $\lambda_R \times \mu$ is a 2-absorbing fuzzy ideal of $R \times R$. □

Definition 3.6. *Let R be a ring. Then the nonconstant fuzzy ideal μ is said to be a 2-absorbing semiprimary fuzzy ideal if $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of R.*

Example 3.7. ***(1)*** *Every prime fuzzy ideal is a 2-absorbing semiprimary fuzzy ideal.*
(2) *Every primary fuzzy ideal is a 2-absorbing semiprimary fuzzy ideal.*
(3) *Every semiprimary fuzzy ideal is a 2-absorbing semiprimary fuzzy ideal.*

Proposition 3.8. *Let μ be a nonconstant fuzzy ideal of R . Then the following assertions are equivalent.*
(i) *μ is a 2-absorbing semiprimary fuzzy ideal.*
(ii) *If $x_r y_s z_t \in \mu$ for any fuzzy points x_r, y_s, z_t of R then $x_r y_s \in \sqrt{\mu}$ or $x_r z_t \in \sqrt{\mu}$ or $y_s z_t \in \sqrt{\mu}$.*

Corollary 3.9. *If μ is 2-absorbing primary fuzzy ideal then μ is 2-absorbing semiprimary fuzzy ideal.*
But, as indicated in the following example, the converse of Corollary 3.9 is not true.

Example 3.10. *Let $R = \mathbb{Z}$, the ring of integers. Define the fuzzy ideal μ of Z by*

$$\mu(x) = \begin{cases} 1 & x \in 36Z, \\ 1/2 & x \in 6Z - 36Z, \\ 0 & otherwise. \end{cases}$$

Since $2_1 3_1 \notin \mu$, $2_1 1_{\frac{1}{2}} \notin \sqrt{\mu}$ and $3_1 1_{\frac{1}{2}} \notin \sqrt{\mu}$ while $2_1 3_1 1_{\frac{1}{2}} \in \mu$, then μ is not 2-absorbing primary fuzzy ideal. However, it is easy to see that μ is 2-absorbing semiprimary fuzzy ideal, since $\sqrt{\mu} = \lambda_{6Z}$ where it is a 2-absorbing fuzzy ideal of Z.

Remark 3.11. *In Example 2.7 [5], it is proved that a 2-absorbing semiprimary ideal is not necessarily a 2-absorbing primary ideal. In the following theorem we show under what conditions a 2-absorbing semiprimary (fuzzy) ideal is a 2-absorbing primary (fuzzy) ideal. Note that if μ is a semiprime fuzzy ideal of R, then we have $\sqrt{\mu} = \mu$.*

Theorem 3.12. *Let R be a ring. Then the following statements hold:*
(1) *Let μ be a semiprime fuzzy ideal of R. Then μ is a 2-absorbing primary fuzzy ideal if and only if it is 2-absorbing semiprimary fuzzy ideal.*
(2) *Let I be a semiprime ideal of R. Then I is a 2-absorbing primary ideal if and only if it is 2-absorbing semiprimary ideal.*

Proof. **(1)** We show that only sufficient conditions. Let μ be semiprime fuzzy ideal. If μ is 2-absorbing semiprimary fuzzy ideal and $x_r y_s z_t \in \mu$ for any x_r, y_s, z_t fuzzy points of R, then $x_r y_s \in \sqrt{\mu}$ or $x_r z_t \in \sqrt{\mu}$ or $y_s z_t \in \sqrt{\mu}$. Since μ is semiprime fuzzy ideal then $\sqrt{\mu} = \mu$ so $x_r y_s \in \mu = \sqrt{\mu}$ or $x_r z_t \in \sqrt{\mu}$ or $y_s z_t \in \sqrt{\mu}$. Hence we get that μ is a 2-absorbing primary fuzzy ideal of R.
(2) We omit the proof since it is clear by (1). □

Theorem 3.13. *Let μ be a fuzzy ideal of R. If μ is a 2-absorbing semiprimary then μ_t is a 2-absorbing semiprimary ideal of R for any $t \in [0, \mu(0)]$.*

Proof. If μ is a 2-absorbing semiprimary then $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of R. By [7, Lemma 3.3], $\sqrt{\mu}_t = \sqrt{\mu_t}$ is also 2-absorbing ideal. Hence μ_t is 2-absorbing semiprimary ideal of R. □

Theorem 3.14. *Let μ_1 be ξ_1-semiprimary fuzzy ideal of R and μ_2 be ξ_2-semiprimary fuzzy ideal of R. Then the following statements hold.*
(i) *$\mu_1 \mu_2$ is a 2-absorbing semiprimary fuzzy ideal of R.*
(ii) *$\mu_1 \cap \mu_2$ is a 2-absorbing semiprimary fuzzy ideal of R.*

Proof. Since $\sqrt{\mu_1} = \xi_1$ and $\sqrt{\mu_2} = \xi_2$ are prime fuzzy ideals then $\sqrt{\mu_1 \mu_2} = \sqrt{\mu_1 \cap \mu_2} = \sqrt{\mu_1} \cap \sqrt{\mu_2}$ is 2-absorbing fuzzy ideal of R. Hence $\mu_1 \cap \mu_2$ and $\mu_1 \mu_2$ are 2-absorbing semiprimary fuzzy ideal. □

Theorem 3.15. *Let μ be a nonconstant fuzzy ideal. If $\mu_* = \{x \in R : \mu(x) > 0\}$ is a 2-absorbing semiprimary ideal of R where $\mu(0) = 1$ and $|Im\mu| = 2$ then μ is a 2-absorbing semiprimary fuzzy ideal of R.*

Proof. Assume that $\mu(0)=1, Im\mu=\{1,\alpha\}$ and μ_* is a 2-absorbing semiprimary ideal.
Let $x_r y_s z_t \in \mu$ but $x_r y_s \notin \sqrt{\mu}$, $y_s z_t \notin \sqrt{\mu}$ and $x_r z_t \notin \sqrt{\mu}$. Then $r\wedge s\wedge t \leq \mu(xyz)$ and $r\wedge s > \sqrt{\mu}(xy)$, $s\wedge t > \sqrt{\mu}(yz)$, $r\wedge t > \sqrt{\mu}(xz)$. Thus for all $n\in\mathbb{Z}^+$, $r\wedge s > \mu(x^n y^n)$, $s\wedge t > \mu(y^n z^n)$ and $r\wedge t > \mu(x^n z^n)$. By our assumption we get that $\mu(x^n y^n)=\mu(y^n z^n)=\mu(x^n z^n)=\alpha$ so $xy, yz, xz \notin \mu_*$. However, $\alpha < r\wedge s\wedge t \leq \mu(xyz)=1$ so $xy, yz, xz \notin \sqrt{\mu_*}$ and $xyz\in\mu_*$. But this contradict that μ_* is 2-absorbing semiprimary ideal. Hence μ is a 2-absorbing semiprimary fuzzy ideal. □

Theorem 3.16. *Let I be a 2-absorbing quasi primary ideal of R and $\alpha\in[0,1)$ be any arbitrary. If μ is the fuzzy ideal of R defined by*

$$\mu(x)=\begin{cases} 1 & x\in I,\\ \alpha & x\notin I,\end{cases}$$

for all $x\in R$, then μ is a 2-absorbing semiprimary fuzzy ideal of R.

Proof. Let I be a 2-absorbing primary ideal of R. Assume that $x_r y_s z_t \in \mu$ but $x_r y_s \notin \sqrt{\mu}$ and $x_r z_t \notin \sqrt{\mu}$ and $y_s z_t \notin \sqrt{\mu}$ for any $x,y,z\in R$. Then $\mu((xy)^n)\leq\sqrt{\mu}(xy) < r\wedge s$ and $\mu((yz)^n)\leq\sqrt{\mu}(yz) < s\wedge t$ and $\mu((xz)^n)\leq\sqrt{\mu}(xz) < r\wedge t$ for all $n\geq 1$. In this case $\mu((xy)^n)=\alpha$ and $(xy)^n\notin I$ so $xy\notin\sqrt{I}$, $\mu((yz)^n)=\alpha$ and $(yz)^n\notin I$ so $yz\notin\sqrt{I}$, $\mu((xz)^n)=\alpha$ and $(xz)^n\notin I$ so $xz\notin\sqrt{I}$. Since I is 2-absorbing semiprimary ideal of R then we get $xyz\notin I$ and so $\mu(xyz)=\alpha$. By our assumption we get $(xyz)_{r\wedge s\wedge t}=x_r y_s z_t\in\mu$ and $r\wedge s\wedge t\leq\mu(xyz)=\alpha$. Thus $\alpha < r\wedge s$, $\alpha < s\wedge t$ and $\alpha < r\wedge t$ so $\alpha < r\wedge s\wedge t$, which is a contradiction. Hence μ is a 2-absorbing semiprimary fuzzy ideal of R. □

Theorem 3.17. *Let $f: R\to S$ be a ring homomorphism . If ξ is a 2-absorbing semiprimary fuzzy ideal of S then $f^{-1}(\xi)$ is a 2-absorbing semiprimary fuzzy ideal of R.*

Proof. Let ξ be a 2-absorbing semiprimary fuzzy ideal of S. We show that $\sqrt{f^{-1}(\xi)}$ is a 2-absorbing fuzzy ideal of R. Since $\sqrt{f^{-1}(\xi)}=f^{-1}(\sqrt{\xi})$ and $\sqrt{\xi}$ is a 2-absorbing fuzzy ideal then the inverse image of $\sqrt{\xi}$ is also 2-absorbing fuzzy ideal by [7, Theorem 31]. Hence $f^{-1}(\xi)$ is a 2-absorbing semiprimary fuzzy ideal of R. □

Theorem 3.18. *Let $f: R\to S$ be a surjective ring homomorphism. If μ is a 2-absorbing semiprimary fuzzy ideal of R which is constant on $Kerf$ then $f(\mu)$ is a 2-absorbing semiprimary fuzzy ideal of S.*

Proof. Assume that μ is a 2-absorbing semiprimary fuzzy ideal of R which is constant on $Kerf$. Then $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of R such that $\sqrt{\mu}$ is also constant on $Kerf$. By [7, Theorem 32], $f(\sqrt{\mu})=\sqrt{f(\mu)}$ is a 2-absorbing fuzzy ideal of S. □

Theorem 3.19. *If f is a homomorphism from a ring R onto a ring S, then the mapping $\mu\to f(\mu)$ defines a one-to-one correspondence between the set of all 2-absorbing semiprimary fuzzy ideals of R which is constant on $Kerf$ and the set of all 2-absorbing semiprimary fuzzy ideals of S.*

Definition 3.20. *Let μ be a 2-absorbing semiprimary fuzzy ideal of R. Then $\gamma=\sqrt{\mu}$ is a 2-absorbing fuzzy ideal. We say that μ is a γ-2-absorbing semiprimary fuzzy ideal of R.*

Theorem 3.21. *Let $\mu_1,\mu_2,...,\mu_n$ be γ-2-absorbing semiprimary fuzzy ideals of R for some 2-absorbing fuzzy ideal γ of R. Then $\mu=\bigcap\limits_{i=1}^{n}\mu_i$ is a γ-2-absorbing semiprimary fuzzy ideal of R.*

Proof. Let μ_i, $i\in\{1,2,...,n\}$ be γ-2-absorbing semiprimary fuzzy ideals of R. Then $\sqrt{\bigcap\limits_{i=1}^{n}\mu_i}=\bigcap\limits_{i=1}^{n}\sqrt{\mu_i}=\gamma=\sqrt{\mu}$ is a 2-absorbing fuzzy ideal. Hence μ is a γ-2-absorbing primary fuzzy ideal of R. □

Theorem 3.22. *Let R_1 and R_2 be commutative rings with nonzero identity and μ be a nonconstant fuzzy ideal of R_1 (of R_2). If μ is a 2-absorbing semiprimary fuzzy ideal of R_1 (of R_2) then $\mu\times\lambda_{R_2}$ ($\lambda_{R_1}\times\mu$) is a 2-absorbing semiprimary fuzzy ideal of $R_1\times R_2$.*

Proof. Assume that μ is a 2-absorbing semiprimary fuzzy ideal. Since $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal then $\sqrt{\mu\times\lambda_{R_2}}=\sqrt{\mu}\times\lambda_{R_2}$ is a 2-absorbing fuzzy ideal of $R_1\times R_2$. Hence we get $\mu\times\lambda_{R_2}$ is a 2-absorbing semiprimary fuzzy ideal of $R_1\times R_2$. □

Corollary 3.23. *Let $R=R_1\times R_2$ where R_1 and R_2 be two rings and μ be a nonconstant fuzzy ideal of R. Then the following statements are equivalent: **(1)** μ is a 2-absorbing semiprimary fuzzy ideal of R. **(2)** Either $\mu=\mu_1\times\lambda_{R_2}$ for some 2-absorbing semiprimary fuzzy ideal μ_1 of R_1, or $\mu=\lambda_{R_1}\times\mu_2$ for some 2-absorbing semiprimary fuzzy ideal μ_2 of R_2, or $\mu=\mu_1\times\mu_2$ for some 2-absorbing semiprimary fuzzy ideal of R_2.*

4. Conclusion

In this paper, we have characterized 2-absorbing semiprimary fuzzy ideals of a ring. Also the notions of 2-absorbing and 2-absorbing primary fuzzy ideals and their properties are proposed. Furthermore, the relationship between 2-absorbing semiprimary fuzzy ideals and 2-absorbing semiprimary ideals. Finally, we have examined that the properties of cartesian product of 2-absorbing semiprimary fuzzy ideals. To extend this study, one could study other algebraic structures and do some further study on the properties them.

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