

$^{32}\text{S}^{+92,94,96,98,100}\text{Mo}$ REAKSİYONLARINDAKİ BARIYER DAĞILIMINA DAYANAN BÜYÜK AÇILI YARI-ESNEK SAÇILMA TESİR KESİTLERİ VE ETKİN AĞIRLIK FONKSİYONU

Zehra Merve CİNAN* , Ahmet Hakan YILMAZ**, Burcu EROL***, Taylan BAŞKAN****

ÖZ

Coulomb bariyeri etrafındaki çok çeşitli bombardıman enerjilerinde eş zamanlı olarak hesaplanan $^{32}\text{S}^{+92,94,96,98,100}\text{Mo}$ reaksiyonlarındaki bariyer dağılımı için yarı esnek saçılma ve etkin ağırlık fonksiyonunu çalıştık. Yarı-esnek açılma dağılım verileri Woods-Saxon potansiyeli olan optik model kodu kullanılarak analiz edilmiştir. Bu reaksiyonların yarı-esnek saçılma tesir kesitlerini ve bariyer dağılımının yapılarını açıklayabildiği hesaba katıldığı gösterilmiştir. Bu sonuçlar, birleşmiş kanalların formalizminin, çeşitli kütle sistemleri için bile geçerli olabileceğini göstermektedir. Geniş açılı yarı-esnek saçılma reaksiyonları, füzyon reaksiyonlarını belirlemek için önerilen özdeş çekirdek-çekirdek potansiyeli ile incelenmiştir. Bariyer dağılımının ve nükleon transferinin Woods-Saxon potansiyeli üzerindeki etkisi göz önüne alındığında, bir dizi reaksiyonun hesaplanan yarı-esnek saçılma tesir kesitlerinin birbiriyle iyi bir uyum içinde olduğu gözlenmiştir.

Anahtar Kelimeler: Tesir Kesiti, Yarı-Esnek Saçılma, Etkin Ağırlık Fonksiyonu, Bariyer Dağılımı

**THE LARGE ANGLE QUASI-ELASTIC SCATTERING CROSS SECTIONS
AND THE EFFECTIVE WEIGHT FUNCTION BASED ON THE BARRIER
DISTRIBUTION FOR $^{32}\text{S}+^{92,94,96,98,100}\text{Mo}$ REACTIONS**

Zehra Merve CİNAN* , Ahmet Hakan YILMAZ** , Burcu EROL*** , Taylan BAŞKAN****

ABSTRACT

We have studied quasi-elastic scattering and the effective weight function for the barrier distribution in $^{32}\text{S}+^{92,94,96,98,100}\text{Mo}$ reactions, which calculated simultaneously in a wide range of bombarding energies around the Coulomb barrier. The quasi-elastic angular distribution data were analyzed using the optical model code with Woods-Saxon potentials. It was shown that the calculations taken into account so that these reactions can explain structures of the quasi-elastic cross-section and the quasi-elastic barrier distribution. These results have indicated that the coupled-channels formalism can still valid even for the various mass systems. The large-angle quasi-elastic scattering reactions have scrutinized with the identical nucleus- nucleus potential recommended for designating fusion reactions. Given the effect of a barrier distribution and nucleon transfer on the Woods-Saxon potential, it was observed that the calculated semi-elastic scattering cross-sections of a series of reactions are in good harmony each other.

Keywords: *Cross Sections, Quasi-Elastic Scattering, Effective Weight Function, Barrier Distribution*

Makale Gönderim Tarihi: 05.08.2019 ; Makale Kabul Tarihi : 26.11.2019 Makale Türü: Araştırma

DOI: 10.20854/bujse.601961

*Sorumlu yazar: Faculty of Science, Department of Physics, Karadeniz Technical University, Trabzon, TURKEY

(m_cinan@ktu.edu.tr)

**Faculty of Science, Department of Physics, Karadeniz Technical University, Trabzon, TURKEY

(hakany@ktu.edu.tr)

*** Faculty of Science, Department of Physics, Recep Tayyip Erdoğan University, Rize, TURKEY

(burcu.karayunus@gmail.com)

**** Faculty of Science, Department of Physics, Karadeniz Technical University, Trabzon, TURKEY

(taylanbaskan@ktu.edu.tr)

Introduction

It is important to understand the nuclear potential to identify nucleus collisions. Nuclear potential can be studied via quasi-elastic scattering and fusion reactions. This scattering process is the sum of elastic, inelastic scattering and transfer channels. These reactions at energies near the Coulomb barrier have greatly discussed in last years. They maintain an impeccable possibility to achieve the knowledge of nuclear structure and nucleon interaction and to analyze the structure of heavy-ion reactions at near barrier energies that is of huge value for the synthesis of super-heavy nuclei. Whence, quasi-elastic scattering and fusion are supplementary to each other [1-11].

In our work, we have tried to illustrate the heavy-ion elastic and quasi-elastic scattering with the identical potential for understanding the fusion reactions. Theoretical model for the description of the elastic and quasi-elastic scattering have determined and then the results have calculated with different programs have been compared with each other. The calculation and results have imparted at the end.

Coupled-Channels Method For Elastic Scattering, Quasi-Elastic Scattering And Fusion

To account for the stimulations of nuclei that interact with each other in fusion and scattering reactions, we have used the following Hamiltonian [12]:

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_{rel}(r) + H_0(\xi) + V_{coup}(r, \xi) \quad (1)$$

r symbolize the coordinate for the relative motion between the bullet and the target nuclei and μ is the reduced mass. $H_0(\xi)$ describes the vibrational excitation spectra of the bullet and projectile nuclei, ξ is the internal degrees of the vibration in the nuclei. $V_{coup}(r, \xi)$ is the potential of coupling between the motion and excitations of the bullet and projectile nuclei. $V_{rel}(r)$ contains the atomic number of the bullet and the target and bare nuclear potential. Optical potential for the reaction is a sum of the Coulomb and nuclear potentials that is deemed to have a Woods-Saxon shape and consists of the real and imaginary parameters [12-13]:

$$V_{rel}(r) = \frac{Z_1 Z_2 e^2}{r} + V_N(r) = \frac{Z_1 Z_2 e^2}{r} + V_0(r) + (W_0(r)) \quad (2)$$

$$= \frac{Z_1 Z_2 e^2}{r} \frac{V_0}{1 + \exp[(r - R_0)/a]} - i \frac{W_0}{1 + \exp[(r - R_w)/a_w]}$$

The coupled-channels equations are attained from the vibrational excitation spectra of the bullet and projectile nuclei in terms of $H_0(\xi)$,

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_{rel}(r) + \epsilon_n - E \right] u_n^J(r) \quad (3)$$

$$+ \sum_m V_{nm}(r) u_n^J(r) = 0$$

ϵ_n is the excitement energy operator of the n th channel and J is the total angular momentum of the reaction channels. This approach significantly minimizes the magnitudes of the coupled-channels problem for heavy ion reactions.

The recommended potential is stand on the density approach. Calculations demonstrate that the potential of the nucleus-nucleus is linked with the incoming energy in the energies near the Coulomb barrier. In our work, we have examined the Woods-Saxon potential for the definition of heavy ion elastic scattering near the Coulomb barrier. Rely on the optical model, we have disentangled the Schrödinger equation for a given nucleus-nucleus potential using the conventional system to attain the partial-wave scattering matrix that is used to illustrate the elastic scattering data. The real and imaginary sections of the optical model potential accepted in the calculations were defined via the Woods-Saxon potential [2].

We have computed the elastic scattering reaction differential cross sections as a function of energy for the reactions $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ at different energies. The computational results were demonstrated in Fig. 1.

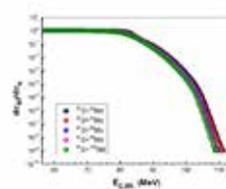


Fig.1. Elastic scattering reaction differential cross sections results for $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ reactions.

At these energy regions, the fusion cross section was generally designated via the conventional equation

$$\sigma_{fus}(E_{c.m.}) = \pi R_f^2 (1 - B/E_{c.m.}) \quad (4)$$

with fusion radius parameter R_f and height of fusion barrier B . Our computational results were demonstrated in Fig. 2.

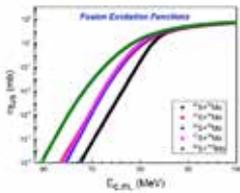


Fig. 2. Fusion excitation functions result for $^{32}\text{S}+^{92,94,96,98,100}\text{Mo}$ reactions.

We have determined that both the fusion and the elastic scattering excitation functions of the five reactions can be adequately well reproduced as shown Fig. 1 and 2. While elastic scattering reaction differential cross-sections have very small differences from one another at high energies, the results of the fusion excitation functions are in harmony at high energies.

Taking B to be the barrier height B_0 of the Woods-Saxon potential, the fusion cross sections cannot be duplicated via the Eq. (4). To characterize cross sections, we have induced an empirical barrier distribution. We have recommended an effective weight function to illustrating the barrier distribution

$$D_{eff}(B) = \begin{cases} D_1(B) & : B < B_x \\ D_{avr}(B) & : B \geq B_x \end{cases} \quad (5)$$

here $D_{avr}(B) = (D_1(B) + D_2(B)) / 2$ and B_x is the left cross point of $D_1(B)$ and $D_2(B)$. $D_1(B)$ and $D_2(B)$ are two Gaussian operators and these operators affiliated with the barrier height B_0 of the Woods-Saxon potential. The effective weight function of the reactions $^{32}\text{S}+^{92,94,96,98,100}\text{Mo}$ was illustrated in the Fig. 3 [11, 14].

Quasi-elastic scattering was considered to be the sum of a number of reactions, such as elastic and inelastic scattering, which can be expressed as an important counterpart of fusion reactions.

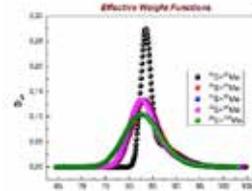


Fig. 3. The effective weight function results for $^{32}\text{S}+^{92,94,96,98,100}\text{Mo}$ reactions.

According to the results in Fig. 1, 2 and 3 from the reaction models studied, a combined description of the scattering and fusion data gives important information about the potential parameters used and the compatibility of the calculation programs. For this reason, it has contemplated that the quasi-elastic scattering is an excellent technique of the one for fusion.

We have found that the Woods-Saxon potential yields good outcomes for heavy ion elastic scattering in the upward barrier energies. However, the fusion section of the identical reaction framework cannot be well illustrated by the potential and it is necessary to familiarize the barrier distribution for the regeneration of the fusion output. Either elastic scattering or fusion outputs may be pleasurablely characterized by the potentials in the energies around the Coulomb barrier. In our study, we have aimed to identify a general nuclear potential that can be used to identify different nucleus-nucleus reactions.

As a decent response to the fusion reaction, we have studied wide-angle quasi-elastic scattering to find out the nucleus-nucleus potential. We have investigated the effect of the proposed barrier distribution on wide-angle quasi-elastic scattering for fusion reactions.

Identical to the characterization of fusion with the empirical barrier distribution, we describe the large-angle quasi-elastic scattering cross section with the effective weight function $D_{eff}(B)$ (Equation 5) at energies around the Coulomb barrier,

$$\frac{d\sigma_{qel}}{d\sigma_R}(E_{c.m.}) = P_{eff} + P_{corr} \quad (6)$$

with

$$P_{eff} = \frac{1}{F_0} \int_0^{\infty} D_{eff}(B) \frac{d\sigma_{el}}{d\sigma_R}(E_{c.m.}, B) dB \quad (7)$$

in these equations, P_{corr} is a small correction term, $\frac{d\sigma_{el}}{d\sigma_R}$ is a proportion of the elastic cross section σ_{el} to the Rutherford cross section σ_R , F_0 is a normalization constant $F_0 = \int D_{eff}(B) dB$. Within the semiclassical perturbation model, a semiclassical notation to the backward scattering ($\theta=\pi$) is bestowed [4, 15].

$$\frac{d\sigma_{el}}{d\sigma_R}(E_{c.m.}, B) = \left(1 + \frac{V_0(R_c)}{E_{c.m.}} \sqrt{\frac{Z_1 Z_2 e^2 a}{E_{c.m.}}} \right) \times \frac{\exp\left[-\frac{2\pi}{a}(E_{c.m.} - B)\right]}{1 + \exp\left[-\frac{2\pi}{a}(E_{c.m.} - B)\right]} \quad (8)$$

hw is an obliqueness of the Woods-Saxon potential. Where the nuclear potential $V_0(R_c)$ was appreciated at the Coulomb rotation point,

$$V_0(R_c) = \left(B - \frac{Z_1 Z_2 e^2}{R_f} \right) \left(\frac{1 + \exp[(R_f - R_c)/a]}{1 + \exp[(R_c - R_c)/a]} \right) \quad (9)$$

by the closest approach distance among two nucleus $R_c = Z_1 Z_2 e^2 / E_{c.m.}$. a is the diffusiveness coefficient of the nuclear potential and Z_1, Z_2 remark bullet and target nucleus. $E_{c.m.}$ is the center-of-mass energy. R_f is the barrier location.

Calculations And Results

In this study, both the fusion and quasi-elastic scattering sections of a series of reactions were investigated. Fig. 4 and 2 show the calculated quasi-elastic scattering and fusion cross sections for the reactions $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$. The results of the calculation programs were also demonstrated for crosscheck.

In order to take into consideration for the structure of the ^{32}S nuclei, we have established the potential between ^{32}S and $^{92,94,96,98,100}\text{Mo}$ with an appropriate procedure, while we have wielded a Wood-Saxon potential with the worldwide Akyüz-Winther parameters for the channels [16-19].

The nuclear component of nucleus-nucleus potential in article [16], U, is elementary parameterized like

$$U(r) = - \frac{V_0}{1 + e^{(r-R_P-R_T)/a_V}} \quad (10)$$

whose components originate in the enlightenment of the nuclear densities. These parameter values have thimbleful regulated owing to a methodical crosscheck of elastic scattering reactions. With the potential amplitude is

$$V_0 = 16\pi\gamma a_V \bar{R}_{TP} \quad (11)$$

In these equations the reduced radius is

$$\bar{R}_{TP} = \frac{R_T R_P}{R_T + R_P} \quad (12)$$

with

$$R_i = (1.20A_i^{1/3} - 0.09)fm, \quad i = T, P \quad (13)$$

The diffuseness coefficient is

$$\frac{1}{a_V} = 1.17[1 + 0.53(A_P^{-1/3} + A_T^{-1/3})]fm^{-1} \quad (14)$$

while the surface tension coefficient is

$$\gamma = 0.95 \left[1 - 1.8 \frac{N_P - Z_P}{A_P} \frac{N_T - Z_T}{A_T} \right] MeV.f m^{-2} \quad (15)$$

where A_i, Z_i and N_i are the mass, charge and neutron numbers of nuclei $i=T,P$.

The radius and diffuseness coefficients of imaginary part R_w and a_w were recommended to be proportional to the real potential parameter part, while imaginary depth W_0 is a quarter of the real depth. This selection is optional, so these parameters should be used in addition to the calculations.

The Coulomb potential radius in our calculations is equal to

$$R_C = 1.30(A_P^{1/3} + A_T^{1/3})fm \quad (16)$$

Table 1. The parameters of quasi-elastic scattering for our calculations

Reactions	V_0 (MeV)	r (fm)	a_c (fm)	γ (MeV fm ⁻²)	R_D (fm)	R_T (fm)	R_{EP} (fm)	R_{E0} (fm)	R_{E1} (fm)
$^{32}\text{S} + ^{92}\text{Mo}$	69.576	1.18	0.565	0.95	3.72	5.33	2.191	9.047	9.996
$^{32}\text{S} + ^{94}\text{Mo}$	69.903	1.15	0.566	0.95	3.72	5.37	2.198	9.096	10.038
$^{32}\text{S} + ^{96}\text{Mo}$	70.062	1.19	0.566	0.95	3.72	5.40	2.203	9.124	10.089
$^{32}\text{S} + ^{98}\text{Mo}$	70.858	1.17	0.567	0.95	3.72	5.44	2.209	9.162	10.121
$^{32}\text{S} + ^{100}\text{Mo}$	70.581	1.18	0.567	0.95	3.72	5.48	2.216	9.200	10.162

We have introduced a collation among the features of the barrier distributions stated by the proposed methods based on the nucleus–nucleus potentials calculated for reactions. As a result of the calculations, it was observed that the differential cross-sections were systematically decreased at high energy values and the results were in good agreement with each program codes in Fig. 4z

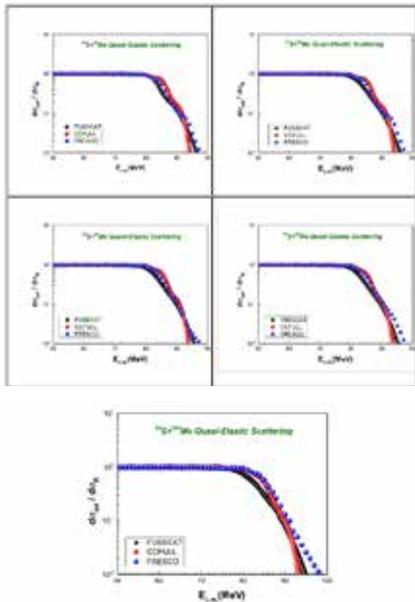


Fig. 4. Theoretical quasi-elastic scattering cross-section results for $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ reactions [2-25].

Summary And Conclusions

In our work, we have calculated the quasielastic barrier distributions and the effective weight function for $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ systems. The shapes of the distribution for $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ reactions

are consistent with the one foreseen by our calculations.

Results were represented that the barrier distributions for the fusion reaction and the quasi-elastic scattering change owing to the excitations at energies above the Coulomb barrier.

The energy dependence of the cross sections, on the other hand, was not affected much by the non-corporate excitations and barrier distributions remains the same.

Therefore we have extracted couplings to the many excitations as a possible source of the hindrance of fusion cross sections at sub-barrier energies and at energies above the Coulomb barrier.

The refinement of the models to achieve a degree of precision and reliability comparable to the data presents an interesting challenge to theory. From this attributive comparison, the quasi-elastic excitation function appears to have some sensitivity to the fusion barrier distribution. It would be favourable if these attributive features could be quantified by displaying the data in the form of a legation of the barrier distribution, identical to that extracted from fusion excitation functions.

However, this important result still leaves the mechanism for fusion hindrance phenomena as an open question.

In brief, we have calculated the quasielastic and elastic scattering excitation functions for the $^{32}\text{S} + ^{92,94,96,98,100}\text{Mo}$ reactions around the Coulomb barrier with high precision in 1 MeV energy steps. Excitation functions were calculated using the code FUSSCAT, FRESKO and CCFULL.

$\frac{d\sigma_{el}}{d\Omega}$ and $\frac{d\sigma_{qe}}{d\Omega}$ have subtracted from the measured excitation functions and compared with barrier distributions extracted from the existing fusion excitation function and the quasi-elastic and elastic scattering excitation functions.

These calculations demonstrate that the information about the fusion barrier distribution for a reaction can be investigate by quasi-elastic and elastic scattering excitation functions. Quasi-elastic scattering were measured to comprehend effects of excitation and deformation of colliding nuclei on the dynamics of fusion reactions.

REFERENCES

- [1] M. L. Inche Ibrahim, M. Zamrun, H. Abu Kassim, Phys. Rev. C 87, 024611 (2013).
- [2] N.Wang, W. Scheid, Phys. Rev. C 78, 014607 (2008).
- [3] Z. F.Muhammad, K. Hagino, S. Mitsuoka, H. Ikezoe, Phys. Rev. C 77, 034604 (2008).
- [4] K. Hagino, N. Rowley, Phys. Rev. C 69, 054610 (2004).
- [5] N. Rowley, G. R. Satchler, P. H. Stelson, Phys. Lett. B254, 25 (1991).
- [6] J. R. Leigh et al., Phys. Rev. C 52, 3151 (1995).
- [7] H. Timmers et al., Nucl. Phys. A584, 190 (1995).
- [8] R. K. Gupta, M. Manhas, G. Munzenberg, W. Greiner, Phys. Rev. C 72, 014607 (2005).
- [9] V. I. Zagrebaev, Phys. Rev. C 64, 034606 (2001).
- [10] G. G. Adamian et al., Nucl. Phys. A633, 409 (1998).
- [11] M. Liu, N.Wang, Z. Li, X.Wu, E. Zhao, Nucl. Phys. A768, 80 (2006).
- [12] S. Yusa, K. Hagino, N.Rowley, Phys. Rev. C 85, 054601 (2012).
- [13] M.Z. Firihi, International Journal of Modern Physics E Vol. 25, No. 8, 1650054 (2016),
- [14] N. Wang, K. Zhao, W. Scheid, X. Wu, Phys. Rev. C 77, 014603 (2008).
- [15] S. Landowne, H.H. Wolter, Nucl. Phys. A351, 171 (1981).
- [16] R.O. Akyuz, A. Winther, Proc. Enrico Fermi Int. School of Physics, “ Nuclear Structure and Heavy-Ion Reactions”, ed. R.A. Broglia, C.H.Dasso, R. Ricci (North-Holland, Amsterdam, 1981), p.491, (1979).
- [17] R.A. Broglia, A. Withner, “Heavy Ion Reactions, Part I and II”, Frontiers in Physics, vol.84, Addison-Wesley, (1991).
- [18] A. Winther, Nucl. Phys. A572, 191-235 (1994).
- [19] G. Pollarolo, A. Winther, Phys. Rev. C 62, 054611 (2000).
- [20] I.J. Thompson, Fresco, <http://www.fresco.org.uk> .
- [21] I.J. Thompson, Getting Started With Fresco User Guide, June (2010).
- [22] Antonio M. Moro, Fresco Notes, October (2004).
- [23] F. Nunes, “Scattering Theory Lecture Notes”, Michigan State University.
- [24] K. Hagino, N. Rowley, M. Dasgupta, “Fusion cross sections at deep sub-barrier energies”, Phys. Rev. C 67, 054603 (2003).



REFERENCES

- [25] K. Hagino, N. Rowley, A.T. Kruppa, “A program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions”, *Comput. Phys. Commun.* 123, 143 (1999).
- [26] A.B. Balantekin, N. Takigawa, “Quantum tunneling in nuclear fusion”, *Rev. Mod. Phys.*, 70, 77 (1998).
- [27] G.R. Satchler, “Direct Nuclear Reactions”, Clarendon Press, (1983).
- [28] G.R. Satchler, “Introduction to Nuclear Reaction”, Palgrave; 2nd edition (February 9, 1990).
- [29] R.A. Broglia and A. Winther, “Heavy-Ion Reactions”, *Phys. Reports*, Vol 4, 153-198, (1972).
- [30] D.M. Brink, “Semi-classical method in nucleus-nucleus collisions”, Cambridge University Press, (1985).
- [31] P. Frobrich and R. Lipperheide, “Theory of Nuclear Reactions”, Clarendon Press (April 30 1999).
- [32] R. Herman, “Fusion”, Cambridge University Press, Cambridge (1990).