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# A Study of Soft Topological Axioms and Soft Compactness by Using Soft Elements

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**ABSTRACT:** In this article, we study the ordinary topology on the collection of soft elements. Some interesting relations between the ordinary topology on the collection of soft elements and the soft topology are discussed. New definitions of soft separation axioms and soft compactness are presented and studied using soft elements.

***Keywords*** — *Soft set, Soft element, Soft topology, Soft separation axioms, Soft compactness*

## 1. Introduction

Soft set theory is an important branch of modern mathematics because it is a tool to handle various types of uncertainties arising from the complicated problems in economics, engineering, environmental sciences, social sciences, medical sciences etc. In Molodtsov (1999) initiated the theory of soft sets. Also in Molodtsov et al. (2006), he applied successfully in directions such as smoothness of functions, game theory, operations research, Riemann-Integration, Perron integration, probability and theory of measurement. Then Maji et al. (2003) defined operations on soft sets in 2003 and many researchers have studied the nature of soft sets and its applications in various real life problems, see (Abbas et al., 2017; Acar et al., 2010; Aktaş and Çağman, 2007; Ali et al., 2009; Çağman and Enginoğlu, 2010; Çağman et al., 2011; Çelik et l., 2011; Feng et al., 2008; Goldar and Ray, 2019; Pei and Miao, 2005; Ray and Goldar, 2017; Shirmohammadi and Rasouli, 2017).

Çağman et al. (2011) introduced soft topology and Shabir and Naz (2011) defined soft topological spaces. Several authors (Al-Khafaj and Mahmood (2014), Aygünoğlu and Aygün (2012), Atmaca (2016), Benchalli and Patil (2017), Babitha and John (2010), Çağman et al. (2015), Georgiou et al. (2013), Georgiou and Megaritis (2014), Goldar and Ray (2017), Hamza and Saad (2017), Hida (2014), Hosny and Abd El-Latif (2016), Hussain (2015), Jalil and Reddy (2017), Kandil et al. (2014), Majumdar and Samanta (2010), Osmanoglu and Tokat (2014), Shabir and Naz (2011), Shah and Shaheen (2014), Subhashinin and Sekar (2014), Tasbozan et al. (2017) Thakur and Rajput (2018), Varol and Aygün (2013), Varol et al. (2012), Yang et al. (2015), Zorlutuna et al. (2012)) have extended the idea of soft topology following the definition of soft topology by Shabir, Naz and Cagman et al. Goldar and Ray (2017) defined the soft topology as a ordinary topology of soft elements and studied soft topological properties by using of soft elements. To define soft set we need two non-empty sets one is called universal set $X$ and other set is parametric set $A$. There are no restriction on $X$ and $A$. A soft set is a function $F : A \to P(X)$ and a soft element $a$ of the soft set $F$ is also a function $a : A \to X$ such that $a(t) \in F(t)$ for all $t \in A$. So to exists a soft element of $F$ we need the condition $F(t) \neq \phi$ for all $t \in A$. If $F$ and $H$ are two soft sets then $H$ is soft subset of $F$ if $H(t) \subseteq F(t)$ for all $t \in A$. We define soft topology on soft subsets of $F$ in usual way and study this soft topology in light of soft elements. As soft elements are functions, the equality of two soft elements is different from classical case. Two soft elements $a$ and $b$ are equal if $a(t) = b(t)$ for all $t \in A$ and so not equal if $a(t) \neq b(t)$ for at least one $t$. Similar thing arise for the definition of a soft element $a$ is not belongs to a soft set $F$. $a$ is not belongs to $F$ if $a(t) \notin F(t)$ for some $t$. Unlike classical case this forces that the statement $a$ is not a soft element of $F$ implies $a$ is a member of compliment of $F$ is false. And obviously the soft topology on soft set will be different from classical one and need special attention to handle it.

In this article, we discuss some interesting relations between soft topology and the topology on soft elements and proposed a new definition of soft $T_i$-space ($i = 1,2,3,4$), soft Hausdorff space, soft regular, soft normal space by using soft elements. These new definition are equivalent to that in literature. The proof of the analogous properties and theorems of soft $T_i$-space ($i = 1,2,3,4$) are much similar if we use these new definitions.

In section 2, we have presented the definitions and preliminary results which are used in the next sections and studied some soft topological properties by using soft elements. In section 3, we have presented a new definition of soft $T_i$-space ($i = 1,2,3,4$), soft Hausdorff space, soft regular space, soft normal space by using soft elements and these new definition has the advantage that all the analogous elementary ordinary separation axioms properties follows easily. In section 4, definition of soft cover and soft compact are given and some interesting properties of soft compact spaces are studied. Also in this section, we have studied soft finite intersection property.

## 2. Preliminaries

Let $X$ denote universal set and $A$ be the set of parameters. Throughout this paper, we will take only one parameter set $A$. The power set of $X$ is denoted by $P(X)$.

**Definition 2.1.** *Molodtsov (1999) A pair $(F,A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \to P(X)$. We write $F$ for the soft set $(F,A)$.*

**Definition 2.2.** *Aygünoğlu and Aygün (2012),Ray and Goldar (2017) Suppose $F$ be a soft set such that $F(t) \neq \phi$ for all $t \in A$. A function $a : A \to X$ is called soft element of $F$ if $a(t) \in F(t)$ for all $t \in A$. In this case we write $a \in_s F$. Axiom of choice is needed to ensure the existence of soft elements if $A$ is infinite.*

**Definition 2.3.** *Ray and Goldar (2017) Let $F : A \to P(X)$ be a soft set. The collection of all soft elements of $F$ is denoted by $SE(F)$.*
*That is $SE(F) = \{ f : f : A \to X, f(t) \in F(t), \forall t \in A \}$. Hence $SE(F)$ is defined for those soft sets $F$ such that $F(t) \neq \phi$ for all $t \in A$.*

**Definition 2.4.** *Ali et al. (2009),Çağman and Enginoğlu (2010),Maji et al. (2003),Pei and Miao (2005) Let $F$ and $H$ be two soft sets over $X$ then $H$ is said to be a soft subset of $F$ if $H(t) \subseteq F(t)$ for all $t \in A$. In this case we denote $H \subset_s F$.*

**Definition 2.5.** *Ali et al. (2009),Çağman and Enginoğlu (2010),Maji et al. (2003),Pei and Miao (2005) Let $F$ and $H$ be two soft sets then the soft union $F \cup_s H$ and the soft* *intersection $F \cap_s H$ are defined by $(F \cup_s H)(t) = F(t) \cup H(t)$ and $(F \cap_s H)(t) = F(t) \cap H(t)$ for all $t \in A$. It can be noted that $SE(F \cup_s H) \supset SE(F) \cup SE(H)$, $SE(F \cap_s H)$ may not be defined for $(F \cap_s H)(t)$ may be an empty set for some $t$. However if it be defined then $SE(F \cap_s H) = SE(F) \cap SE(H)$.*

**Definition 2.6.** *Ray and Goldar (2017) Let $\{F_i : i \in I\}$ be a non-empty family of soft sets then (i)the intersection $\cap_s F_i$ is a soft set defined by $(\cap_s F_i)(t) = \cap F_i(t)$ for all $t \in A$.*
*(ii)and the union $\cup_s F_i$ is also a soft set defined by $(\cup_s F_i)(t) = \cup F_i(t)$ for all $t \in A$.*

**Definition 2.7.** *Ray and Goldar (2017) For each $a \in_s F$, a singleton soft set $\{a\}$ is defined by $\{a\} : A \to P(X)$ such that $\{a\}(t) = \{a(t)\}$. Clearly a soft set $F$ is singleton if $F(t)$ is a singleton set for every $t$. A singleton soft set contains only one soft element.*

**Definition 2.8.** *Maji et al. (2003) Two soft sets $F$ and $H$ are said to be soft equal if $F(t) = H(t)$ for all $t \in A$ and it is denoted by $F =_s H$. Also $a, b \in_s F$ are said to be soft equal if $a(t) = b(t)$ for all $t \in A$ and $a \neq_s b$ if $\exists\, t_1 \in A$ such that $a(t_1) \neq b(t_1)$.*

**Theorem 2.9.** *Ray and Goldar (2017) For two soft set $F$ and $H$, $H \subset_s F$ if and only if $SE(H) \subset SE(F)$.*

**Definition 2.10.** *Çağman et al. (2011) Let $F : A \to P(X)$ be a soft set and $F(t) \neq \phi$ for all $t \in A$. Let $\tau$ be a collection of some soft subsets of $F$. $\tau$ is called soft topology on $F$ if*
*(1) $\Phi, F \in \tau$*
*(2) $\tau$ is closed under arbitrary soft union and finite soft intersection.*
*Note that $\Phi : A \to P(X)$ is a soft set defined by $\Phi(t) = \phi$ for all $t \in A$.*

**Definition 2.11.** *Çağman et al. (2015),Çağman et al. (2011) Let $< F, \tau >$ be a soft topological space. Every elements of $\tau$ is called soft open sets. A soft set $S \subset_s F$ is called soft closed if the soft complement of $S$, $S^c \in \tau$ where $S^c(t) = F(t) - S(t)$ for all $t \in A$.*

**Theorem 2.12.** *Shabir and Naz (2011) If $< F, \tau >$ is a soft topological space then $< F(t), \tau_t >$ is a topological space for all $t \in A$, where $\tau_t = \{H(t) : H \in \tau\}$.*

**Definition 2.13.** *Goldar and Ray (2017) If $T \subset SE(F)$ then we can think $T$ as a soft set, $T : A \to P(X)$ with $T(t) = \{a(t) : a \in T\}$.*

**Theorem 2.14.** *Let $F$ be a soft set such that $F(t) \neq \phi$ for all $t \in A$. Let $\tau_t$ be a topology in $F(t)$ for all $t \in A$. Let $\tau$ be a collection of soft sets $H$ such that $H(t) \in \tau_t$ for all $t \in A$. Then $\tau$ is a soft topology on $F$.*

*Proof.* Let $\{H_i : i \in I\} \subset \tau$ then $H_i(t) \in \tau_t$ for all $t \in A$. So by Definition 2.6, $(\bigcup_{i \in I} H_i)(t) = \bigcup_{i \in I} H_i(t) \in \tau_t$ for all $t \in A$ and $(H_1 \cap_s H_2)(t) = H_1(t) \cap H_2(t) \in \tau_t$ for all $t \in A$. The function $\Phi : A \to P(X)$, $\Phi(t) = \phi$ for all $t \in A$, is in $\tau$ and also $F \in \tau$. Hence $< F, \tau >$ is a soft topological space. $\square$

**Example 2.15.** *Suppose that $A = \{t_1, t_2\}$ is a parameter set, $X = \{u_1, u_2, u_3, u_4\}$ is the universal set and $F = \{(t_1, \{u_1, u_2, u_3\}), (t_2, \{u_1, u_2, u_3\})\}$ is a soft set over $X$. Let $\tau_{t_1} = \{\phi, \{u_1, u_2, u_3\}, \{u_2\}, \{u_2, u_3\}, \{u_1, u_2\}\}$ and $\tau_{t_2} = \{\phi, \{u_1, u_2, u_3\}, \{u_1\}, \{u_1, u_2\}, \{u_1, u_3\}\}$.*

*Then it is clear that $\tau_{t_1}$ and $\tau_{t_2}$ are topologies on $F(t_1) = \{u_1, u_2, u_3\}$, and $F(t_2) = \{u_1, u_2, u_3\}$, respectively.*
*Now by Theorem 2.14, $\tau = \{H \subset_s F : H(t) \in \tau_t\}$. Clearly $\Phi = \{(t_1, \phi), (t_2, \phi)\}$ and $F = \{(t_1, \{u_1, u_2, u_3\}), (t_2, \{u_1, u_2, u_3\})\}$ belongs to $\tau$. The other members of $\tau$ are*
$F_1 = \{(t_1, \{u_1, u_2, u_3\}), (t_2, \{u_1\})\}$
$F_2 = \{(t_1, \{u_1, u_2, u_3\}), (t_2, \{u_1, u_2\})\}$
$F_3 = \{(t_1, \{u_1, u_2, u_3\}), (t_2, \{u_1, u_3\})\}$
$F_4 = \{(t_1, \{u_2\}), (t_2, \{u_1, u_2, u_3\})\}$
$F_5 = \{(t_1, \{u_2\}), (t_2, \{u_1\})\}$
$F_6 = \{(t_1, \{u_2\}), (t_2, \{u_1, u_2\})\}$
$F_7 = \{(t_1, \{u_2\}), (t_2, \{u_1, u_3\})\}$
$F_8 = \{(t_1, \{u_2, u_3\}), (t_2, \{u_1, u_2, u_3\})\}$
$F_9 = \{(t_1, \{u_2, u_3\}), (t_2, \{u_1\})\}$
$F_{10} = \{(t_1, \{u_2, u_3\}), (t_2, \{u_1, u_2\})\}$
$F_{11} = \{(t_1, \{u_2, u_3\}), (t_2, \{u_1, u_3\})\}$
$F_{12} = \{(t_1, \{u_1, u_2\}), (t_2, \{u_1, u_2, u_3\})\}$
$F_{13} = \{(t_1, \{u_1, u_2\}), (t_2, \{u_1\})\}$
$F_{14} = \{(t_1, \{u_1, u_2\}), (t_2, \{u_1, u_2\})\}$
$F_{15} = \{(t_1, \{u_1, u_2\}), (t_2, \{u_1, u_3\})\}$
*Thus $\tau = \{\Phi, F, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}\}$ is a soft topology on $F$.*

**Note 2.16.** *By Example 2.15 we see that for two different $\tau$'s*
*(i)$\{\Phi, F, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}\}$ and*
*(ii) $\{\Phi, F, F_5, F_7, F_{10}, F_{14}\}$ there are same $\tau_{t_1}$ and $\tau_{t_2}$. First $\tau$ is a soft topology but second one is not a soft topology on $F$.*

**Remark 2.17.** *From Theorems 2.12 and 2.14 we get two important observations.*
*(i) If $H$ is soft open then $H(t)$ is open in $\tau_t$ for all $t \in A$, by Theorem 2.12. Conversely if $H(t)$ is open in $\tau_t$ for all $t \in A$ then by Theorem 2.14, $H \in \tau$.*
*(ii) If $H$ is soft closed then $H(t)$ is closed in $\tau_t$ for all $t \in A$, by Theorem 2.12. Conversely if $H(t)$ is closed in $\tau_t$ for all $t \in A$ then by Theorem 2.14, $H$ is soft closed in $\tau$.*

**Theorem 2.18.** *Goldar and Ray (2017) Let $F : A \to P(X)$ be a soft set and $< F, \tau >$ is a soft topological space. Define $\tau^* = \{T \subseteq SE(F) : T(t) \text{ is open in } (F(t), \tau_t)\ \}$. Then $\tau^*$ is a topology on $SE(F)$.*

**Lemma 2.19.** *Goldar and Ray (2017) For $N \subset_s F$, $SE(N)(t) = N(t)$ for all $t \in A$.*

**Remark 2.20.** *Goldar and Ray (2017) If $H \in \tau$ and $SE(H)$ exists then $H(t) \in \tau_t$ for all $t \in A$ so $SE(H)(t) \in \tau_t$ for all $t \in A$, hence $SE(H) \in \tau^*$.*
*Again if $F -_s H \in \tau$ and assuming $SE(F -_s H)$ exists then $SE(F -_s H) = SE(F) - SE(H) \in \tau^*$. So $H$ is soft open in $\tau$ implies $SE(H)$ is open in $\tau^*$ and $H$ is soft closed in $\tau$ implies $SE(H)$ is closed in $\tau^*$.*

**Definition 2.21.** *Çağman et al. (2011) Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. Then the collection $\tau^H = \{K \cap_s H : K \in \tau\}$ is called soft subspace topology on $H$.*

**Proposition 2.22.** *Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. Then $< H(t), (\tau^H)_t >$ is subspace topological space of $< F(t), \tau_t >$ for each $t \in A$.*

*Proof.* Since $< H, \tau^H >$ is a soft topological space then clearly $< H(t), (\tau^H)_t >$ is a topological space for all $t \in A$. As $H(t) \subset F(t)$ then $(\tau_t)^{H(t)}$ is subspace topology of $H(t)$ for all $t \in A$. We want to show that $(\tau_t)^{H(t)} = (\tau^H)_t$.
Let $\alpha \in (\tau^H)_t$. Since $\alpha \subset H(t)$ then $\exists\, B \in \tau^H$ such that $B(t) = \alpha$ for all $t \in A$. Now $B = K \cap_s H$ where $K \in \tau$ which implies that $B(t) = K(t) \cap H(t)$ and $K(t) \in \tau_t$. So $\alpha = B(t) \in (\tau_t)^{H(t)}$. Therefore $(\tau^H)_t \subset (\tau_t)^{H(t)}$.
Conversely, let $t_0 \in A$ be fixed. Let $\alpha \in (\tau_{t_0})^{H(t_0)}$ then $\alpha = B \cap H(t_0)$ where $B \in \tau_{t_0}$. Define $S : A \to P(X)$ by

$$\begin{aligned} S(t) &= B \quad \text{if} \quad t = t_0 \\ &= F(t) \quad \text{if} \quad t \neq t_0 \end{aligned}$$

Hence $S \in \tau$ and $\alpha = S(t_0) \cap H(t_0) = (S \cap_s H)(t_0)$. So $S \cap_s H \in \tau^H$ which implies that $\alpha = (S \cap_s H)(t_0) \in (\tau^H)_{t_0}$. Since $t_0 \in A$ is arbitrary. Therefore $\alpha \in (\tau^H)_t$ for all $t \in A$. Hence $(\tau_t)^{H(t)} \subset (\tau^H)_t$. Thus $< H(t), (\tau^H)_t >$ is a subspace of $< F(t), \tau_t >$ for each $t \in A$. ☐

**Proposition 2.23.** *Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. Then $< SE(H), (\tau^H)^* >$ is subspace topological space of $< SE(F), \tau^* >$.*

*Proof.* Since $< H, \tau^H >$ is a soft topological space then clearly $< SE(H), (\tau^H)^* >$ is a topological space. As $SE(H) \subset SE(F)$ then $(\tau^*)^{SE(H)}$ is subspace topology of $SE(H)$. We want to show that $(\tau^*)^{SE(H)} = (\tau^H)^*$.
Let $\alpha \in (\tau^H)^*$ so $\alpha \subset SE(H)$ and $\alpha(t) \in (\tau^H)_t$ for all $t \in A$. Clearly $\alpha(t) \in (\tau_t)^{H(t)}$ for all $t \in A$. Therefore $\alpha(t) = H(t) \cap K_t$ for some $K_t \in \tau_t$. Define $T : A \to P(X)$ by $T(t) = H(t) \cap K_t$ for all $t \in A$. So $T(t) \in \tau_t$ for all $t \in A$ as $H$ is soft open. Hence $T \in \tau^*$. Thus $\alpha \in SE(F)$, we can write $\alpha = SE(H) \cap T$ (consider $T$ as a collection of soft elements see Definition 2.13). This shows that $\alpha \in (\tau^*)^{SE(H)}$. Thus $(\tau^H)^* \subset (\tau^*)^{SE(H)}$.
Conversely, suppose $\beta \in (\tau^*)^{SE(H)}$ so $\beta = SE(H) \cap T$ where $T \in \tau^*$ which implies that $\beta(t) = H(t) \cap T(t)$ where $T(t) \in \tau_t$. Therefore $\beta(t) \in (\tau_t)^{H(t)} = (\tau^H)_t$ for all $t \in A$. Hence $\beta \in (\tau^H)^*$. Therefore $(\tau^*)^{SE(H)} \subset (\tau^H)^*$. This complete the proof. ☐

## 3. Soft Separation Axioms

Soft separation axioms is defined in (Shabir and Naz, 2011; Cagman et al., 2011). But in this section, we have presented a new definition of soft $T_0$, $T_1$, $T_2$, $T_3$ by using of soft elements and these new definitions has the advantage that all the analogous elementary classical separation axioms properties follows easily.

**Definition 3.1.** *Let $H$ and $K$ be two soft sets. Then $\alpha \in_s H \cap_s K$ means $\alpha(t) \in (H \cap_s K)(t) = H(t) \cap K(t)$ for all $t \in A$. So $H(t) \cap K(t) \neq \phi$ for all $t \in A$. Hence $H \cap_s K = \Phi$ means $H(t) \cap K(t) = \phi$ for some $t \in A$.*
*Also let $a, b$ be two soft elements then $a =_s b$ means $a(t) = b(t)$ for all $t \in A$. So $a \neq_s b$ means $a(t) \neq b(t)$ for some $t \in A$. Here when we write $H \cap_s K = \Phi$ we are assuming $\Phi$ as a soft set. So no contradiction with Definition 2.8.*

**Proposition 3.2.** *Let $F$ be a soft set. $a \in_s F^c$ implies that $a \notin_s F$ but the converse is not true.*

*Proof.* Let $a \in_s F^c$
$\Rightarrow a(t) \in F^c(t) = [F(t)]^c$ for all $t \in A$

$\Rightarrow a(t) \notin F(t)$ for all $t \in A$
$\Rightarrow a \notin_s F$.
Now let $a \notin_s F$. Then at least one $t_1 \in A$ such that $a(t_1) \notin F(t_1)$ and $a(t) \in F(t)$ for $t \neq t_1$. Thus the converse is not true. $\square$

**Definition 3.3.** *Let $< F, \tau >$ be a soft topological space. Then $< F, \tau >$ is called soft $T_0$-space if for every $a, b \in_s F$ with $a \neq_s b$ $\exists$ $M, N \in \tau$ such that*
*$a \in_s M$ and $b \notin_s M$ or $b \in_s N$ and $a \notin_s N$.*

**Proposition 3.4.** *(i) Let $< F, \tau >$ be a soft topological space. If $< F, \tau >$ is a soft $T_0$-space then $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$.*
*(ii) Let $< F(t), \tau_t >$ be a topological space. If $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$ then $< F, \tau >$ is a soft $T_0$-space, where $\tau$ is defined in Theorem 2.14.*

*Proof.* (i) Let $< F, \tau >$ is a soft $T_0$-space. suppose $t_1 \in A$ is fixed and $\alpha, \beta \in F(t_1)$. Let $a, b \in_s F$ be such that $a(t) = b(t)$ for all $t \in A$ except $t = t_1$ and $a(t_1) = \alpha$, $b(t_1) = \beta$. So $a \neq_s b$. Now since $< F, \tau >$ is a soft $T_0$-space then there is $H \in \tau$ such that $a \in_s H$ and $b \notin_s H$. Clearly $a(t) = b(t) \in H(t)$ for all $t \in A$ except $t = t_1$. Hence $b(t_1) \notin H(t_1)$. This shows that $< F(t_1), \tau_{t_1} >$ be $T_0$-space. Since $t_1 \in A$ is arbitrary. Therefore $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$.
(ii) Let $< F(t), \tau_t >$ be $T_0$-space. Also let $a, b \in_s F$ with $a \neq_s b$. Then at least one $t_1 \in A$ such that $a(t_1) \neq b(t_1)$. Let $M_{t_1}, N_{t_1} \in \tau_t$ such that $a(t_1) \in M_{t_1}$ and $b(t_1) \notin M_{t_1}$ or $b(t_1) \in N_{t_1}$ and $a(t_1) \notin N_{t_1}$. Define $H, K : A \to P(X)$ such that

$$\begin{aligned} H(t) &= M_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} K(t) &= N_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Clearly $H, K$ are soft sets and $H, K \in \tau$ by Theorem 2.14. Hence $a \in_s H$ and $b \notin_s H$ or $b \in_s K$ and $a \notin_s K$. Therefore $< F, \tau >$ is a soft $T_0$-space. $\square$

**Example 3.5.** *Suppose that $A = \{t_1, t_2\}$ is a parameter set, $X = \{u_1, u_2\}$ is the universal set and $F = \{(t_1, \{u_1, u_2\}), (t_2, \{u_1, u_2\})\}$ is a soft set over $X$. Let $\tau = \{\Phi, F, F_1, F_2, F_3\}$ be a soft topology on $F$, where*
$\Phi = \{(t_1, \phi), (t_2, \phi)\}$
$F_1 = \{(t_1, \{u_1, u_2\}), (t_2, \{u_2\})\}$
$F_2 = \{(t_1, \{u_1\}), (t_2, \{u_1, u_2\})\}$
$F_3 = \{(t_1, \{u_1\}), (t_2, \{u_2\})\}$
*Clearly $< F, \tau >$ is a soft $T_0$-space. Since $a = \{(t_1, u_1), (t_2, u_1)\}$, $b = \{(t_1, u_1), (t_2, u_2)\}$, $c = \{(t_1, u_2), (t_2, u_1)\}$ and $d = \{(t_1, u_2), (t_2, u_2)\}$ are soft elements of $F$ and for every $a, b \in_s F$ with $a \neq_s b$ $\exists$ $M, N \in \tau$ such that*
*$a \in_s M$ and $b \notin_s M$ or $b \in_s N$ and $a \notin_s N$.*
*Now we have $\tau_{t_1} = \{\phi, \{u_1, u_2\}, \{u_1\}\}$ and $\tau_{t_2} = \{\phi, \{u_1, u_2\}, \{u_2\}\}$. Also it is clear that $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$.*

**Example 3.6.** *Suppose that $A = \{t_1, t_2\}$ is a parameter set, $X = \{u_1, u_2, u_3\}$ is the universal set and $F = \{(t_1, \{u_1, u_2\}), (t_2, \{u_2, u_3\})\}$ is a soft set over $X$. Let $\tau_{t_1} = \{\phi, \{u_1, u_2\}, \{u_2\}, \{u_1\}\}$ and $\tau_{t_2} = \{\phi, \{u_2, u_3\}, \{u_2\}, \{u_3\}\}$. Then it is clear that $\tau_{t_1}$ and $\tau_{t_2}$ are topologies on $F(t_1) =$*

*$\{u_1,u_2\}$, and $F(t_2) = \{u_2,u_3\}$, respectively. Also $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$.*
*Now by Theorem 2.14, $\tau = \{H \subset_s F : H(t) \in \tau_t\}$. Clearly $\Phi = \{(t_1,\phi),(t_2,\phi)\}$ and $F = \{(t_1,\{u_1,u_2\}),(t_2,\{u_2,u_3\})\}$ belongs to $\tau$. Now the other soft subsets of $F$ which belongs to $\tau$ are*
$F_1 = \{(t_1,\{u_1\}),(t_2,\{u_3\})\}$
$F_2 = \{(t_1,\{u_1\}),(t_2,\{u_2,u_3\})\}$
$F_3 = \{(t_1,\{u_1\}),(t_2,\{u_2\})\}$
$F_4 = \{(t_1,\{u_2\}),(t_2,\{u_2,u_3\})\}$
$F_5 = \{(t_1,\{u_2\}),(t_2,\{u_3\})\}$
$F_6 = \{(t_1,\{u_2\}),(t_2,\{u_2\})\}$
$F_7 = \{(t_1,\{u_1,u_2\}),(t_2,\{u_2\})\}$
$F_8 = \{(t_1,\{u_1,u_2\}),(t_2,\{u_3\})\}$
*Thus $\tau = \{\Phi,F,F_1,F_2,F_3,F_4,F_5,F_6,F_7,F_8\}$ is a soft topology on $F$. Now $a = \{(t_1,u_1),(t_2,u_2)\}$, $b = \{(t_1,u_1),(t_2,u_3)\}$, $c = \{(t_1,u_2),(t_2,u_2)\}$ and $d = \{(t_1,u_2),(t_2,u_3)\}$ are soft elements of $F$ and for every $a,b \in_s F$ with $a \neq_s b$ $\exists$ $M,N \in \tau$ such that*
*$a \in_s M$ and $b \notin_s M$ or $b \in_s N$ and $a \notin_s N$.*
*Hence $< F, \tau >$ is a soft $T_0$-space.*

**Example 3.7.** *Let $< F, \tau >$ be a soft topological space where $F(t) = \{0,1\}$ for all $t \in A$ and $\tau_t$ be Sierpinski topology that is $\tau_t = \{\{0,1\},\{1\},\phi\}$. Clearly $< F(t), \tau_t >$ is $T_0$-space for all $t \in A$. Now we will show that $< F, \tau >$ is a soft $T_0$-space where $\tau = \{H \subset_s F : H(t) \in \tau_t\}$. Suppose $\alpha, \beta \in_s F$ with $\alpha \neq \beta$. Then $\exists$ $t_1 \in A$ such that $\alpha(t_1) \neq \beta(t_1)$. Suppose $\alpha(t_1) = 0$ and $\beta(t_1) = 1$ (say). Define a soft set $H$ such that*

$$\begin{aligned} H(t) &= \{1\} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

*Then clearly $H \in \tau$ and $H$ contains $\beta$ but not $\alpha$. Thus $< F, \tau >$ is a soft $T_0$-space.*

**Example 3.8.** *Let $< F, \tau >$ be a soft topological space where*
*$F(t) = \{0,1\}$ with $\tau_t = indiscrete\, topology$ for all $t \in A$ except $t = t_1$ and*
*$F(t) = \{0,1\}$ with $\tau_t = discrete\, topology$ for $t = t_1$.Clearly $< F(t), \tau_t >$ is not $T_0$-space for all $t \in A$. We will show that $< F, \tau >$ is not a soft $T_0$-space, where $\tau_t = \{E \in \tau : E(t) \in \tau_t\}$. Let*

$$\begin{aligned} a(t) &= \{1\} \quad \text{if} \quad t = t_1 \\ &= \{0\} \quad \text{if} \quad t \neq t_1 \end{aligned}$$

*and $b(t) = \{1\}$ for all $t \in A$. Clearly $a \neq_s b$. If a soft open set $H$ contains $a$ then $a(t) \in H(t)$ for all $t \in A$. Now $H(t) = \{0,1\}$ for all $t \in A$ except $t = t_1$. So $b(t) \in H(t)$ for $t \neq t_1$ and $a(t_1) = b(t_1) \in H(t_1)$. This shows that $< F, \tau >$ is not soft $T_0$-space.*

**Definition 3.9.** *Let $< F, \tau >$ be a soft topological space. Then $< F, \tau >$ is called soft $T_1$-space if for every $a,b \in_s F$ with $a \neq_s b$ $\exists$ $M,N \in \tau$ such that*
*$a \in_s M$ and $b \notin_s M$ and $b \in_s N$ and $a \notin_s N$.*

**Proposition 3.10.** *(i) Let $< F, \tau >$ be a soft topological space. If $< F, \tau >$ is a soft $T_1$-space then $< F(t), \tau_t >$ is $T_1$-space for all $t \in A$.*
*(ii) Let $< F(t), \tau_t >$ be a topological space. If $< F(t), \tau_t >$ is $T_1$-space for all $t \in A$ then $< F, \tau >$ is a soft $T_1$-space, where $\tau$ is defined in Theorem 2.14.*

*Proof.* (i) $< F, \tau >$ is a soft $T_1$-space. suppose $t_1 \in A$ is fixed and $\alpha, \beta \in F(t_1)$. Let $a, b \in_s F$ be such that $a(t) = b(t)$ for all $t \in A$ except $t = t_1$ and $a(t_1) = \alpha$, $b(t_1) = \beta$. So $a \neq_s b$. Now since $< F, \tau >$ is a soft $T_0$-space then there are $H, K \in \tau$ such that $a \in_s H$ and $b \notin_s H$ and $b \in_s K$ and $a \notin_s K$. Clearly $a(t) = b(t) \in H(t)$ for all $t \in A$ except $t = t_1$. Hence $b(t_1) \notin H(t_1)$ and similarly $a(t_1) \notin K(t_1)$. This shows that $< F(t_1), \tau_{t_1} >$ be $T_1$-space. Since $t_1 \in A$ is arbitrary. Therefore $< F(t), \tau_t >$ is $T_1$-space for all $t \in A$.
(ii) Let $< F(t), \tau_t >$ be $T_1$-space. Also let $a, b \in_s F$ with $a \neq_s b$. Then at least one $t_1 \in A$ such that $a(t_1) \neq b(t_1)$. Let $M_{t_1}, N_{t_1} \in \tau_t$ such that $a(t_1) \in M_{t_1}$ and $b(t_1) \notin M_{t_1}$ and $b(t_1) \in N_{t_1}$ and $a(t_1) \notin N_{t_1}$. Define $H, K : A \to P(X)$ such that

$$\begin{aligned} H(t) &= M_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} K(t) &= N_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Clearly $H, K$ are soft sets and $H, K \in \tau$ by Theorem 2.14. Hence $a \in_s H$ and $b \notin_s H$ and $b \in_s K$ and $a \notin_s K$. Therefore $< F, \tau >$ is a soft $T_1$-space. □

**Theorem 3.11.** *Let $< F, \tau >$ be a soft topological space. Every single point soft set in $F$ is soft closed if and only if $< F, \tau >$ is soft $T_1$-space.*

*Proof.* Let $a \in_s F$. We know that $\{a\} : A \to P(X)$ is a single point soft set if and only if $\{a\}(t) = \{a(t)\}$ is a single point set for every $t \in A$. Since $< F(t), \tau_t >$ is a topological space, every single point set in $F(t)$ is closed if and only if $< F(t), \tau_t >$ is $T_1$-space. Hence from Remark 2.17 and Proposition 3.10 the theorem is proved. □

**Definition 3.12.** *Let $< F, \tau >$ be a soft topological space. Then $< F, \tau >$ is called soft $T_2$-space or soft Hausdorff space if for every $a, b \in_s F$ with $a \neq_s b$ $\exists$ $M, N \in \tau$ such that $a \in_s M$, $b \in_s N$ and $M \cap_s N =_s \Phi$.*

**Proposition 3.13.** *(i) Let $< F, \tau >$ be a soft topological space. If $< F, \tau >$ is a soft Hausdorff space then $< F(t), \tau_t >$ is Hausdorff space for all $t \in A$.*
*(ii) Let $< F(t), \tau_t >$ be a topological space. If $< F(t), \tau_t >$ is Hausdorff space for all $t \in A$ then $< F, \tau >$ is a soft Hausdorff space, where $\tau$ is defined in Theorem 2.14.*

*Proof.* (i) Let $< F, \tau >$ is a soft Hausdorff space. Also let $t_0 \in A$ be fixed and $\alpha, \beta \in F(t_0)$ with $\alpha \neq \beta$. Take a soft element $x$ of the soft set $F$. Define $a, b : A \to X$ such that

$$\begin{aligned} a(t) &= \alpha \quad \text{if} \quad t = t_0 \\ &= x(t) \quad \text{if} \quad t \neq t_0 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} b(t) &= \beta \quad \text{if} \quad t = t_0 \\ &= x(t) \quad \text{if} \quad t \neq t_0 \end{aligned}$$

Clearly $a, b \in_s F$ and $a \neq_s b$. Now since $< F, \tau >$ is a soft Hausdorff space then $\exists$ $M, N \in \tau$ such that $a \in_s M$, $b \in_s N$ and $M \cap_s N =_s \Phi$. This implies that $a(t_o) = \alpha \in M(t_0)$, $b(t_0) = \beta \in N(t_0)$. Now $x(t) \in (M \cap_s N)(t)$ for $t \neq t_0$ this shows that $M(t) \cap N(t) \neq \phi$ for $t \neq t_0$ but $M \cap_s N =_s \Phi$.

Hence $M(t_0) \cap N(t_0) = \phi$. Since $t_0$ is arbitrary. Hence $< F(t), \tau_t >$ be Hausdorff space for all $t \in A$.
(ii)Let $< F(t), \tau_t >$ be Hausdorff space. Also Let $a, b \in_s F$ with $a \neq_s b$. Then $\exists\ t_1 \in A$ such that $a(t_1) \neq b(t_1)$. Let $M_{t_1}, N_{t_1}$ be disjoint open sets in $\tau_t$ such that $a(t_1) \in M_{t_1}$ and $b(t_1) \in N_{t_1}$. Define $H, K : A \to P(X)$ such that

$$\begin{aligned} H(t) &= M_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} K(t) &= N_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Then $H \cap_s K = \Phi$. Also by Theorem 2.14 $M$ and $N$ are soft open sets containing the soft elements $a$ and $b$ respectively. Therefore $< F, \tau >$ is a soft Hausdorff space. □

**Definition 3.14.** *Let $< F, \tau >$ be a soft topological space. Then $< F, \tau >$ is called soft regular space if for every $a \in_s F$ and a soft closed set $E \subset_s F$ with $a \notin_s E$ $\exists\ M, N \in \tau$ such that $a \in_s M$, $E \subset_s N$ and $M \cap_s N =_s \Phi$.*

**Proposition 3.15.** *(i) Let $< F, \tau >$ be a soft topological space. If $< F, \tau >$ is a soft regular space then $< F(t), \tau_t >$ is regular space for all $t \in A$.*
*(ii) Let $< F(t), \tau_t >$ be a topological space. If $< F(t), \tau_t >$ is regular space for all $t \in A$ then $< F, \tau >$ is a soft regular space, where $\tau$ is defined in Theorem 2.14.*

*Proof.* (i) Let $< F, \tau >$ be a soft regular space and $x \in_s F$. Let $t_1 \in A$ be fixed and $S$ be a closed set in $F(t_1)$ with $\alpha \in F(t_1)$ such that $\alpha \notin S$. Define $a : A \to X$ and $E : A \to P(X)$ such that

$$\begin{aligned} a(t) &= \alpha \quad \text{if} \quad t = t_1 \\ &= x(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} E(t) &= S \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Then $E$ is soft closed as $E(t)$ is closed for all $t \in A$. Clearly $a \notin_s E$ as $a(t_1) \notin E(t_1)$. Hence $\exists\ M, N \in \tau$ such that $a \in_s M$, $E \subset_s N$ and $M \cap_s N =_s \Phi$. Since $(M \cap_s N)(t) = \phi$ for $t = t_1$, we conclude that $M(t_1) \cap N(t_1) = \phi$. This completes the proof for first part.
(ii) Let $< F(t), \tau_t >$ be regular space. Also Let $a \in_s F$ and $E$ be a soft closed subset of $F$ with $a \notin_s E$. So there is $t_1 \in A$ such that $a(t_1) \notin E(t_1)$. Let $M_{t_1}, N_{t_1}$ be disjoint open sets in $\tau_t$ such that $a(t_1) \in M_{t_1}$ and $E(t_1) \subset N_{t_1}$. Define $H, K : A \to P(X)$ such that

$$\begin{aligned} H(t) &= M_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} K(t) &= N_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Then $H$ and $K$ are two disjoint soft sets containing $a$ and $E$ respectively. Hence $< F, \tau >$ is a soft regular space. □

**Note 3.16.** *A soft regular space which is also soft $T_1$, is called soft $T_3$-space.*

**Definition 3.17.** *Let $< F,\tau >$ be a soft topological space. Then $< F,\tau >$ is called soft normal space if for every soft closed sets $E_1,E_2 \subset_s F$ with $E_1 \cap_s E_2 =_s \Phi$ $\exists$ $M,N \in \tau$ such that $E_1 \subset_s M$, $E_2 \subset_s N$ and $M \cap_s N =_s \Phi$.*

**Proposition 3.18.** *(i) Let $< F,\tau >$ be a soft topological space. If $< F,\tau >$ is a soft normal space then $< F(t),\tau_t >$ is normal space for all $t \in A$.*
*(ii) Let $< F(t),\tau_t >$ be a topological space. If $< F(t),\tau_t >$ is normal space for all $t \in A$ then $< F,\tau >$ is a soft normal space, where $\tau$ is defined in Theorem 2.14.*

*Proof.* Let $< F,\tau >$ be a soft normal space. Let $t_1 \in A$ be fixed and $S_1$, $S_2$ be two closed sets in $F(t_1)$ with $S_1 \cap S_2 = \phi$. Define $E_1 : A \to P(X)$ and $E_2 : A \to P(X)$ such that

$$\begin{aligned} E_1(t) &= S_1 \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} E_2(t) &= S_2 \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Then $E_1$ and $E_2$ is soft closed as $E_1(t)$ and $E_2(t)$ is closed for all $t \in A$. Clearly $E_1 \cap_s E_2 =_s \Phi$. Hence $\exists$ $M,N \in \tau$ such that $E_1 \subset_s M$, $E_2 \subset_s N$ and $M \cap_s N =_s \Phi$. Since $(M \cap_s N)(t) = \phi$ for $t = t_1$, we conclude that $M(t_1) \cap N(t_1) = \phi$. This completes the proof for first part.

Conversely, let $< F(t),\tau_t >$ be normal space. Also Let $E_1$ and $E_2$ be two soft closed subset of $F$ with $E_1 \cap_s E_2 =_s \Phi$. So there is $t_1 \in A$ such that $E_1(t_1) \cap E_2(t_1) = \phi$. Let $M_{t_1}, N_{t_1}$ be disjoint open sets in $\tau_t$ such that $E_1(t_1) \subset M_{t_1}$ and $E_2(t_1) \subset N_{t_1}$. Define $H,K : A \to P(X)$ such that

$$\begin{aligned} H(t) &= M_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \qquad \textit{and} \end{aligned}$$

$$\begin{aligned} K(t) &= N_{t_1} \quad \text{if} \quad t = t_1 \\ &= F(t) \quad \text{if} \quad t \neq t_1 \end{aligned}$$

Then $H$ and $K$ are two disjoint soft sets containing $E_1$ and $E_2$ respectively. Hence $< F,\tau >$ is a soft normal space. □

**Note 3.19.** *A soft normal space which is also soft $T_1$, is called soft $T_4$-space.*

**Remark 3.20.** *Soft $T_4$-space $\Rightarrow$ Soft $T_3$-space $\Rightarrow$ Soft $T_2$-space $\Rightarrow$ Soft $T_1$-space $\Rightarrow$ Soft $T_0$-space.*

**Proposition 3.21.** *Let $< F,\tau >$ be a soft topological space and $H \subset_s F$. If $< F,\tau >$ is soft $T_i$-space then $< H,\tau^H >$ is a soft $T_i$-space for $i = 0,1,2,3,4$.*

*Proof.* We prove only for soft $T_0$ space. The others are similar.
Let$< F,\tau >$ be soft $T_0$-space and $H \subset_s F$. Let $a,b \in_s H$ with $a \neq_s b$ then $\exists$ soft open sets $M,N \in \tau$ such that $a \in_s M$ and $b \notin_s M$ or $b \in_s N$ and $a \notin_s N$. Now $a \in_s H$ and $a \in_s M$ implies $a \in_s H \cap_s M \in \tau^H$ where $M \in \tau$.

Consider, $b \notin_s M$ this means that $b(t) \notin M(t)$ for some $t \in A$. Then $b(t) \notin H(t) \cap M(t) = (H \cap_s M)(t)$ for some $t \in A$ implies that $b \notin_s H \cap_s M \in \tau^H$. Similarly it can be proved that if $b \in_s N$ and $a \notin_s N$ then $b \in_s H \cap_s N \in \tau^H$ and $a \notin_s H \cap_s N \in \tau^H$. Thus $< H, \tau^H >$ is a soft $T_0$-space. □

# 4. Soft Compactness

**Definition 4.1.** *Aygünoğlu and Aygün (2012),Zorlutuna et al. (2012) Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. A collection $\mathscr{C}_s$ of soft subsets of $F$ is said to be a soft cover of $H$ if $H \subset_s \cup_s \{K : K \in \mathscr{C}_s\}$.*
*A soft cover $\mathscr{C}_s$ of $H$ is said to be a soft open cover of $H$ if every members of $\mathscr{C}_s$ is a soft open set in $< F, \tau >$.*

**Definition 4.2.** *Aygünoğlu and Aygün (2012),Zorlutuna et al. (2012) Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. Then $H$ is said to be soft compact in $< F, \tau >$ if every soft open cover of $H$ has a finite soft subcover of $H$.*
*If $H =_s F$ then $F$ is said to be soft compact space.*

**Theorem 4.3.** *Let $< F, \tau >$ be a soft topological space and $H \subset_s F$. If $H$ is a soft compact in $< F, \tau >$ then $H(t)$ is compact in $< F(t), \tau_t >$.*

*Proof.* Let $H$ is soft compact then every soft open cover of $H$ has a finite soft subcover of $H$. Let $t_0 \in A$ be fixed. Let $\mathscr{C}_{t_0} = \{B : B \in \tau_{t_0}\}$ is a open cover of $H(t_0)$. For each $B \in \mathscr{C}_{t_0}$ define $K_B : A \to P(X)$ such that

$$\begin{aligned} K_B(t) &= B \quad \text{if} \quad t = t_0 \\ &= F(t) \quad \text{if} \quad t \neq t_0 \end{aligned}$$

Then $\{K_B : B \in \mathscr{C}_{t_0}\}$ is a soft open cover of $H$. As $H$ is soft compact there is $B_1, B_2, ...., B_n$ such that $K_{B_1}, K_{B_2}, ...., K_{B_n}$ will cover $H$. Hence $\{B_1, B_2, ...., B_n\}$ is a finite subcover of $\mathscr{C}_{t_0}$ which covers $H(t_0)$. Since $t_0 \in A$ is arbitrary. Hence the theorem is proved. □

**Theorem 4.4.** *Let $< F, \tau_1 >$ and $< F, \tau_2 >$ be two soft topological space and $\tau_1 \subseteq \tau_2$. If $< F, \tau_2 >$ is a soft compact topological space then $< F, \tau_1 >$ is also soft compact topological space.*

*Proof.* Suppose $\mathscr{C}_s = \{K : K \subset_s F \text{ and } K \in \tau_1\}$ is a soft open cover in $< F, \tau_1 >$. Since $\tau_1 \subseteq \tau_2$ so $\mathscr{C}_s$ is also a soft open cover in $< F, \tau_2 >$. Now $< F, \tau_2 >$ is soft compact then $\exists$ a finite soft subcover $\mathscr{C}_s'$ of $\mathscr{C}_s$ which is also a finite soft cover in $< F, \tau_1 >$. Hence $< F, \tau_1 >$ is also soft compact. □

**Theorem 4.5.** *Let $< F, \tau >$ be a soft topological space and $H_i$ are soft compact subset of $F$ where $i = 1, 2, ...., n$ then $\bigcup_{i=1}^{n} H_i$ is a soft compact subset of $F$.*

*Proof.* Let $< F, \tau >$ be a soft topological space and $H_1, H_2, ...., H_n$ are soft compact subset of $F$. Assume $H =_s \bigcup_{i=1}^{n} H_i$ and $\mathscr{C}$ is a soft open cover of $H$. For each $i$ define $\mathscr{C}_i = \{H_i \cap K : K \in \mathscr{C}\}$. So $\mathscr{C}_i$ is a soft open cover of $H_i$. Since $H_i$ is soft compact then there is a finite collection

$\{H_i \cap K_1^i, H_i \cap K_2^i, ......, H_i \cap K_m^i\}$ which also cover $H_i$. Clearly $\{K_j^i : i = 1,2,...,n; j = 1,2,...,m\}$ is a finite subcover of $\mathscr{C}$ which also cover $H$. Thus $\bigcup_{i=1}^{n} H_i$ is a soft compact subset of $F$. □

**Theorem 4.6.** *Let $< F, \tau >$ be a soft topological space and the collection $\mathscr{C}_s = \{K : K$ is a soft compact closed subset of $F\}$ then $\bigcap_{K \in \mathscr{C}_s} K$ is a soft compact subset of $F$.*

*Proof.* Let $< F, \tau >$ be a soft topological space and the collection $\mathscr{C}_s = \{K : K$ *is a soft compact closed subset of F*$\}$. Now $\bigcap_{K \in \mathscr{C}_s} K$ is a soft closed subset of $F$, since for all $K \in \mathscr{C}_s$ is soft closed. Again $\bigcap_{K \in \mathscr{C}_s} K \subset_s K$. Hence $\bigcap_{K \in \mathscr{C}_s} K$ is soft compact subset of $F$. □

**Theorem 4.7.** *Atmaca (2016) Every soft closed subset of a soft compact space is soft compact.*

**Theorem 4.8.** *Let $< F, \tau >$ be a soft Hausdorff space, $a \in_s F$ and $H$ be soft compact with and $a \notin_s H$. Then $\exists$ soft open sets $U$ and $V$ such that $a \in_s U$, $H \subset_s V$ and $U \cap_s V = \Phi$.*

*Proof.* Let $< F, \tau >$ be a soft Hausdorff space. Let $H$ be soft compact set and $a \notin_s H$. For each $x \in_s H$ there are soft open sets $S_x$ and $T_x$ such that $x \in_s S_x$, $a \in_s T_x$ and $S_x \cap_s T_x =_s \Phi$. The family $\{S_x : x \in_s H\}$ is a soft open cover of $H$. As $H$ is soft compact then there is a finite subcollection of soft sets $\{S_{x_1}, S_{x_2}, ...., S_{x_n}\}$ which also cover $H$. Let $U = \bigcap_{i=1}^{n} T_{x_i}$ and $V = \bigcup_{i=1}^{n} S_{x_i}$, then $U$ and $V$ are disjoint soft open sets containing $a$ and $H$ respectively and $U \cap_s V = \Phi$. □

**Theorem 4.9.** *Let $< F, \tau >$ be a soft Hausdorff space and $H$ be any soft compact subset of $F$. Then $H$ is soft closed set.*

*Proof.* Let $< F, \tau >$ be a soft Hausdorff space and $H$ be any soft compact subset of $F$. Then by Theorem 4.8, for any $a \in_s H^c$ $\exists$ soft open soft sets $U$ and $V$ such that $a \in_s U$, $H \subset_s V$ and $U \cap_s V = \Phi$. In particular $U \cap_s H = \Phi$ and hence $U \subset_s H^c$. Thus $H^c$ is a soft neighbourhood of each of its points. So $H^c$ is soft open and $H$ is soft closed. □

**Theorem 4.10.** *Every soft compact Housdorff space is soft regular.*

*Proof.* Let $< F, \tau >$ be a soft compact Hausdorff space. Then every soft closed subset of $F$ is soft compact and so the space $< F, \tau >$ is soft regular by Theorem 4.8. □

**Theorem 4.11.** *Let $< F, \tau >$ be a soft regular space, $H$ be a soft closed subset of $F$ and $K$ be soft compact subset of $F$ with $H \cap_s K = \Phi$. Then $\exists$ soft open sets $U$ and $V$ such that $H \subset_s U$, $K \subset_s V$ and $U \cap_s V = \Phi$.*

*Proof.* Let $< F, \tau >$ be a soft regular space, $H$ a soft closed subset of $F$ and $K$ be soft compact subset of $F$ with $H \cap_s K = \Phi$. Let $a \in_s K$ then $a \notin_s H$. By regularity of $< F, \tau >$ we get, for each $a \in_s K$ there are soft open sets $S_a$ and $T_a$ such that $a \in_s S_a$, $H \subset_s T_a$ and $S_a \cap_s T_a =_s \Phi$. The family $\{S_a : a \in_s K\}$ is a soft open cover of $K$. As $K$ is soft compact then there is a finite subcollection of soft sets $\{S_{a_1}, S_{a_2}, ...., S_{a_n}\}$ which also cover $K$. Let $U = \bigcap_{i=1}^{n} T_{a_i}$ and $V = \bigcup_{i=1}^{n} S_{a_i}$, then $U$ and $V$ are disjoint soft open sets containing $H$ and $K$ respectively and $U \cap_s V = \Phi$. □

**Theorem 4.12.** *Every soft compact Housdorff space is soft normal.*

*Proof.* The proof of the theorem is follows from Theorems 4.10 and 4.11 □

**Definition 4.13.** *A family $\mathfrak{F}$ of soft subset of $F$ is said to posses finite intersection property (FIP) if soft intersection of any finite soft subcollection of $\mathfrak{F}$ is non-empty.*

**Theorem 4.14.** *Let $< F, \tau >$ be a soft topological space and $\mathfrak{F}$ be a family of soft subset of $F$ . If $\mathfrak{F}$ has FIP then the collection $\mathfrak{F}_t = \{H(t) : H \in \mathfrak{F}\}$ of $F(t)$ has FIP also for all $t \in A$.*

*Proof.* Let $\mathfrak{F}$ has FIP and $t_0 \in A$ be fixed. Let $\{B_i : i \in I\}$ be a collection of sets of $F(t_0)$ in $\mathfrak{F}_{t_0}$. Define $F_i : A \to P(X)$ by

$$\begin{aligned} F_i(t) &= B_i \quad \text{if} \quad t = t_0 \\ &= F(t) \quad \text{if} \quad t \neq t_0. \end{aligned}$$

Hence $F_i$ is soft set and $F_i \in \mathfrak{F}$. Since $\mathfrak{F}$ has FIP then for every finite $F_1, F_2, ....., F_n$ such that $\bigcap_{i=1}^{n} F_i \neq_s \Phi$ which implies that $(\bigcap_{i=1}^{n} F_i)(t_0) = \bigcap_{i=1}^{n} F_i(t_0) = \bigcap_{i=1}^{n} B_i \neq \Phi(t_0) = \phi$. Since $t_0 \in A$ is arbitrary. Thus the theorem is true for all $t \in A$. □

**Theorem 4.15.** *A soft topological space $< F, \tau >$ is soft compact if and only if for every family of soft closed sets having FIP has nonempty soft intersection.*

*Proof.* A soft topological space $< F, \tau >$ is soft compact $\Leftrightarrow$ no finite sub collection of a collection of soft open sets $\mathscr{A}$ cover $F$ then $\mathscr{A}$ does not cover $F$.
Let $\mathscr{C}_s = \{F - G : G \in \mathscr{A}\}$. Then $\mathscr{C}_s$ is a collection of soft closed sets and $\mathscr{C}_s$ has FIP is equivalent to above statement. This completes the proof. □

## References


Abbas, M. Ali, M. I., Romaguera, S., 2017. Generalized Operations in Soft Set Theory via Relaxed Conditions on Parameters. Filomat, 19: 5955-5964.

Acar, U., Koyuncu, F., Tanay, B., 2010. Soft Sets and Soft Rings. Computers and Mathematics with Applications, 59: 3458-3463.

Aktaş, H., Çağman, N., 2007. Soft Sets and Soft Groups. Information Sciences, 177: 2726-2735.

Al-Khafaj, M. A. K., Mahmood, M. H., 2014. Some Properties of Soft Connected Space and Soft Locally Connected Spaces. IOSR Journal of Mathematics, 10: 102-107.

Ali, M. I., Feng, F., Liu, X., Min, W. K., Shabir, M., 2009. On Some New Operation in Soft Set Theory. Computers and Mathematics with Applications, 9: 1547-1553.

Aygünoğlu, A., Aygün, H., 2012. Some Notes on Soft Topological Spaces. Neural Computing and Applications, 21: 113-119.

Atmaca, S., 2016. Compactification of Soft Topological Spaces. Journal of New Theory, 12: 23-28.

Benchalli, S. S., Patil, P. G., 2017. Some Properties of Soft $\beta$-Compact and Related Soft Topological Spaces. Italian Journal of Pure and Applied Mathematics, 38: 487-496.

Babitha, K. V., John, S. J., 2010. Studies on Soft Topological Spaces. Journal of Intelligent and Fuzzy Systems, 28: 1713-1722.

Çağman, N., Enginoğlu, S., 2010. Soft Set Theory and Uni-Int Decision Making. European Journal of Operational Research, 207: 848-855.

Çağman, N., Enginoğlu, S., Karataş, S., Aydın, T., 2015. On Soft Topology. El-Cezeri Journal of Science and Engineering, 2: 23-38.
Çağman, N., Karataş, S., Enginoğlu, S., 2011. Soft Topology. Computers and Mathematics with Applications, 61: 351-358.
Çelik, Y., Ekiz, C., Yamak, S., 2011. A New View on Soft Rings. Hacettepe Journal of Mathematics and Statistics, 40: 273-286.
Feng, F., Jun, Y. B., Zhao, X., 2008. Soft Semirings. Computers and Mathematics with Applications, 56: 2621-2628.
Georgiou, D. N., Megaritis, A. C., Petropoulos, V. I., 2013. On Soft Topological Spaces, Applied Mathematics and Information Sciences 7: 1889-1901.
Georgiou, D. N., Megaritis, A. C., 2014. Soft Set Theory and Topology. Applied General Topology 15: 93-109.
Goldar, S., Ray, S., 2017. A Study of Soft Topology from Classical View Point, Proceedings of IMBIC, 6: 108-116.
Goldar, S., Ray, S., 2019. On Soft Ring And Soft Ideal, Journal of Applied Science and Computations,6: 1457-1467.
Hamza, A. M., Saad, M., 2017. On The Open Limit Point Compactness and Soft Open Limit Point Compactness. Applied Mathematical Sciences, 11: 2697-2706.
Hida, T., 2014. A Comparison of Two Formulations of Soft Compactness. Annals of Fuzzy Mathematics and Informatics, 8: 511-525.
Hosny, R. A., Abd El-Latif, A. M., 2016. Soft G-compactness in Soft Topological Spaces. Annals of Fuzzy Mathematics and Informatics, 11: 973-987.
Hussain, S., 2015. A Note on Soft Connectedness. Journal of the Egyptian Mathematical Society, 23: 6-11.
Jalil, S., Reddy, B. S., 2017. On Soft Compact, Sequentially Compact and Locally Compactness. Advances in Fuzzy Mathematics, 12: 835-844.
Kandil, A., Tantawy, O. A. E., El-Sheikh, S. A., Abd El-Latif, A. M., 2014. Soft Connectedness via Soft Ideals. Journal of New Results in Sciuences, 4: 90-108.
Maji, P. K., Biswas, R., Roy, A. R., 2003. Soft Set Theory, Computers and mathematics with Applications, 45: 555-562.
Majumdar, P., Samanta, S. K., 2010. On Soft Mapping. Computers and mathematics with Applications, 60: 2666-2672.
Molodtsov, D., 1999. Soft set theory-First results. Computers and Mathematics with Applications, 37(4-5): 19-31.
Molodtsov, D., Leonov, V. Y., Kovkov, D. V., 2006. Soft Sets Technique and Its Application. Nechetkic Sistemy i Myagkie Vychisteniya, 1: 8-39.
Osmanoglu, I., Tokat, D., 2014. Compact Soft Multi Spaces. European Journal of Pure and Applied Mathematics, 7: 97-108.
Pei, D., Miao, D., 2005. From Soft Sets to Information Systems. IEEE International Conference on Granular Computing, 2: 617-621.
Ray, S., Goldar, S., 2017. Soft Set and Soft Group from Classical View Point. The Journal of the Indian Mathematical Society, 84: 273-286.
Shabir, M., Naz, M., 2011. On Soft Topological Spaces, Computers and Mathematics with Applications, 61: 1786-1799.
Shah, T., Shaheen, S., 2013. Soft Topological Groups and Rings. Annals of Fuzzy Mathematics and Informatics, 7(5): 725-743.
Shirmohammadi, N., Rasouli, H., 2017. A Categorical Approach to Soft S-Acts. Filomat, 31: 4185-4198.
Subhashinin, J., Sekar, C., 2014. Soft P-Connectedness via Soft P-Open Sets. International Journal of Mathematics Trends and Technology, 6: 203-214.
Tasbozan, H., Icen, V., Bagirmaz, N., Ozcan, A. F., 2017. Soft Sets and Soft Topology on Nearness Approximation Space. Filomat, 31: 4117-4125.
Thakur, S. S., Rajput, A. S., 2018. Connectedness Between Soft Sets. New Mathematics and Natural Computation, 14: 53-71.
Varol, B. P., Aygun, H., 2013. On Soft Hausdorff Spaces. Annals of Fuzzy Mathematics and Informatics, 5: 15-24.
Varol, B. P., Shostak, A., Aygün, H., 2012. A New Approach To Soft Topology. Hacettepe Journal of Mathematics and Statistics, 41: 731-741.
Yang, H. L., Liao, X., Li, S. G., 2015. On Soft Continuous Mappings and Soft Connectedness of Soft Topological Spaces. Hacettepe Journal of Mathematics and Statistics, 44: 385-398.
Zorlutuna, İ., Akdağ, M., Min, W. K., Atmaca, S., 2012 Remarks on Soft Topological Spaces. Annals of Fuzzy Mathematics and Informatics, 3: 171-185.