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A Case Study about Computational Estimation Strategies of Seventh Graders

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ABSTRACT. This study examined computational estimation strategies of the seventh grade students. A case study methodology was used and data were collected from five high achievers via an interview session. During the interview participants were required to solve fifteen computational estimation numerical questions which were presented in whole number, fraction and decimal and they were asked to explain their reasoning. A theme oriented matrix was used to analyze the data. The results of the study found that reformulation was the most preferred strategy and translation and compensation strategies were relatively less preferred strategies by the interviewees. The least preferred strategy was compensation which was known as adjusting the estimated results according to exact answer.

Keywords: computational estimation, strategy, case study, seventh grade

INTRODUCTION

Until about thirty years ago, being able to perform paper-pencil calculations or mental calculation quickly, neatly and accurately was a valuable skill. Now, with advances in technology, society not only needs the exactness of computers but also need people who can estimate the reasonableness of the exact answers they obtain. Therefore, computational estimation has become important when determining the reasonableness of an answer, particularly when using a calculator (Hope, 1986). According to Reys (1986), the current emphasis on estimation in the mathematics curriculum has been fostered by the widespread availability and use of technology. Additionally, Usiskin (1986) stated that the computation of a single correct answer covers only a part of mathematics; other problems may require estimation. Moreover, Reys (1992: 142) suggested that "over 80% of all mathematical *real world* applications call for estimation, rather than exact computation."

Global reports by mathematical councils and curriculum reform initiatives, deems computational estimation as an important component of students becoming proficient in mathematics (The Turkish Ministry of National Education-TMNE, 2005; National Council of Teachers of Mathematics-NCTM, 2000; Segovia and Castro, 2009; The England Department for Education and Skills- EDCTM, 2007; Australian Education Council-AEC, 1991). The National Council of Teachers of Mathematics (NCTM, 2000) acknowledged that students should develop and adapt procedures for mental computation and computational estimation with fractions, decimals, and integers. Similarly, The Turkish Ministry of National Education (TMNE, 2004) asserted that students should be able to perform mental computation and have computational estimation ability besides exact computation skills. Mathematics students today are expected to learn estimation as a means of checking answers in computation and problem-solving situations where an exact answer may not be needed (Usiskin, 1986). 1

In Turkey, although estimation had been an important part of problem solving before 2005, by the 6-8th grade mathematics education new curriculum (TMNE, 2005) estimation gained great importance not only for problem solving procedure but also it is important as a basic mathematical ability. The 6-8th grade mathematics education new curriculum contains fourteen objectives to improve students' computational estimation through 6 to 8 grades such as "Estimate the result of fraction operations by using strategies; estimate the results of the decimal operations by using strategies and estimate the square root of the numbers which has not exact square root". Although all the mathematics education curriculum gives great importance to learning and teaching estimation, research studies show that students are not successful about

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estimation (Boz, 2004; Çilingir & Türnüklü, 2009). To understand why students are so weak about computational estimation and how mathematics educators can help them to improve their abilities we first tried understand how students solve estimation questions and what their perspectives about estimation are. If we clearly categorized the strategies that students use while solving estimation questions we can concentrate on these strategies and improve it. Because of this we try to identify seventh grade students' computational estimation strategies in the current study. The overarching question of the study is "What computational estimation strategies are used by seventh grade students?" Using qualitative inquiry, we examined participants' strategies and thoughts about computational estimation in an effort to illuminate variety of strategies that are used for the same questions by different students. We believe the findings of this study make a significant contribution to understanding of general strategies for computational estimation problems.

Literature Review

Researchers have identified different types of strategies used in computational estimation (Çilingir & Türnüklü, 2009; Dowker, 1992; Levine, 1982; Reys, Rybolt, Bestgen, & Wyatt, 1982; Reys B., 1986; Reys, Reys, & Penafiel, 1991; Reys, Reys, Nohda, Ishida, Yoshikawa & Shimizu, 1991).

Levine (1982) interviewed 89 undergraduates to observe the estimation strategies. In her study, Levine (1982) identified eight common estimation strategies used to estimate solutions to numerical problems. These are fraction, exponents, rounding both numbers, rounding one number, powers of 10, known numbers, incomplete partial products and proceeding algorithmically. In a related study, using the same test protocol from Levine`s (1982) study but with a more mathematically wise population, Dowker (1992) identified seven strategies, three of which were also identified by Levine (1982). Both Levine (1982) and Dowker (1992) commonly identified the following strategies: use of fractions, rounding, and use of algorithms as processes. Beside the studies that were conducted in other countries, there are some research studies on computational estimation in Turkey by Boz and Bulut (2002), Boz (2004), Çilingir and Türnüklü (2009). Çilingir and Türnüklü (2009) investigated 6-8th graders' estimation ability and strategies for both computational estimation and measurement estimation. They conducted semi structured interviews with thirty students. The researchers identified strategies for both computational and measurement related problems and listed as "depended on the existing knowledge and experience, visualizing, decomposition, comparison, estimation through experiments, rounding, accommodating, distribution, the use of front-end orders, grouping, mental calculation, and randomly made estimates". The researchers did not report any differences among the grade level through the strategy usage.

In general, the strategies identified by research studies are compatible numbers, truncation, front-end strategies, reformulation, compensation, translation, nice numbers, matching pairs, comparing whole numbers, comparing fractions with a whole and a half, grouping, averaging or clustering, and standard computation procedure. Among the studies that have been done, the most extensive research studies have been conducted by Reys and his colleagues (Bestgen, Reys, Rybolt, &Wyatt; 1980; Reys, et al., 1991[a]; Reys, et al., 1991[b]). They tested a large group of students from USA, Japan and Mexico using the same instrument "Computational Estimation Test". According to the test results; the most successful students were selected and interviewed with the almost the same interview protocol to identify their strategies. According to these studies strategies gathered under three main types; reformulation, translation, and compensation.

Reformulation is defined as changing the numerical data into more mentally manageable form (Reys et al., 1982). One example of reformulation, which is the best known, is rounding numbers. This is the simplest strategy, and therefore, it is often the only strategy taught in the classroom (Levine, 1982; Trafton, 1986). Rounding numbers can be conducted by two methods; by considering rules and by considering the requirement of the situation. For example, while computing 28 x 17 by considering rules; both numbers should be rounded to upper tens and the multiplication becomes 30 x 20, since both the last digits of the numbers are bigger than five. On the other hand, using the same example, the multiplication can simplified as 30 x 15 by using compatible numbers.

The use of "nice" numbers or "compatible numbers" is another example of reformulation strategy. Levine (1982) and Dowker (1992) called this strategy "known numbers" in their studies. Compatible numbers are proposed by many researchers as a reformulation strategy (e.g., Murphy, 1989; Reys et al., 1982; Reys, 1986). While performing the compatible numbers strategy the groups of numbers, which are used in combination, are put together and then operated in the procedure. Murphy (1989) provided some examples for use of compatible numbers in her research: in order to estimate the division of 5657÷28, the operation could be changed to 6000÷30 or 5000/25, and in another example 15% of $ 28.75 could be changed by $\frac{1}{7}$ of $ 28 or $\frac{1}{6}$ of $ 30.

Besides whole numbers, fraction and decimal related questions, reformulation could be performed by converting the numbers to fractions and/or decimal equivalents. Reys (1986) called this kind of reformulation strategy into the decimal or fraction a "special numbers strategy." For example, rounding fractions could be done by utilizing three important benchmarks, namely, 1, $\frac{1}{2}$ or 0, so that $3\frac{5}{12}$ might be rounded to $3\frac{1}{2}$ or the operation 3.65 x 0.78 might be rounded to $3\frac{1}{2} \times \frac{3}{4}$. According to researchers, a reformulation strategy was used by students at different achievement levels for problems both in numerical and application formats (Dowker, 1992; Levine, 1982; Reys et al., 1982; Reys, et al., 1991[a]; Reys, et al., 1991[b]).

The other computational estimation strategy used in the computational estimation questions is "translation." *Translation* means changing the equation or mathematical structure of the problem to a more mentally manageable form (Reys et al., 1982). The type of operation may be changed to make the problem easier; for example, addition may be converted to multiplication. For instance, the addition of the five numbers, 253 + 248 + 198 + 204 + 186 can be converted to multiplication of 200 x 5 using the translation strategy. Translation is a more sophisticated technique than reformulation. Reys et al. (1982) observed that translation is more flexible than reformulation and may require an advanced level of conceptual understanding.

Among the three strategies, the last one, *compensation strategy*, is the most complex one and generally the percentage of the usability of this strategy is lower than the others. This strategy is used for finding answers that are closer to an exact answer by manipulating the estimated result. For instance, a two quick, yet reasonable estimate for 2124 x 13, would be 21 000 (2100x10) or 21240 (2124x 10). Yet each of these are clearly an underestimate. Compensation might be accomplished by considering the numbers that were dropped in rounding. The 3 drop from the second number (13) could be used to multiply by 2100, so that the result, 6300 (2100 x 3) would be added to 21 000. The compensated result 27 300 is closer than the first two estimated results (21 000 or 21240) to the exact answer which is 27 612.

Compensation is the process of the adjustments that is made during or at the end of the process. According to Reys et al. (1982), good estimators used compensation frequently and identified it as an essential strategy to successful estimation. Lemaire, Lecacheur and Farioli (2000) concluded that the fastest strategy was reformulation and the slowest was the compensation strategy. Similarly, Reys et al. (1991) claimed that the most common process applied by Japanese and American students was reformulation and to a lesser extent was compensation. Sowder and Wheeler (1989) found that most fifth graders recognized the value of compensation but did not use it when generating computational estimates. The researchers

(Sowder & Wheeler, 1989) stated that as grade levels increased, the use of the compensation strategy also increased.

METHODS

The aim of the research study is identifying the computational estimation strategies and understanding the participants' point of view about how they are using the computational estimation strategies. Since the nature of the current study required the use of qualitative data collection and analysis techniques, case study design was used in the research study.

Participants

The participants in the study were determined by a two-step sampling process. First, the elementary school was chosen randomly among forty-six elementary schools in the Aegean region of Turkey. Qualitative research studies are not aimed at generalizing the result of the study, however case-to-case transfer is one of the purpose of the sampling process (Collins, Onwuegbuzie, & Jiao, 2012). Miles and Huberman (1994) underlined that case-to-case transfer relate to conceptual power of the research studies. In the current study random selection is used to prevent biased sampling. Moreover, to conduct the case-to-case transfer, the researchers first used random sampling of the elementary school and then made precise explanations of the interview group. As a second step purposive sampling was used to select high achieving seventh graders according to the results of the Computational Estimation Test (CET). According to Merriam (1998) "purposive sampling emphasizes a criterion based selection of information rich cases from which a researcher can discover, understand and gain more insight on issues crucial for the study" (p. 61). This study employed getting high scores from the CET as a criterion for purposive sampling. Hence our study sought only high achieving students on CET because we wanted students who were knowledgeable of estimation, estimation strategies and used it affectively during the estimation required questions in the interview sessions.

The scores gathered from CET were ranked and the top seven performing students were asked to participate in the study. When participants' computational estimation ability performances are examined according to numeric and word CET, all interviewees showed similar levels of achievement to each other (see Table 1).

Table 1. *Interviewees' scores on CETs*

Interviewee	According to Numeric CET Score (out of 15)	Interviewee	According to Word CET Score (out of 15)
Ayşe	9	Mert	11
Mert	9	Ayşe	9
Deniz	9	Sergen	9
Nevzat	9	Deniz	9
Sergen	7	Nevzat	8

Two students were excluded from interview group on basis of their incompleteness of responses to the questions during the interview sessions. Hence the participants for this study were five seventh grade students who were Deniz, Mert, Sergen, Nevzat and Ayşe (all names are pseudonyms). Ayşe was the only female participant. Furthermore, the participants selected were enrolled in the same mathematics class, which resulted in them having the same mathematics teacher (Teacher A). The participants' previous year (6^{th} grade) mathematics grades and the current year mathematics exams scores are relatively high comparing to classmates. Therefore the participants are five high achievers in their mathematics class.

Data Collection Tools

CET

The CET was administered to 116 seventh grade students in word and numeric formats with the same numbers of items. CET consists of 15 open-ended questions that combined computational estimation tasks from different research studies (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys, Rybolt, Bestgen & Wyatt, 1982). Reliability of the word and numeric formats of CET were examined in a pilot study. As a result some questions were revised for the main study. The Croanbach's Alpha of word format and numeric format tests were .76 and .78, respectively. In the CET there were equal numbers of whole numbers, decimals and fractions related questions (see some example from CET in Appendix).

Interview protocol. The first researcher collected data via an individual interview session of each participant. During the interview, students were asked each of the numeric questions of the CET, without anytime restrictions. Only numeric format of CET was used for interview because interviewer might warn to students to use estimation rather than exact computation. Students were asked to explain their reasoning and process used to generate the responses provided.

Procedure

A pilot study of the CET –word and numeric format, and interview protocol were conducted during the Fall of 2008. Based on the responses, possible themes for data analysis were identified. Subsequently, the data for this study were collected in Spring 2009 (February – April). We administered the CET-word format and CET-numeric format over a two-week period, to 116 seventh grade students. The 15 items from the CET-word format were administered first, and a week later the same classes were assessed using the CET-numeric format. Each question was projected on the wall one by one and for each question, 10 seconds was given in numeric format questions and 15 seconds for word format questions to students to find out estimated answers. Before starting the test, students were informed about the restricted time and were told, "To use time effectively, not to copy the problem but do the work in their heads." They were provided with answer sheets to record their answers. A week later, the 15 item numeric form of the CET was administered to 116 students in the same way by using an overhead projector.

Students received 1 point for each answer that was within a specified interval. The accepted interval ranged from 25% below the exact answer to 25% above the exact answer. Students' responses that were not in the specified interval received no points. Students' scores were totaled out of 15 possible points per test. Descriptive statistics of students' performance on the CET – word and numeric format were derived. Students who scored exceptionally high on both the CET word and numeric format were invited to participate in the study.

Two weeks later the five students who got the highest scores on both forms of the CET participated in the interview sessions. The interview sessions were video-recorded and transcribed verbatim. Interviews were conducted by two people: the first author who asked the estimation problems and follow up questions and an assistant graduate student who used the video camera. The first author spent some time with interviewees before the interviews to establish rapport with them.

The conducted interviews provided the means of learning about processes and strategies participants used to solve different estimation problems. Interviewees were asked to explain how they arrived at their estimate. The students were presented each question on a card, in the same order. No time limit was imposed, but students were instructed to estimate their answers rather than use algorithms to compute exact answers. Interviewees were asked to explain aloud their thoughts for finding estimated solutions. To ensure that the students were mentally active while solving the questions, no paper or pencil was provided for students during the interviews. The first researcher probed students for depth of understanding, and clarity. These interview sessions took approximately 30-45 minutes for each interviewee.

Data Analysis

Interview data were coded based on key words and phrases that were identified in the review of the literature and pilot study. The coding outline consisted of three main themes, namely; *reformulation, translation and compensation.* The themes and related codes were placed into a three-way matrix, which essentially involved crossing the three dimensions to see how they interact with each other (Miles & Huberman, 1994). Therefore, we placed interviewees, themes and codes into the theme-matrix, conducted a theme-oriented analysis, and subsequently expanded a more holistic case-oriented analysis. Each interviewee's transcription was carefully examined through the themes and codes individually to observe uniqueness, and then collectively in an effort to observe similarities and differences among participants.

The first author coded the interview transcriptions, which was validated by a second coder. The second coder was a researcher with expertise in mathematics education. A three round coding procedure was applied by two researchers. In the first round they were assigned to independently code one of the interviewee's transcripts. To increase the coding reliability, in the second round, they compared the coded transcripts and negotiated any discrepancies. Subsequently, in the last round common coding language was produced and five interviewee's transcriptions were coded according to constructed theme-matrix which is given in Table 2.

Table 2. *Theme-Matrix*

Participants	Main Themes	Codes
Deniz Sergen Mert Ayşe Nevzat	Reformulation	• Rule based rounding- numbers are rounded up when they end with five or more and they are rounded down when they end with less than five. • Situation based rounding-conducting rounding procedure depending on the context and situation of the problem. • Compatible numbers- a set of numbers that can be easily "fit together" • Truncation-changing the number with a lower form of itself.
	Translation	• Convert addition to multiplication-changing addition computation into multiplication • Convert division to fraction –changing division computation into fraction form
	Compensation	• Intermediate compensation-making arrangements about rounded numbers during the solution of the problem • Final compensation –making arrangements at the end of the solution about the approximate results

RESULTS

We present our findings around three themes: reformulation, translation and compensation. Reformulation is presented through the lens of rule based rounding, situation based rounding, compatible numbers and truncation. Translation is discussed via two basic codes (converting addition to multiplication and converting division to fraction). Finally compensation is examined by intermediate and final compensation.

Reformulation

According to Table 3, reformulation strategy was used in four different ways (rule based rounding, situation based rounding, compatible numbers and truncation) by the interviewees. All five students undoubtedly used reformulation at least once. Not surprisingly among the reformulation strategies rule based rounding is the most frequently used one (37 times).

Table 3. *Reformulation Strategy Table*

Reformulation Codes	How many times that are used (n)	Example
• rule based rounding	37	835.67-0.526→836-1
• situation based rounding	11	835.67-0.526→835-0.5
• compatible numbers	13	16. 272 ÷36→ 16÷36 → 16 is almost half of 36 so answer is 0,5
• Truncation	13	$3\frac{1}{2} \times 10\frac{1}{8} \rightarrow 3 \times 10$

Rule based rounding is a school taught rule which is conducted as the number is rounded up when it ends with five or more, and it is rounded down when it ends with a digit less than five. In the following excerpt Sergen who used the rule based rounding explained how he used it in question three(Q3: 7 465—572);

> ***Sergen:*** *In the first number "four hundred sixty five" should be rounded to "five hundred" (465 →500), since sixty-five (4**65**) is more than fifty, and in the second number seventy two (5**72**) should be rounded up (572 → 600) because of the same reason.*

Besides the rule based rounding, interviewees conducted compatible numbers and truncation which were the second most used way of reformulation (13 times). Compatible numbers is conducted by finding out the numbers which fit together. Sergen preferred to use it in question fifteen (Q15: 87 419 + 92 765 + 90 045 + 81 974 + 98 102). Interestingly, Sergen was the only interviewee who used compatible numbers strategy, the other interviewees chose situation based and rule based rounding. In the following excerpt, Sergen explains how he used the compatible numbers strategy;

> ***Sergen:*** *I round the first number, 87 419 to 87 000. Then the second one, 92 765 can be rounded to 93 000.*
>
> ***Researcher:*** *Why did you round these numbers like that?*
>
> ***Sergen:*** *Because, I want to get ten by adding first rounded number's (87 000) seven and the second rounded number's (93 000) three. Seven comes from at the end of the first number and three comes from at the end of the second number.*

Truncation, which is a bit of a different strategy from other reformulation strategies, is conducted by rounding the numbers only below. According to gathered data truncation appeared into two different ways of usage. These are "ignore fraction" and "ignore too small decimal". Considering Table 1, it has the same rate of usage (13 times) with the compatible numbers strategy (13 times). For example in question thirteen (Q13: $3\frac{1}{2}$ x $10\frac{1}{8}$), three of the five interviewees (Ayşe, Nevzat and Sergen) used ignore fraction as a truncation strategy. Ayşe explained her solution as follows;

Ayşe: *I could omit the fraction part of the numbers* ($3\frac{1}{2}$ and $10\frac{1}{8}$) *and the operation becomes 3 times 10. Therefore, the result is 30.*

Researcher: *Why did you omit the fraction parts, Ayşe?*

Ayşe: *Since they are very small factions, especially* $\frac{1}{8}$*. If I don't omit the first fraction* $\frac{1}{2}$*, I can round it to four* ($3\frac{1}{2} \rightarrow 4$) *and the result is forty now.*

Ignore too small decimal is another type of truncation strategy. Two of the five students solved question five (Q5: 0.7 + 0.002 + 0.81) by using ignore too small decimal strategy where the second decimal is ignored and not involved in the addition process. The following is an excerpt from an interview with Mert;

Mert: *The first number* (0.7) *is seen as 1 and also the third number* (0.81) *is seen as 1.*

Researcher: *Why do you think so?*

Mert: *Since they are very close to 1.Therefore, the addition is almost two.*

Researcher: *Ok. But what about the second decimal number* (0.002)?

Mert: *Oh yes, it doesn't need to be used in this addition.*

Researcher: *Why?*

Mert: *It is so small, almost zero. It does not have a big effect on the result.*

Among the reformulation strategies, situation based rounding was the least frequently used (11 times). However, Nevzat was one of the two students that were using the situation based rounding in the questions during the interview. For example, in question three (Q3: 7465—572), he used this strategy which he explained as shown in the following excerpt:

Nevzat: *The answer is seven thousand.*

Researcher: *How did you find?*

Nevzat: *I round the first number (7465) to seven thousand five hundred (7500).*

Researcher: *Why did you round like this?*

__Nevzat:__ Because when I round the first number (7465) to seven thousand five hundred it will consisted of five hundred (7500) and the second one (572) also has five hundred. At the end the operation becomes easier.

Translation

Besides reformulation strategies, interviewees used translation strategy for the computational estimation questions. According to Table 4, it is divided into two sub strategies; changing addition into multiplication and changing division into fraction. Among the interviewees only two students (Nevzat and Mert) performed the changing addition into multiplication strategy in all types of numbers among the questions.

Table 4. *Translation strategy table*

Translation Codes	How many times that are used (n)	Example
• convert addition to multiplication	5	87 419 + 92 765 + 90 045 + 81 974 + 98 102$\rightarrow$ 90 000 x 5
• convert division to fraction	6	$713 \div 8 \rightarrow \frac{720}{8}$

In the following excerpt, Mert explains his perspective for question one (Q1: 87 419 + 92 765 + 90 045 + 81 974 + 98 102). He changed the addition operation to multiplication after rounding processes.

***Mert:** The answer is five hundred thousand (500 000).*

***Researcher:** How did you find that?*

***Mert:** All numbers could be rounded to a hundred thousand (100 000) there are five of them. So the result is five hundred thousand (500 000).*

Changing division into fraction strategy was used in question eleven (Q11: 474 257 ÷ 8 127) by Nevzat. In question eleven, he performed rounding procedure for both of the numbers and then converted the division algorithm into the fraction version of the numbers. That is, he produced $\frac{480}{8}$ and then conducted the simplification of this fraction.

***Nevzat:** The first number (474 257) rounded to 470 000 and the second one is to 8 000. hımm. How can I divide these?*

***Researcher:** Why did you round the first number like this?*

***Nevzat:** Because it is closer to 470 000. But yeah.. It can also be rounded to 480 000 and I think it is more useful.*

***Researcher:** What does useful mean?*

***Nevzat:** When I removed the three zeros, it became 480 over 8. Then I could simplify this fraction to find the approximate answer.*

Compensation

The last and most sophisticated strategy was compensation, which means that rethinking the result of the question and making some changes to get a closer answer to exact one. It could be done while performing the operation or at the end of the operation. For example, one can round the first number and compensate it, approximately the same amount of truncating is performed to the other number; this type of compensation is called *intermediate compensation*. The other type is called the *final compensation* since the rounding or truncating the number is done at the end of the procedure. According to Table 5, among all strategies it was the least used and both sub strategies were used almost in the same amount by interviewees.

Table 5. *Compensation strategy table*

Compensation Codes	How many times that are used (n)	Example
• Intermediate compensation	4	835.67-0.526→835.67 →835.70-0.50
• Final compensation	3	835.67-0.526→836-0.5=835.5 a little bit less than 835.5

Actually, there were only three interviewees (Mert, Ayşe and Nevzat) that could use them in a few questions (Q1: 87 419 + 92 765 + 90 045 + 81 974 + 98 102, Q3: 7465—572, Q8: 713÷ 8, and Q11: 474 257 ÷ 8 127). Among these three interviewees, Mert was a competent strategy user, since he used compensation strategies in three questions; Q1 (31 x 68 x 296), Q8 (713÷ 8) and Q11 (474 257 ÷ 8 127). For example, in the following excerpt, Mert confidently applied the intermediate compensation strategy for Q1 (31 x 68 x 296);

Mert: *The result is approximately 600 000.*

Researcher: *Could you explain how did you find it?*

Mert: *First I get 2100, then multiply 20 by 3.*

Researcher: *Ok. But how did you find 2100?*

Mert: *Firstly, I rounded 68 to 70, and then rounded 296 to 300. I removed the three zeros while multiplication of 70 and 300 and get firstly 21 then added to three zeros at the end of 21. So I found 2100. After this step, I should multiply 2100 and 30. Thirty came from the first number (31). I rounded the 31 to 30. However, to compensate the result I chose the multiplication of 20x3 rather than the multiplication of 21x3. Then the answer is 600 000.*

Researcher: *Why did you need to compensate the result?*

Mert: *Actually, it is not so big deal, but when I rounded up all the numbers, the estimated result appeared bigger than the exact answer. Therefore to balance this interval between exact answer and estimated one I truncate the result of the first multiplication (70x30=2100). So that I can found an answer this was closer to exact result.*

Similar to Mert, Nevzat conducted intermediate compensation strategy only in Q3 (7465- 572) which is given in the following excerpt;

Nevzat: *The result is approximately 7000.*

Researcher: *How did you find the answer?*

Nevzat: *I rounded the first one (7465) to 7500. Since I rounded it up, the second one should be subtracted some to balance. Therefore, the second number (572) became 500. As a result, 7500 minus 500 is 7000.*

Among the three interviewees who used the compensation strategy, Mert was the one who used final compensation. The following excerpt is from the interview with Mert for Q8 (713÷ 8);

Mert: *The first number (713) could be 720. hımm. The multiplication of 8 there should be 72… Well, it is 9… yeah 90 but no, it should be 89.*

Researcher: *Why not 90?*

Mert: *Since the first number (713) rounded up so the result should be a bit smaller than 90, so it may be 89.*

Similarly, he gave the answer 3 for question four ($7\frac{1}{6} - 4\frac{1}{4}$) and applied final compensation. He was asked "Is 3 OK for the answer of the question?" he answered that "Actually, a bit less than three but it does not matter, since the remaining part is too small".

According to the interview data three people used the compensation in only three questions; the other two interviewees did not use this strategy in any questions. Among the strategy users Mert is the one who used it more frequently than the other two interviewees.

CONCLUSIONS AND DISCUSSIONS

In this study our aim was to discover seventh grade students' computational estimation strategies and how they explained their strategies. This research revealed that students used some common strategies to solve the given estimation questions. Their strategies were grouped into three main titles: reformulation, translation and compensation.

All five interviewees preferred to use reformulation strategy for all questions. This situation was also observed by Reys (1993) in his research study and he identified reformulation strategy as the most frequently observed strategy. The reason for this situation might be that this strategy is the only known estimation strategy by most of the students and also most of the teachers. In the current study, one of the interviewees was asked why he always used rounding in his estimation process and he explained that he was only taught a rounding strategy for estimation. In addition, some students thought estimation and rounding were the same things. A similar finding was observed in Reys' (1993) research study where students generally thought estimation and rounding were synonymous. Among our participants rule based rounding which was one of the reformulation strategies was used more often than others. The reason of this result might be students' dependency on exact computation and rules of mathematics. Because students thought that mathematics has strict rules and someone who made mathematics should follow these rules. According to Sowder-Threadgill (1984) some students could go so far as to use a combination of finger writing and visual imagery to perform the computation although they asked to estimate the question.

Translation was the next most used strategy and included the sub strategies of converting addition to multiplication and division to fraction. However, except one student, the other students who preferred to use translation strategy used it in one decimal and one whole number question. Reys et al. (1982) observed that translation is more flexible than reformulation. Although this flexibility let students use this strategy more often, in this study the seventh graders used translation strategies less frequently than reformulation strategies. The reason may be related with the requirement of the strategy which was explained by Reys et al. (1982), that is translation strategy needs an advanced level of conceptual knowledge of estimation and number sense. This strategy helps students convert operations to more manageable forms. To be able to convert the operations from one form to another, students should have good number sense. According to Yang (1995) good number sense leads to the use of multiple representations of numbers and use of translation. Reys and Yang (1998) maintain that number sense is one of the basics of using estimation and using different kinds of strategies. The proficiency of these topics improves estimation ability.

This study found compensation to be the least used strategy. Students who wanted to make their estimation closer to exact answers used either intermediate compensation or final compensation. Nevzat and Mert were the only two students who used them. Since the most sophisticated strategy is compensation (Reys, et al., 1982; Reys et al., 1991[a]; Sowder, 1992), it is used less frequently. Among the five interviewees, Mert was the only students who used a compensation strategy more than once. According to his answers Mert might have a high tolerance for error which is why he did not feel uncomfortable with some computational error. Although most of the participants seemed giving great importance to exact computation, three of them did not even try to adjust their estimation by using compensation strategy. We believe that it is related with the poor number sense ability because they could not realize how far from their estimated answer from the exact one. Although some interviewees were willing to use a compensation strategy, they were not very successful in its use. Their willingness may be related with their dependency on the exact results. For example, Sergen and Ayşe were reliant on the idea of finding exact results for any mathematical questions; they were not satisfied with estimated answers. They were both eager to use exact computation rather than using the computational estimation during the interviews for all questions. Even in one question where the estimated answer was asked to Ayşe, she said that "I could find the exact answer for the question, why do I have to find the approximate one?" Also she confessed that estimated answers made her uncomfortable since according to her, mathematics should give exact and clear results for all questions.

Gliner (1991) claimed that people more dependent on exact computation might have lower success on computational estimation. This claim supports the situation of Ayşe and Sergen, since they were not that much good estimators like Mert and Nevzat.

RECOMMENDATIONS

The specified strategies revealed that students preferred using mostly reformulation strategies. This may be related to the fact that these are the only taught strategies in the mathematics classes in Turkey. Especially, rule based rounding is emphasized by teachers more than other strategies. Therefore, teachers should give equal emphases to the computational estimation strategies. Moreover, teachers should use and appreciate the language of estimation in arithmetic classes. While doing this, they should not require too much precision. They should emphasize there is not one best estimate, but a range of acceptable estimates. At the same time, they should emphasize and give feedback about the reasonableness of the estimated answers. So that students understand that estimation could be acceptable in mathematical applications. Especially, compensation strategies should be emphasized in the classroom applications, since by using either final or intermediate compensation, students make adjustments to produce more precise estimates. Therefore improving compensation strategy might help students become better computation estimators. More time should be spent helping student understand the importance and power of compensating during the estimation process.

It might be desirable in a further study to investigate the strategies used by teachers and their performance on computational estimation. Their perspectives on computational estimation in mathematics classes also might be investigated by the researchers to understand what they think and feel about computational estimation. To what extent do their perspectives influence their students' strategy choice and use for estimation questions?

APPENDIX

Table. *Some questions from CET word and numeric formats*

	Numerical Format of CET	Words Format of CET(In Turkish)
Question 7	$16{,}272 \div 36$	Approximately how much garden mold does each flowerpots hold, when 16,272 liters of mold shared for 36 flowerpots?
Question 9	$14\frac{3}{4} \div \frac{5}{8}$	If the area of the kitchen wall is $14\frac{3}{4}$ cm^2 then approximately how many $\frac{5}{8}$ cm^2 sized wall tiles can be placed on that wall?
Question 8	$713 \div 8$	Burak is moving on another city and he wants to distribute 713 marbles to 8 of his friends. Approximately how many marbles does each friend will have?

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Yedinci Sınıfların Hesaplamalı Tahmin Stratejileri ile ilgili Bir Örnek Olay Çalışması

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ÖZET

Amaç ve Önem: Bu çalışmanın amacı ilköğretim yedinci sınıf öğrencilerinin hesaplamalı tahmin sorularını cevaplarken kullandıkları yöntem ve stratejileri incelemektir. Hesaplamalı tahmin becerisi 2005 yılından bu yana kullanılan yenilenmiş matematik öğretimi programında önemle vurgulanan bir beceridir. Çağımızda gelişen teknoloji yardımıyla hesaplamalarda kesin sonuçlara kolaylıkla ulaşmamız mümkün iken ulaşılan sonuçların mantıklılığı ve duruma uygunluğu konusunda insan beyninin yeteneklerine başvurmak gerekmektedir. Bu nedenle akıl yürütme ve tahmin becerileri matematik dersi içerisinde önemli bir yer kaplamaktadır. Önemi hem yurtdışı hem de yurtta yapılan çalışmalarla vurgulanmış bu beceriyi geliştirmeden önce öğrencilerin hâlihazırda tahmin gerektiren sorulara yaklaşımlarını ve bu soruları yaparken kullandıkları yöntemleri incelemek matematik eğitimi araştırmacıları için temel teşkil etmektedir.

Yöntem: Örnek durum çalışması olarak tasarlanan çalışmaya beş tane yedinci sınıf öğrenci katılmıştır. Bu öğrenciler kademeli olarak seçilmiştir. Birinci aşamada Muğla il merkezinde bulunan ilköğretim okullarından rastgele seçim yöntemleri ile bir okul belirlenmiş olup bu okulda okumakta olan 116 yedinci sınıf öğrencisine sayısal ve sözel olarak paralel form şeklinde hazırlanmış "Tahmin Beceri Testi" uygulanmıştır. Bu testlerin sonuçlarına göre 116 öğrenci sıralanmış ve ilk 7 öğrenci çalışmanın görüşme kısmı için seçilmiştir. Ancak görüşmeler sırasında 2 öğrencinin görüşme sorularına tam olarak cevap vermeyişi nedeniyle veri analizi aşamasında 5 öğrencinin sonuçları değerlendirmeye alınmıştır. 5 adet yedinci sınıf öğrenci ile bireysel görüşmeler yapılmış ve görüşmede "Tahmin Beceri Testi"nin sayısal soruları katılımcılara yeniden sorulmuştur. Klinik mülakat protokolü temel alınarak katılımcıların kullandığı yöntemler tespit edilmeye çalışılmıştır.

Bulgular: Çalışmanın verilerinden elde edilen sonuçlar üç başlık altında incelenmiştir. Birincisi "Sayıların Yeniden Düzenlenmesi" adlı strateji olup bu strateji kendi içinde üç alt başlıkta gözlemlenmiştir. Bunların içinde "kurallara bağlı yuvarlama yapma" stratejisi katılımcılar tarafından en sık kullanılan yöntem olmuştur. İkinci ana başlık olarak "İşlemlerin Yeniden Düzenlenmesi" stratejisi ele alınmış ve sayıların yeniden düzenlenmesi stratejisine göre daha nadir kullanıldığı tespit edilmiştir. "Toplamayı Çarpmaya Çevirme" ve "Bölmeyi Kesre Çevirme" olarak karşılaştığımız İşlemlerin Yeniden Düzenlenmesi stratejisi ise "Düzenleme-Düzeltme" stratejisine oranla ikinci sık kullanılan yöntem olmuştur. İşlemlerin ortasında düzenleme düzeltme ve işlemlerin sonunda düzenleme düzeltme yapan katılımcılar bu stratejiyi çok az sayıda kullanmışlardır. Katılımcılar tarafından sadece 7 kere kullanılan bu strateji diğer stratejilere göre tahmin becerisi daha iyi olan öğrencilerin tercih ettiği bir yöntem olmuştur.

Tartışma, Sonuç ve Öneriler: Elde edilen sonuçlara göre katılımcılar soruların cevaplarını tahmin ederek bulmaları istenildiğinde çoğunlukla Sayıları Yeniden Düzenleme stratejisini kullanmaktadır. Hatta tahmin ediniz denildiğinde ilk önce Kurallara Bağlı Yuvarlama yapmayı tercih etmektedirler. Bunun sebebi olarak matematik derslerinde tahmin gerektiren soruların az kullanılması ve tahmin etmek için kullanılabilecek diğer yöntemlerin ders içinde sıklıkla kullanılmaması öngörülebilir. Diğer dikkat çeken bulgulardan biri olan net cevaplara olan düşkünlük ise öğrencilerin matematiğe dair inançlarına ve hataya olan duyarlılıklarına bağlı olabilir. Öğrencilerin farklı tahmin stratejilerini kullanmaları ve özellikle "Düzenleme-Düzeltme Stratejisinde" daha iyi olmalarını sağlamak için matematik öğretmenlerinin tahmin gerektiren sorulara daha fazla zaman vermeleri ve sınıf içi etkinliklerinde farklı stratejiler kullanarak aritmetik sorularını ve problemleri hesaplamalı tahmin yöntemleri ile de çözmeleri gerekmektedir. 2

Bu araştırma birinci yazarın doktora tezine dayanmaktadır.

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