



The Relationship Between Middle School Students' Problem Posing Skills And Algebraic Thinking Levels¹

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Abstract

The purpose of this study, to determine problem-posing skills and algebraic thinking levels and whether there is a relationship between problem-posing skills and algebraic thinking levels. In this research, correlation was used from quantitative research methods. In the fall semester of the 2016-2017 academic year, 308 students (151 girls, 157 boys) attending 7th and 8th grade in three secondary schools in the province of Konya were participated in the research. In the research, to determine the algebraic thinking levels of students, developed by Hart et al. (1998) and "Algebraic Thinking Level Test" adapted to Turkish by Altun (2005) was used. In addition, to measure the problem-posing skills of the students, Problem Posing Test was used as a data collection tool. The Spearman's Rank Correlation coefficient technique was used to determine the relationship between problem-posing skills and algebraic thinking levels. As a result of the research, while there is an accumulated at level 0 and level 1 among 7th grade students, it is accumulated at level 2 and level 4 among 8th grade students. In addition, it was determined that there is a strong correlation between students' algebraic thinking level and problem-posing skills in the positive direction.

Keywords

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Ortaokul Öğrencilerinin Problem Kurma Becerileri İle Cebirsel Düşünme Düzeyleri Arasındaki İlişki

Öz

Bu çalışmanın amacı, ortaokul 7. ve 8. sınıf öğrencilerinin problem kurma becerilerini ve cebirsel düşünme düzeylerini belirleyerek aralarında bir ilişki olup olmadığını tespit etmektir. Çalışmada nicel araştırma yöntemlerinden korelasyon türü ilişkisel tarama modeli kullanılmıştır. Araştırmaya, Konya ili Akşehir ilçesinde bulunan, MEB'e bağlı üç ortaokulda, 7. ve 8. sınıf düzeyinden 308 öğrenci (151 kız, 157 erkek) katılmıştır. Araştırmada, öğrencilerin cebirsel ifadeleri anlama ve düşünme düzeylerini belirlemek amacıyla Hart vd. (1998) tarafından geliştirilen ve Altun (2005) tarafından Türkçe kullanıma uygun hale getirilen "Cebirsel Düşünme Düzeyi Testi (CDDT)" kullanılmıştır. Ayrıca öğrencilerin problem kurma becerisini ölçmek için problem kurmaya yönelik "Problem Kurma Testi (PKT)" veri toplama aracı olarak kullanılmıştır. Problem kurma becerileri ile cebirsel düşünme düzeyleri arasındaki ilişkiyi tespit etmek

Anahtar Kelimeler

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için Spearman Sıra Farkları korelasyon katsayısı tekniği kullanılmıştır. Araştırma sonucunda 7. sınıf öğrencilerinde Düzey 0 ile Düzey 1 seviyesinde yığılma yaşanırken, 8. sınıf öğrencilerinde Düzey 2 ve Düzey 4 seviyelerinde yığılma yaşanmıştır. Ayrıca öğrencilerin cebirsel düşünme düzeyleri ile problem kurma becerileri arasında pozitif yönde güçlü bir ilişki olduğu tespit edilmiştir.

Introduction

Problem posing from the first years of primary education is an activity frequently encountered by students. Problem posing is a problem-solving activity involving questions to be investigated or perceived for a given situation, and the creation of new problems. Problem posing; throughout the problem solving process, including problem rearrangement and pattern searching (Akay et al., 2006: 139-140). Algebraic thinking is the use of mathematical symbols and tools by extracting information from a situation, representing that information mathematically in words, diagrams, tables, graphs and equations; and interpreting and applying mathematical findings, testing conjectures, and identifying functional relationships, to the same situation and to new, related situations (Herbert and Brown, 1997: 123-124).

When the literature is examined, many researches on algebraic thinking and the development of algebraic thinking have been encountered (Kaya and Keşan, 2004; Erbaş et al., 2009; Steele and Johanning, 2004). In our country studies, in general, students' errors and misconceptions related to algebraic concepts have been investigated, but no studies have been found attribution to algebraic thinking level and problem posing. The focus of the research with these movements has been to determine the relationship between problem-posing skills and algebraic thinking levels of secondary school students.

What is the Problem Posing?

Problem posing defined as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems (Stoyanova and Ellerton, 1996: 518). According to NCTM (2000), problem posing; to create a new problem in a given situation or phrase. Thanks to problem-posing activity; learn mathematical reasoning, have the ability to explore mathematical situations and express mathematical situations properly (Akay et al., 2006: 145; Silver, 1994: 20).

According to Stoyanova and Ellerton (1996: 520), problem posing situations can be classified as free, semi-structured or structured:

1. *Free Problem-Posing Situations:* It is not given any problem situation to student here. Students are asked to set up problems that are appropriate to the natural situation given without limitation. In the case of free problem posing, students use a situation in daily life inside or outside the school and produce a new problem by producing some questions. Students are asked to create a new problem with incentives such as "create an easy or difficult problem", "arrange a suitable problem for math competitions or tests" or "generate a problem you want" (Akay, 2006: 85).

2. *Semi-Structured Problem-Posing Situations:* Semi-structured problem-posing situations involve giving the students unfinished problem structures, and asking them to describe what kind of problems could be created on the basis of the information given. The unfinished problem structures can be given either by a picture, equation, calculation or inequality (Stoyanova and Ellerton, 1996: 523).

3. *Structured Problem-Posing Situations:* Teachers develop specific problem solving strategies and ask their students to set up problems that they need to use in solving these strategies. As an example; "Last night, there was a party at your cousin's house and the doorbell rang 10 times. When the doorbell rang the first time, only one guest arrived. If there would be 3 more guests every bell rings, how many guests would have come to the house when the 10th doorbell rang? Using the information contained here is you can build as many problems as you can."

What is the Algebraic Thinking?

Algebra; using a number and a symbol, a mathematical expression that transforms the relationship or relationships studied into generalized equations (Akkaya and Durmuş, 2006: 13).

Algebraic thinking includes; problem solving, reasoning, using representations, understanding variables, expressing the meaning of symbolic representations, working with models for the development of mathematical ideas, transforming between representations (Kaf, 2007, as cited in Kaya et al., 2016: 143). Algebraic thinking is a special form of mathematical thinking, not limited only to algebraic studies (Çelik, 2007: 8). Algebraic thinking that implies symbols as a reflection of mental activity; establishing relationships between algebraic situations, manifesting thoughts through different and multiple representations, describing concrete, semi-concrete and abstract concepts in algebraic relations and represents reaching the conclusion by reasoning (Kaya and Keşan, 2014: 42).

The development of algebraic thinking is accelerated by abstract processes and consists of four successive levels (Hart et al., 1998, as cited in Altun, 2015: 292-293).

Level 1, finding a letter value by arithmetic operations, reaching a result by treating the letters as object names or the ability to finalizing a process without having to value these letters, despite the letters in the content of the problem.

Level 2, the part that is different from the first level, is a bit more complicated than the questions that belong to this level. Second-level students can solve more complex questions because they are accustomed to algebraic expressions.

Level 3, the letters are considered as an unknown and can be processed through these unknowns. It is difficult for a child who understands unknowns as an object to go to the right conclusion. Level 4 is like to the third level, but more complex expressions can be meaningful and results of operations can be reached. In these questions, students should perceive letters as unknowns, use them in an unknown relation or equation and see a letter as a representative of more than one number.

Method

The purpose of this study, to determine problem-posing skills and algebraic thinking levels and whether there is a relationship between problem-posing skills and algebraic thinking levels. Therefore in this research, correlation was used from quantitative research methods.

In the fall semester of the 2016-2017 academic year, 308 students attending 7th and 8th grade in three secondary schools in the province of Konya were participated in the research. In the research, to determine the algebraic thinking levels of students, developed by Hart et al. (1998) and "Algebraic Thinking Level Test" adapted to Turkish by Altun (2005) was used. In addition, to measure the problem-posing skills of the students, suitable for each grade level "Problem Posing Test" consisting of 3 sections and each section 2 sub-items was used as a data collection tool.

When the distribution of the algebraic thinking levels of the students was determined, it was required to have answered 2/3 of the questions of the related level correctly in the first stage. Secondly, considering that the levels of algebraic thinking have a sequential structure, it has been sought to be successful at earlier levels in order to assign the student to a level. Moreover, students who failed to answer 2/3 of the questions in the level 1 level were evaluated as level 0 (Altun, 2005, Kaş, 2010: 76, Yaprak Ceyhan, 2012: 68).

The data obtained from the application were analyzed using the SPSS 24.0 package program. For data analysis, descriptive statistical methods (frequency, percentage calculations), Mann Whitney U-Test and Spearman's Rank-Order correlation coefficient technique were used and significance (p) was tested at 0.05 level.

Results

1. Interpretation of Data Obtained from Problem Posing Test

The highest score to be taken from Problem Posing Test is 30 and the lowest score is 0. The mean score of Problem Posing Test was 15.77 and the median score was 17. In addition, mode 18 was determined for the scores of the test items.

The mean score of the students in the first part (A) of the Problem Posing Test was found to be 3,78 and the median score was 1. In this part, the mode of the scores obtained was determined as 0. In

the second part (B) of Problem Posing Test, the average score of the students was 3.85 and the median score was 4. The mode of the scores obtained in this section was determined as 6. The mean score of the students in the third part (C) of Problem Posing Test was found to be 8,14 and the median was found to be 9. The mode of the scores obtained in this section was determined as 12.

According to entire of the Problem Posing Test, 7 students (4%) of 7th grade and 22 students (14%) of 8th grade have received full marks from all questions, 8 students (5%) of 7th grade students and 12 students (7%) of the 8th grade students were not able to score any questions.

Table 1. Mann Whitney U-Test findings depending on class level Mann Whitney U-Test findings -Part A of problem posing test-

Group	N	Mean Rank	Sum of Ranks	U	p	z
7	156	139,81	21810,50	9564,500	,002	-3,045
8	152	169,58	25775,50			

According to the Mann Whitney U-Test results, there is a significant difference between the scores of 7th and 8th grade students in the part A of the Problem Posing Test ($U = 9564,500$ $p=0,002<0,05$ $z=-3,045$). According to the findings, in the part A consisting of the questions “create the appropriate problem for the given equation” 8th grade students are more successful. When the effect size is calculated for z value, $r = 0.17$ is reached. Cohen (1988), according to the effect size values determined, for the Mann Whitney U-Test $r < .20$ was considered as a low impact. This result shows that there is no big difference between the scores of 7th and 8th grade students in the part A of Problem Posing Test.

Table 2. Mann Whitney U-Test findings depending on class level Mann Whitney U-Test findings -Part B of problem posing test-

Group	N	Mean Rank	Sum of Ranks	U	p	z
7	156	139,29	21729,50	9483,500	,001	-3,249
8	152	170,11	25856,50			

According to the Mann Whitney U-Test results, there is a significant difference between the scores of 7th and 8th grade students in the part A of the Problem Posing Test ($U = 9483,500$ $p=0,001<0,05$ $z=-3,249$). According to the findings, in the part B consisting of the questions “create a similar problem to a given problem” 8th grade students are more successful. When the effect size is calculated for z value, $r = .18 < .20$ is reached. According to Cohen (1988) effect size values, this value was evaluated as low effect. This result shows that there is no big difference between the scores of 7th and 8th grade students in the part B of Problem Posing Test.

Table 3. Mann Whitney U-Test findings depending on class level Mann Whitney U-Test findings -Part C of problem posing test-

Group	N	Mean Rank	Sum of Ranks	U	p	z
7	156	145,71	22731,50	10485,500	,060	-1,879
8	152	163,52	24854,50			

According to the Mann Whitney U-Test results, there is no significant difference between the scores of the 7th and 8th grade students in the part C which consists of the questions that “create a problem that matches the shape or table” ($U = 10485,500$ $p=0,06>0,05$ $z=-1,879$).

2. Interpretation of Data Obtained from Algebraic Thinking Level Test

Levels of 7th grade students; 35% is Level 0, 24% is Level 1, 21% is Level 2, 16% is Level 3 and 3% is Level 4 and levels of 8th grade students; 14% is Level 0, 18% Level 1, 24% Level 2, 19% Level 3 and 25% Level 4. In all students, 25% is Level 0, 21% is Level 1, 22% is Level 2, 18% is Level 3 and 14% is Level 4.

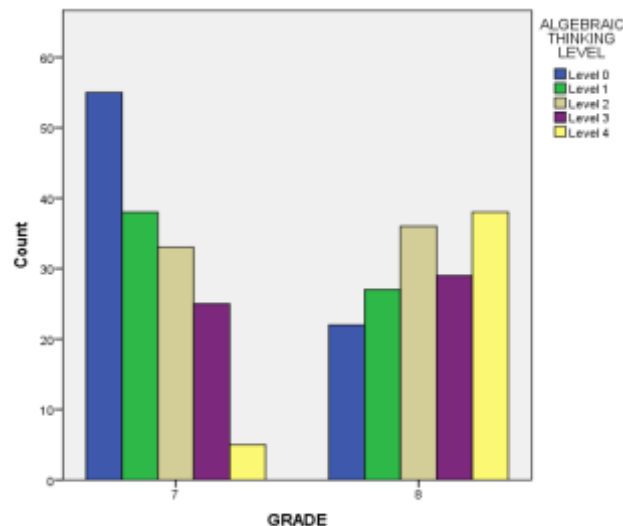


Figure 1. Distribution of algebraic thinking levels of students by class level

According to the Figure 1, while there is an accumulated at level 0 and level 1 among 7th grade students, it is accumulated at level 2 and level 4 among 8th grade students.

3. Correlation Test Results

The findings of “Is there a meaningful relationship between the level of algebraic thinking and the problem-posing test?” were reached with Spearman Rank Correlation coefficient test. The results are shown in Table 4.

Table 4. Spearman Rank correlation analysis results

Variables	N	Correlation Coefficient r	Sig. (2-tailed) p
Algebraic Thinking Level	308		
Problem Posing Test Total Score	308	,776**	,000

** Correlation is significant at the 0.01 level (2-tailed).

As a result of the research, according to Table 4, it was determined that there is a strong correlation between students' algebraic thinking level and problem-posing skills in the positive direction ($r(308) = .776, p < .05$).

Conclusions and Recommendations

The focus of this study was to determine algebraic thinking -a special form of mathematical thinking- and its relation to problem posing. According to the findings obtained from the level of algebraic thinking test, 35% of grade 7th students are in Level 0, 24% in Level 1, 21% in Level 2, 16% in Level 3, 3% in Level 4 level. In the 8th grade students, 14% is Level 0, 18% is Level 1, 24% is Level 2, 19% is Level 3 and 25% is Level 4. While there is a accumulated at level 0 and level 1 among 7th grade students, it is accumulated at level 2 and level 3 among 8th grade students. Considering that the development of algebraic thinking accelerates in the course of abstract moral operations (Altun, 2015: 285); it is expected that students at different levels of cognitive development will have different levels of algebraic thinking. This result Kaya et al. (2016), Oral et al. (2013) and Karşlıgil Ergin (2015) also coincides with the findings in their research.

Despite these findings, Gülpek (2006) found that the distribution of algebraic thinking levels in the 8th grade students was equal in the study called “The development of algebraic thinking levels of 7th and 8th grade students”. According to the results of Dikkartın and Mert Uyangör (2017) it was found that 21% of eighth grade students and 37% of seventh grade students reached Level 4.

In the research, a high rate of 35% of the 7th grade students was found to be at level 0 in terms of algebraic thinking. This result may be related to 7th grade students the meaning of the concept of variable and their difficulty in understanding the function of this concept because in this level the letters are not considered as an object and cannot make any sense of the letters as objects. This result is supported by research findings and theoretical information in the literature (Akgün, 2006; Dede and Argün, 2003).

According to the results of the study, there is a strong relationship between students' level of algebraic thinking and problem-posing skills. In the light of this, to development students' level of algebraic thinking, the field of algebra learning and problem-posing exercises can be used together to teach. In addition, problem-posing studies can be used to improve students' level of algebraic thinking. Thus, it can contribute to the increase in academic achievement of students.

In the next studies,

- ✓ This study can be repeated with a different study group in order to better predict the relationship between problem-building ability and level of algebraic thinking.
- ✓ The study can be expanded using mixed methods with qualitative and quantitative content.

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