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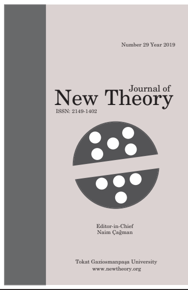

On Some Hyperideals in Ordered Semihypergroups

Abul Basar[1], Shahnawaz Ali[2], Mohammad Yahya Abbasi[3], Bhavanari Satyanarayana[4], Poonam Kumar Sharma[5]

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Abstract − In this paper, we study ordered hyperideals in ordered semihypergroups. Also, we study (m, n)-regular ordered semihypergroups in terms of ordered (m, n)-hyperideals. Furthermore, we obtain some ideal theoretic results in ordered semihypergroups.

Keywords − *Ordered semihypergroup, regular ordered semihypergroup, ordered bi-hyperideal, ordered (m, n)-hyperideal*

1. Introduction and Basic Definitions

The concept of the hypergroup introduced by the French Mathematician Marty at the 8th Congress of Scandinavian Mathematicians [1]. The concept of a semihypergroup is a generalization of the concept of a semigroup. Algebraic hyperstructures are a standard generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Many authors studied different aspects of semihypergroups, for instance, Davvaz [2], De Salvo et al. [3], Fasino and Freni [4], Gutan [5]. The monograph on application of hyperstructures to various area of study has been written by Corsini and Leoreanu [6]. Heideri and Davvaz studied ordered hyperstructures [7]. For semihypergroups, we refer [2, 8, 9]. Hila et al. studied quasi-hyperideals of ordered semihypergroups [10]. Corsini also studied hypergroup theory [11], [12]. Changphas and Davvaz [13] studied properties of hyperideals in ordered semihypergroups. Most recently, Basar et al. [14–16] investigated different types of hyperideals in ordered hypersemigroups, ordered LA-Γ-semigroups and LA-Γ-semihypergroups.

Let H be a nonempty set, then the mapping $\circ : H \times H \to H$ is called hyperoperation or join operation on H, where $P^\star(H) = P(H) \setminus \{0\}$ is the set of all nonempty subsets of H. Let A and B be two nonempty sets. Then, a hypergroupoid $(S, \circ)$ is called a semihypergroups if for every $x, y, z \in S$,

$$x \circ (y \circ z) = (x \circ y) \circ z$$

i.e.,

$$\bigcup_{u \in y \circ z} x \circ u = \bigcup_{v \in x \circ y} v \circ z$$

[1]basar.jmi@gmail.com (Corresponding Author);
[2]shahnawazali57@gmail.com; [3]mabbasi@jmi.ac.in [4]bhavanari2002@yahoo.co.in [5]pksharma@davjalandhar.com
[1,2]Department of Natural and Applied Science, Glocal University, Saharanpur(UP)-247 121, India
[3]Department of Mathematics, Jamia Millia Islamia, New Delhi-110 025, India
[4]Department of Mathematics, Acharya Nagarjuna University, Guntur, Andhra Pradesh-522 510, India
[5]Department of Mathematics, D. A. V. College, Jalandhar, Punjab-144 008, India

A semihypergroup $(S, \circ)$ together with a partial order " $\leq$ " on S that is compatible with semihypergroup operation such that for all $x, y, z \in S$, we have

$$x \leq y \Rightarrow z \circ x \leq z \circ y$$

and

$$x \circ z \leq y \circ z$$

is called an ordered semihypergroup. For subsets A, B of an ordered semihypergroup S, the product set $A \circ B$ of the pair (A, B) relative to S is defined as below:

$$A \circ B = \{a \circ b : a \in A, b \in B\}$$

and for $A \subseteq S$, the product set $A \circ A$ relative to S is defined as $A^2 = A \circ A$. For $M \subseteq S$, $(M] = \{s \in S \mid s \leq m$ for some $m \in M\}$. Also, we write $(s]$ instead of $(\{s\}]$ for $s \in S$. Let $A \subseteq S$. Then for a non-negative integer m, the power of A is defined by $A^m = A \circ A \circ A \circ A \cdots$, where A occurs m times. Note that the power vanishes if $m = 0$. So, $A^0 \circ S = S = S \circ A^0$. In what follows we denote ordered semihypergroup $(S, \leq)$ by S unless otherwise specified.

Suppose S is an ordered semihypergroup and I is a nonempty subset of S. Then, I is called an ordered right (resp. left) hyperideal of S if

(i) $I \circ S \subseteq I (resp.\ S \circ I \subseteq I)$

(ii) $a \in I, b \leq a$ for $b \in S \Rightarrow b \in I$

Definition 1.1. Suppose B is a sub-semihypergroup (resp. nonempty subset) of an ordered semihypergroup S. Then B is called an (resp. generalized) (m, n)-hyperideal of S if (i) $B^m \circ S \circ B^n \subseteq B$, and (ii) for $b \in B$, $s \in S$, $s \leq b \Rightarrow s \in B$.

Note that in the above Definition 1.1, if we set $m = n = 1$, then B is called a (generalized) bi-hyperideal of S.

Definition 1.2. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and m, n are nonnegative integers. Then S is called (m, n)-regular if for any $s \in S$, there exists $x \in S$ such that $s \leq s^m \circ x \circ s^n$. Equivalently: $(S, \circ, \leq)$ is (m, n)-regular if $s \in (s^m \circ S \circ s^n]$ for all $s \in S$.

2. Preliminary

We begin with the following:

Lemma 2.1. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and $s \in S$. Let m, n be non-negative integers. Then, the intersection of all ordered (generalized) (m, n)-hyperideals of S containing s, denoted by $[s]_{m,n}$, is an ordered (generalized) (m, n)-hyperideal of S containing s.

Proof. Let $\{A_i : i \in I\}$ be the set of all ordered (generalized) (m, n)-hyperideals of S containing s. Obviously, $\bigcap_{i \in I} A_i$ is a sub-semihypergroup of S containing s. Let $j \in I$. As, $\bigcap_{i \in I} A_i \subseteq A_j$, we have

$$\begin{aligned} (\bigcap_{i \in I} A_i)^m \circ S \circ (\bigcap_{i \in I} A_i)^n &\subseteq A_j^m \circ S \circ A_j^n \\ &\subseteq A_j \end{aligned}$$

Therefore, $(\bigcap_{i \in I} A_i)^m \circ S \circ (\bigcap_{i \in I} A_i)^n \subseteq \bigcap_{i \in I} A_i$ as $\bigcap_{i \in I} A_i$ is a sub-semihypergroup of S containing s. Let $a \in \bigcap_{i \in I} A_i$ and $b \in S$ so that $b \leq a$. Therefore, $b \in \bigcap_{i \in I} A_i$. Hence, $\bigcap_{i \in I} A_i$ is an ordered (generalized) (m, n)-hyperideal of S containing s.

Theorem 2.2. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and $s \in S$. Then, we have the following:

(i) $[s]_{m,n} = (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$ for any positive integers m, n

(ii) $[s]_{m,0} = (\bigcup_{i=1}^{m} s^i \cup s^m \circ S]$ for any positive integer m

(iii) $[s]_{0,n} = (\bigcup_{i=1}^{n} s^i \cup s^n]$ for any positive integer n

Proof. (i) $(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n] \neq \emptyset$. Let $a, b \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$ be such that $a \leq x$ and $b \leq y$ for some $x, y \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$. If $x, y \in s^m \circ S \circ s^n$ or $x \in \bigcup_{i=1}^{m+n} s^i$, $y \in s^m \circ S \circ s^n$ or $x \in s^m \circ S \circ s^n$, $y \in \bigcup_{i=1}^{m+n} s^i$, then

$$x \circ y \subseteq s^m \circ S \circ s^n$$

and therefore,

$$x \circ y \subseteq \bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n$$

It follows that $a \circ b \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$. Let $x, y \in \bigcup_{i=1}^{m+n} s^i$. Then, $x = s^p, y = s^q$ for some $1 \leq p, q \leq m + n$.
Now two cases arise: If $1 \leq p + q \leq m + n$, then $x \circ y \subseteq \bigcup_{i=1}^{m+n} s^i$.
If $m + n < p + q$, then $x \circ y \subseteq s^m \circ S \circ s^n$. So, $x \circ y \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$. This implies that $(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$ is a sub-semihypergroup of S. Moreover, we have

$$\begin{aligned}
(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^m \circ S &= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S] \circ S \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \circ S \cup s^m \circ S \circ S] \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (s \circ S] \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S] \circ (s \circ S] \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-2} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ (s \circ S]] \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-2} \circ (s^2 \circ S] \\
&\vdots \\
&\subseteq (s^m \circ S]
\end{aligned}$$

In a similar fashion, we have

$$S \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^n \subseteq (S \circ s^n]$$

Therefore,

$$\begin{aligned}
(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^m \circ S \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^n &\subseteq (s^m \circ S \circ s^n] \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]
\end{aligned}$$

So, $(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$ is an (m, n)-hyperideal of S containing s; hence, $[s]_{m,n} \subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$.

For the reverse inclusion, suppose $a \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$ is such that $a \leq t$ for some $t \in (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$. If $t = s^j$ for some $1 \leq j \leq m + n$, then $t \in [s]_{m,n}$, therefore, $a \in [s]_{m,n}$. If $t \in s^m \circ S \circ s^n$, by

$$s^m \circ S \circ s^n \subseteq ([s]_{m,n})^m \circ S \circ ([s]_{m,n})^n \subseteq [s]_{m,n}$$

then $t \in [s]_{m,n}$; hence, $a \in [s]_{m,n}$. This implies that $(\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n] \subseteq [s]_{m,n}$. Hence, $[s]_{m,n} = (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]$.
(ii) and (iii) can be proved in a similar fashion.

Lemma 2.3. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and $s \in S$. Suppose m, n are positive integers. Then, we have the following:

(i) $([s]_{m,0})^m \circ S \subseteq (s^m \circ S]$

(ii) $S \circ ([s]_{0,n})^n \subseteq (S \circ s^n]$

(iii) $([s]_{m,n})^m \circ S \circ ([s]_{m,n})^n \subseteq (s^m \circ S \circ s^n]$

Proof. (i)Using Theorem 2.2, we have

$$\begin{aligned}
([s]_{m,0})^m \circ S &= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^m \circ S \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S] \circ S \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \circ S \cup s^m \circ S \circ S] \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S]^{m-1} \circ (s \circ S] \\
&\vdots \\
&\subseteq (s^m \circ S]
\end{aligned}$$

Hence, $([s]_{m,0})^m \circ S \subseteq (s^m \circ S]$. (ii) can be proved similarly as (i).
(iii) Applying Theorem 2.2, we have

$$\begin{aligned}
([s]_{m,n})^m \circ S &= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^m \circ S \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n] \circ S \\
&\subseteq (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^{m-1} \circ (\bigcup_{i=1}^{m+n} s^i \circ S \cup s^m \circ S \circ s^n \circ S] \\
&= (\bigcup_{i=1}^{m+n} s^i \cup s^m \circ S \circ s^n]^{m-1} \circ (s \circ S] \\
&\vdots \\
&= (s^m \circ S]
\end{aligned}$$

Therefore, $([s]_{m,n})^m \circ S \subseteq (s^m \circ S]$. In a similar way, $S \circ ([s]_{m,n})^n \subseteq (S \circ s^n]$. Therefore,

$$\begin{aligned}
([s]_{m,n})^m \circ S \circ ([s]_{m,n})^n &\subseteq (s^m \circ S] \circ ([s]_{m,n})^n \\
&\subseteq (s^m \circ (S \circ ([s]_{m,n})^n)] \\
&\subseteq (s^m \circ (S \circ s^n]] \\
&\subseteq (s^m \circ S \circ s^n]
\end{aligned}$$

Hence, (iii) holds.

Theorem 2.4. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and m, n are positive integers. Let $\mathcal{R}_{(m,0)}$ and $\mathcal{L}_{(0,n)}$ be the set of all ordered $(m, 0)$-hyperideals and the set of all ordered $(0, n)$-hyperideals of S, respectively. Then:

(i) S is $(m, 0)$-regular if and only if for all $R \in \mathcal{R}_{(m,0)}, R = (R^m \circ S]$

(ii) S is $(0, n)$-regular if and only if for all $L \in \mathcal{L}_{(0,n)}, L = (S \circ L^n]$

Proof. (i) Suppose S is $(m, 0)$-regular. Then,

$$\forall s \in S, s \in (s^m \circ S]. \tag{1}$$

Suppose $R \in \mathcal{R}_{(m,0)}$. As, $R^m \circ S \subseteq R$ and $R = (R]$, we have $(R^m \circ S] \subseteq R$. If $s \in R$, by (1), we obtain $s \in (s^m \circ S] \subseteq (R^m \circ S]$, therefore, $R \subseteq (R^m \circ S]$. So, $(R^m \circ S] = R$.
Conversely, suppose

$$\forall R \in \mathcal{R}_{(m,0)}, R = (R^m \circ S] \tag{2}$$

Suppose $s \in S$. Therefore, $[s]_{m,0} \in \mathcal{R}_{(m,0)}$. By (2), we obtain

$$[s]_{m,o} = (([s]_{m,0})^m \circ S]$$

Applying Lemma 2.3, we obtain

$$[s]_{m,o} \subseteq (s^m \circ S]$$

Therefore, $s \in (s^m \circ S]$. Hence, S is $(m, 0)$-regular.
(ii) It can be proved analogously.

Theorem 2.5. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and m, n are non-negative integers. Suppose $\mathcal{A}_{(m,n)}$ is the set of all ordered (m, n)-hyperideals of S. Then,

$$S \ is \ (m, n) - regular \Longleftrightarrow \forall A \in \mathcal{A}_{(m,n)}, A = (A^m \circ S \circ A^n] \tag{3}$$

Proof. Consider the following four conditions:
Case(i): $m = 0$ and $n = 0$. Then (3) implies
S is (0, 0)-regular $\Longleftrightarrow \forall A \in \mathcal{A}_{(0,0)}, A = S$ because $\mathcal{A}_{(0,0)} = \{S\}$ and S is (0, 0)-regular.
Case (ii): $m = 0$ and $n \neq 0$. Therefore, (3) implies
S is (0, n)-regular$\Longleftrightarrow \forall A \in \mathcal{A}_{(0,n)}, A = (S \circ A^n]$. This follows by Theorem 2.4(ii).
Case (iii): $m \neq 0$ and $n = 0$. This can be proved applying Theorem 2.4(i).
Case (iv): $m \neq 0$ and $n \neq 0$. Suppose S is (m, n)-regular. Therefore,

$$\forall s \in S, s \in (s^m \circ S \circ s^n] \tag{4}$$

Let $A \in \mathcal{A}_{(m,n)}$. As $A^m \circ S \circ A^n \subseteq A$ and $A = (A]$, we obtain $(A^m \circ S \circ A^n] \subseteq A$. Suppose $s \in A$. Applying (4), $s \in (s^m \circ S \circ s^n] \subseteq (A^m \circ S \circ A^n]$. Therefore, $A \subseteq (A^m \circ S \circ A^n]$. Hence, $A = (A^m \circ S \circ A^n]$.
Conversely, suppose $A = (A^m \circ S \circ A^n]$ for all $A \in \mathcal{A}_{(m,n)}$. Suppose $s \in S$. As $[s]_{m,n} \in \mathcal{A}_{(m,n)}$, we have

$$[s]_{m,n} = (([s]_{m,n})^m \circ S \circ ([s]_{m,n})^n]$$

Applying Lemma 2.3(iii), we obtain $[s]_{m,n} \subseteq (s^m \circ S \circ s^n]$, therefore, $s \in (s^m \circ S \circ s^n]$. Hence, S is (m, n)-regular.

Theorem 2.6. Suppose $(S, \circ, \leq)$ is an ordered semihypergroup and m, n are nonnegative integers. Suppose $\mathcal{R}_{(m,0)}$ and $\mathcal{L}_{(0,n)}$ is the set of all $(m, 0)$-hyperideals and $(0, n)$-hyperideals of S, respectively. Then,

$$\begin{aligned} S \ is \ (m, n)\text{-regular ordered semihypergroup} \Longleftrightarrow \forall \mathrm{R} \in \mathcal{R}_{(\mathrm{m},0)} \forall \mathrm{L} \in \mathcal{L}_{(0,\mathrm{n})}, \\ R \cap L = (R^m \circ L \cap R \circ L^n] \end{aligned} \tag{5}$$

Proof. Consider the following four cases:
Case (i): $m = 0$ and $n = 0$. Therefore, (5) implies
S is (0, 0)-regular $\Longleftrightarrow \forall R \in \mathcal{R}_{(0,0)} \forall L \in \mathcal{L}_{(0,0)}, R \cap L = (L \cap R]$ because $\mathcal{R}_{(0,0)} = \mathcal{L}_{(0,0)} = \{S\}$ and S is $(0, 0)$-regular.
Case (ii): $m = 0$ and $n \neq 0$. Therefore, (5) implies S is $(0, n)$-regular $\Longleftrightarrow \forall R \in \mathcal{R}_{(0,n)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L = (L \cap R \circ L^n]$. Suppose S is $(0, n)$-regular. Suppose $R \in \mathcal{R}_{(0,0)}$ and $L \in \mathcal{L}_{(0,n)}$. By Theorem 2.4(ii), $L = (S \circ L^n]$. As $R \in \mathcal{R}_{(0,0)}$, we have $R = S$, therefore, $R \cap L = L$. Therefore,

$$(L \cap R \circ L^n] = (L \cap S \circ L^n] = ((S \circ L^n] \cap S \circ L^n] = (S \circ L^n] = L = R \cap L$$

Conversely, suppose

$$\forall R \in \mathcal{R}_{(0,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L = (L \cap R \circ L^n]. \tag{6}$$

If $R \in \mathcal{R}_{(0,0)}$, then $R = S$. If $L \in \mathcal{L}_{(0,n)}$, $S \circ L^n \subseteq L$ and $L = (L]$. Therefore, (6) implies

$$\forall L \in \mathcal{L}_{(0,n)}, L = (S \circ L^n]$$

Applying Theorem 2.4(ii), S is $(0,n)$-regular.
Case (iii): $m \neq 0$ and $n = 0$. This can be proved as before.
Case (iv): $m \neq 0$ and $n \neq 0$. Suppose that S is (m,n)-regular. Suppose $R \in \mathcal{R}_{(m,0)}$ and $L \in \mathcal{L}_{(0,n)}$. To prove that $R \cap L \subseteq (R^m \circ L] \cap (R \circ L^n]$, suppose $s \in R \cap L$. We have

$$s \in (s^m \circ S \circ s^n] \subseteq (s^m \circ L] \subseteq (R^m \circ L]$$

and

$$s \in (s^m \circ S \circ s^n] \subseteq (R \circ s^n] \subseteq (R \circ L^n]$$

Hence, $R \cap L \subseteq (R^m \circ L] \cap (R \circ L^n]$. As

$$(R^m \circ L] \subseteq (R^m \circ S] \subseteq (R] = R$$

and

$$(R \circ L^n] \subseteq (S \circ L^n] \subseteq (L] = L$$

This implies that $(R^m \circ L] \cap (R \circ L^n] \subseteq R \cap L$, therefore, $R \cap L = (R^m \circ L] \cap (R \circ L^n]$.
Conversely, suppose

$$\forall R \in \mathcal{R}_{(m,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L = (R^m \circ L \cap R \circ L^n] \tag{7}$$

Suppose $R = [s]_{m,0}$ and $L = S$. Applying (7), we obtain $[s]_{m,0} \subseteq (([s]_{m,0})^m \circ S]$. Applying Lemma 2.3, we obtain

$$[s]_{m,0} \subseteq (s^m \circ S] \tag{8}$$

In a similar fashion, we obtain

$$[s]_{0,n} \subseteq (S \circ s^n] \tag{9}$$

As $R^m \subseteq R$ and $L^n \subseteq L$, by (7), we have

$$\forall R \in \mathcal{R}_{(m,0)} \forall L \in \mathcal{L}_{(0,n)}, R \cap L \subseteq (R \circ L]$$

As $(s^m \circ S] \in \mathcal{R}_{(m,0)}$ and $(S \circ s^n] \in \mathcal{L}_{(0,n)}$, we obtain

$$(s^m \circ S] \cap (S \circ s^n] \subseteq ((s^m \circ S] \circ (S \circ s^n]] \subseteq (s^m \circ S \circ s^n]$$

Applying (8) and (9), we obtain

$$[s]_{m,0} \cap [s]_{0,n} \subseteq (s^m \circ S \circ s^n]$$

Hence, S is (m,n)-regular.

3. Conclusion

In this article, we investigated ordered hyperideals in ordered semihypergroups. Also, we studied (m,n)-regular ordered semihypergroups in terms of ordered (m,n)-hyperideals. Moreover, we characterized ordered semihypergroups by some results based on ideal theory.

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