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Convex and Concave Sets Based on Soft Sets and Fuzzy Soft Sets

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Abstract — In this study, after given some basic definitions of soft sets and fuzzy soft sets we firstly define convex-concave soft sets. Then, we investigate their properties and give some relations between convex and concave soft sets. Furthermore, we define fuzzy convex-concave soft sets and give some properties for the sets.

Keywords — *Fuzzy set, soft sets, convex sets, concave sets, strictly convex, strongly convex*

1. Introduction

In 1999, Molodtsov [1] proposed a completely new approach so-called *soft set theory* for modeling vagueness and uncertainty which may not be successfully modeled by the classical mathematics, probability theory, fuzzy sets [2], rough sets [3], and other mathematical tools. In the last decade, properties and applications on the soft set theory solidly enriched (e.g. [4–12]), including the extension of soft set theory (e.g. [13–25]). Along with them, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets, rough sets, intuitionistic fuzzy sets, vague sets, interval-valued fuzzy sets (e.g. [26–32]). Then, A method with unknown data in soft sets and in fuzzy soft sets is introduced by Deng and Wang [33], Gong et al. [34] gave two parameters reduction algorithms, Yang et al. [35] proposed the concept of multi-fuzzy soft sets with a few operations, Mao et al. [36] gave multi-experts group decision making problems by using intuitionistic fuzzy soft matrices, Feng and Lie [37] studied subsets and various relations deal with soft set theory, Wang et al. [38] built a new decision-making method by introducing the concept of fuzzy soft sets for the virtual machine startup problems, Agarwal et al. [39] introduced a new score function, similarity measure, relations with applications for generalized intuitionistic fuzzy sets.

Different definitions of convex fuzzy and concave fuzzy sets have defined but the first definition of convex fuzzy sets introduced by Zadeh [2] and then concave fuzzy sets introduced by Chaudhuri [40]. After Zadeh [2], concavoconvex fuzzy sets proposed by Sarkar [41], with some properties. Moreover, works on convex (concave)fuzzy sets in theories and applications has been progressing rapidly by many autor, for example, [42–47].

Convex and concave fuzzy sets play important roles in optimization theory. A significant definition of convex fuzzy sets introduced by Zadeh [2] and concave fuzzy sets introduced by Chaudhuri [40]. The concavoconvex fuzzy sets proposed by Sarkar [41] which is convex and concave fuzzy sets together conceived by combining. The works on convex and concave fuzzy sets, in theories and applications, have been progressing rapidly (e.g. [42, 46, 47]).

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The present expository paper is a condensation of part/extension of the dissertation [48]. In this work, we introduce the soft and fuzzy soft version of fuzzy convex and concave sets and also investigate their some properties. The plan of the paper is as follows. In Section 2, we give some notations, definitions used throughout the paper In section 3, after we give convex soft sets, we define strictly convex soft sets and strongly convex soft sets and then give desired some properties. In Section 4, we define fuzzy soft convex sets and fuzzy soft concave sets and then we show some properties.

2. Preliminary

In this section, we present the basic definitions and some operations of fuzzy sets [2], soft set theory [1] and fuzzy soft set [26]. More detailed explanations related to this subsection may be found in [1, 2, 7, 26, 30].

Throughout this paper E will denote the n-dimensional Euclidean space R^n. U denotes the arbitrary set, I denotes the interval $[0,1]$, and I° denotes $(0,1)$.

Definition 2.1. [2] Let U be the universe. Then, a fuzzy set X over U is defined by a set of ordered pair

$$X = \{(\mu_X(x)/x) : x \in U\}$$

where

$$\mu_X : U \to [0,1]$$

is called membership function of X. The value $\mu_X(x)$ is called the membership value or the grade of membership of $x \in U$. The membership value represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [41] A fuzzy set in R^n is defined to be convex if for all $p, q \in R^n$ and all r on the line segment $\overline{pq}$ the following condition with respect to its characteristic function μ is satisfied:

$$\mu(r) \geq min\{\mu(p), \mu(q)\}$$

Conversely, a fuzzy set in R^n is defined to be concave if for $p, q \in R^n$ and all r on the line segment $\overline{pq}$ the following condition with respect to its characteristic function μ is satisfied:

$$\mu(r) \leq max\{\mu(p), \mu(q)\}$$

Definition 2.3. [1] Let U be a universe, $P(U)$ be the power set of U and E be a set of parameters that are describe the elements of U. A soft set S over U is a set defined by a set valued function f_S representing a mapping

$$f_S : E \to P(U)$$

It is noting that the soft set is a parametrized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$S = \{(x, f_S(x)) : x \in E\}$$

Here, f_S is called approximate function of the soft set S and $f_S(x)$ is called x-approximate value of $x \in E$. The subscript S in the f_S indicates that f_S is the approximate function of S.

Generally, f_S, f_T, f_V, ... will be used as an approximate functions of S, T, V, ..., respectively.

Note that if $f_S(x) = \emptyset$, then the element $(x, f_S(x))$ is not appeared in S.

Definition 2.4. [7] Let S and T be two soft sets. Then,

1. If $f_S(x) = \emptyset$ for all $x \in E$, then S is called a empty soft set, denoted by S_Φ.

2. If $f_S(x) \subseteq f_T(x)$ for all $x \in E$, then S is a soft subset of T, denoted by $S \tilde{\subseteq} T$.

3. Complement of S is denoted by $S^{\tilde{c}}$. Its approximate function $f_{S^{\tilde{c}}}$ is defined by

$$f_{S^{\tilde{c}}}(x) = U \setminus f_S(x) \quad \text{for all } x \in E$$

4. Union of S and T is denoted by $S \tilde{\cup} T$. Its approximate function $f_{S \tilde{\cup} T}$ is defined by

$$f_{S \tilde{\cup} T}(x) = f_S(x) \cup f_T(x) \quad \text{for all } x \in E$$

5. Intersection of S and T is denoted by $S\tilde{\cap}T$. Its approximate function $f_{S\tilde{\cap}T}$ is defined by

$$f_{S\tilde{\cap}T}(x) = f_S(x) \cap f_T(x) \quad \text{for all } x \in E$$

Definition 2.5. [8] Let S be a soft set over U and α be a subset of U. Then, α-inclusion of the soft set S, denoted by S^α, is defined as

$$S^\alpha = \{x \in E : f_S(x) \supseteq \alpha\}$$

Definition 2.6. [26] Let U be an initial universe, $F(U)$ be all fuzzy sets over U. E be the set of all parameters and $A \subseteq E$. An fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows,

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}$$

where $\gamma_A : E \to F(U)$ such that $\gamma_A(x) = \emptyset$ if $x \notin A$, and for all $x \in E$

$$\gamma_{A(x)} = \{\mu_{\gamma_{A(x)}}(u)/u : u \in U, \mu_{\gamma_{A(x)}}(u) \in [0,1]\}$$

is a fuzzy set over U.

The subscript A in the γ_A indicates that γ_A is the approximate function of Γ_A.
Note that if $\gamma_A(x) = \emptyset$, then the element $(x, \gamma_A(x))$ is not appeared in Γ_A.

Definition 2.7. [26] Let Γ_A and Γ_B be two fuzzy soft sets. Then,

1. If $\gamma_A(x) = \emptyset$ for all $x \in E$, then Γ is called a empty fuzzy soft set, denoted by Γ_Φ.
2. Complement of Γ_A is denoted by $\Gamma_A^{\tilde{c}}$. Its approximate function $\gamma_{A^{\tilde{c}}}$ is defined by

$$\gamma_{A^{\tilde{c}}}(x) = \gamma_A^c(x), \text{ for all } x \in E$$

3. Union of Γ_A and Γ_B is denoted by $\Gamma_A \tilde{\cup} \Gamma_B$. Its fuzzy approximate function $\gamma_{A\tilde{\cup}B}$ is defined by

$$\gamma_{A\tilde{\cup}B}(x) = \gamma_A(x) \cup \gamma_B(x) \quad \text{for all } x \in E$$

4. Intersection of Γ_A and Γ_B is denoted by $\Gamma_A \tilde{\cap} \Gamma_B$. Its fuzzy approximate function $\gamma_{A\tilde{\cap}B}(x)$ is defined by

$$\gamma_{A\tilde{\cap}B}(x) = \gamma_A(x) \cap \gamma_B(x) \quad \text{for all } x \in E$$

5. Γ_A is an fuzzy soft subset of Γ_B, denoted by $\Gamma_A \tilde{\subseteq} \Gamma_B$, if $\gamma_A(x) \subseteq \gamma_B(x)$ for all $x \in E$.

3. Convex Soft sets

In this section, after we give convex soft sets, we define strictly convex soft sets and strongly convex soft sets and then give desired some properties. Some of it is quoted from [2, 40–42, 46–48].

Definition 3.1. The soft set S on E is called a convex soft set, is shown in Figure 1, if

$$f_S(ax + (1-a)y) \supseteq f_S(x) \cap f_S(y)$$

for every $x, y \in E$ and $a \in I$.

Definition 3.2. The soft set S on E is called a concave soft set if

$$f_S(ax + (1-a)y) \subseteq f_S(x) \cup f_S(y)$$

for every $x, y \in E$ and $a \in I$.

Definition 3.3. The soft set S on E is called a strongly convex soft set if

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y)$$

for every $x, y \in E$, $x \neq y$ and $a \in I^\circ$.

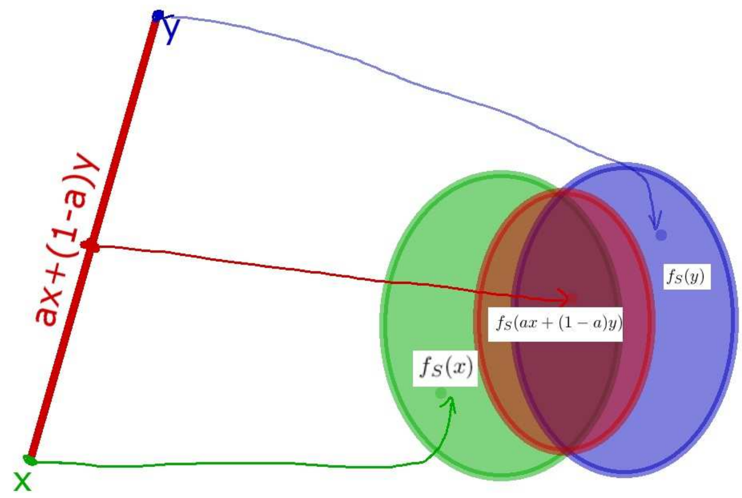

Fig. 1. The convex soft set

Definition 3.4. The soft set S on E is called a strictly convex soft set if

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y)$$

for every $x, y \in E$, $f_S(x) \neq f_S(y)$ and $a \in I^\circ$.

Note 3.5. A convex soft set is not necessarily a strongly convex soft set and a strictly convex soft set is not necessarily a strongly convex soft set.

Theorem 3.6. If $\{S_i : i \in \{1, 2, ...\}\}$ is any family of convex soft sets, then,

1. the intersection $\tilde{\cap}_{i\in I} S_i$ is a convex soft set but union of any family $\{S_i : i \in I = \{1, 2, ...\}\}$ of convex soft sets is not necessarily a convex soft set.

2. the union $\tilde{\cup}_{i\in I} S_i$ is a concave soft set and the intersection of any family $\{S_i : i \in I = \{1, 2, ...\}\}$ of concave soft sets is concave soft set.

Theorem 3.7. S is a convex soft set $\Leftrightarrow$ $S^{\tilde{c}}$ is a concave soft sets.

PROOF. $\Rightarrow$ Suppose that there exist $x, y \in E$, $a \in I$ and S be a convex soft set.
Then, since S is convex,

$$f_S(ax + (1-a)y) \supseteq f_S(x) \cap f_S(y) \tag{1}$$

or

$$U \setminus f_S(ax + (1-a)y) \subseteq U \setminus \{f_S(x) \cap f_S(y)\} \tag{2}$$

we have

$$U \setminus f_S(ax + (1-a)y) \subseteq \{U \setminus f_S(x) \cup U \setminus f_S(y)\} \tag{3}$$

So, $S^{\tilde{c}}$ is a concave fuzzy soft set.
$\Leftarrow$ $S^{\tilde{c}}$ be a concave soft set.
Since $S^{\tilde{c}}$ is concave, we have

$$U \setminus f_S(ax + (1-a)y) \subseteq \{U \setminus f_S(x) \cup U \setminus f_S(y)\} \tag{4}$$

Then,

$$U \setminus f_S(ax + (1-a)y) \subseteq U \setminus \{f_S(x) \cap f_S(y)\} \tag{5}$$

or

$$f_S(ax + (1-a)y) \supseteq f_S(x) \cap f_S(y) \tag{6}$$

So, S is a convex soft set. □

Theorem 3.8. $S\tilde{\cap}T$ is a strictly convex soft set when both S and T are strictly convex soft sets.

PROOF. Suppose that there exist $x, y \in E$ and $a \in I^\circ$ and $W = S\tilde{\cap}T$. Then,

$$f_W(ax + (1-a)y) = f_S(ax + (1-a)y) \cap f_T(ax + (1-a)y) \tag{7}$$

Now, since S and T strictly convex sets,

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y) \textit{ such that } f_S(x) \neq f_S(y) \tag{8}$$

$$f_T(ax + (1-a)y) \supset f_T(x) \cap f_T(y) \textit{ such that } f_S(x) \neq f_S(y) \tag{9}$$

and hence,

$$f_W(ax + (1-a)y) \supset (f_S(x) \cap f_S(y)) \cap (f_T(x) \cap f_T(y))) \textit{ such that } f_S(x) \neq f_S(y) \tag{10}$$

and thus

$$f_W(ax + (1-a)y) \supset f_W(x) \cap f_W(y)) \textit{ such that } f_S(x) \neq f_S(y) \tag{11}$$

□

Theorem 3.9. If $\{S_i : i \in \{1, 2, ...\}\}$ is any family of strictly convex soft sets, then the intersection $\tilde{\cap}_{i\in I} S_i$ is a strictly convex soft set.

Remark 3.10. The union of any family $\{S_i : i \in I = \{1, 2, ...\}\}$ of strictly convex soft sets is not necessarily a strictly convex soft set.

Theorem 3.11. Let S be a strictly convex soft set on E.

1. If there exists $a \in I^\circ$, for every $x, y \in E$ such that

$$f_S(ax + (1-a)y) \supseteq f_S(x) \cap f_S(y) \tag{12}$$

   Then S is a convex soft set on E.

2. If there exists $a \in I$, such that for every pair of distinct points $x \in E$, $y \in E$, we have

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y) \tag{13}$$

   Then S is a strongly convex soft set on E.

PROOF. The proof is straightforward. □

Theorem 3.12. Let S be a convex soft set on E.

1. If there exists $a \in I$, for every pair of distinct points $x \in E$, $y \in E$ implies that

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y) \tag{14}$$

   Then S is a strongly convex soft set on E.

2. If there exists $a \in I$, for every $x \in E$, $y \in E$, $f_S(x) \neq f_S(y)$ implies,

$$f_S(ax + (1-a)y) \supset f_S(x) \cap f_S(y) \tag{15}$$

   Then S is a strictly convex soft set on E.

Definition 3.13. The fuzzy soft set Γ_A on E is called a convex fuzzy soft set, is shown in Figure 2, if

$$\gamma_A(ax + (1-a)y) \supseteq \gamma_A(x) \cap \gamma_A(y)$$

for every $x, y \in E$ and $a \in I$.

Theorem 3.14. $\Gamma_A\tilde{\cap}\Gamma_B$ is a fuzzy convex soft set when both Γ_A and Γ_B are fuzzy convex soft sets.

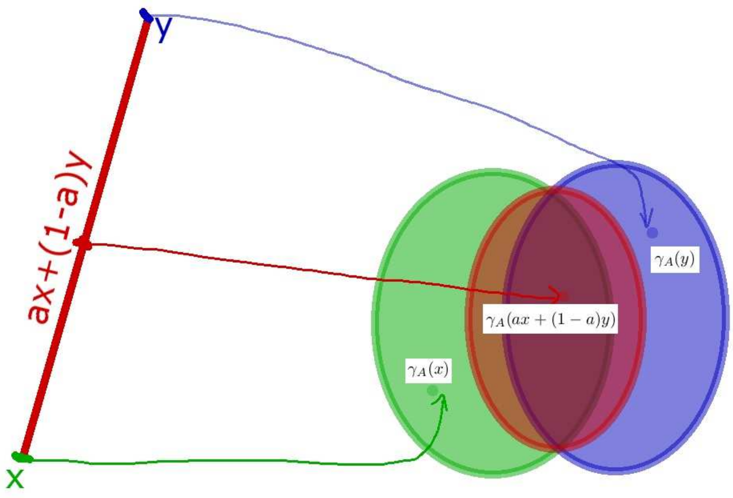

Fig. 2. The fuzzy convex soft set

PROOF. Suppose that there exist $x, y \in E$ and $a \in I$ and $C = S \cap T$. Then,

$$\gamma_C(ax + (1-a)y) = \gamma_S(ax + (1-a)y) \cap \gamma_T(ax + (1-a)y) \tag{16}$$

Now, since S and T convex,

$$\gamma_S(ax + (1-a)y) \supseteq \gamma_S(x) \cap \gamma_S(y) \tag{17}$$

$$\gamma_T(ax + (1-a)y) \supseteq \gamma_T(x) \cap \gamma_T(y) \tag{18}$$

and hence,

$$\gamma_C(ax + (1-a)y) \supseteq (\gamma_S(x) \cap \gamma_S(y)) \cap (\gamma_T(x) \cap \gamma_T(y)) \tag{19}$$

and thus

$$\gamma_C(ax + (1-a)y) \supseteq \gamma_C(x) \cap \gamma_C(y) \tag{20}$$

□

Definition 3.15. The soft set Γ_A on E is called a concave fuzzy soft set if

$$\gamma_A(ax + (1-a)y) \subseteq \gamma_A(x) \cup \gamma_A(y)$$

for every $x, y \in E$ and $a \in I$.

Theorem 3.16. $\Gamma_A \tilde{\cup} \Gamma_B$ is a concave fuzzy soft set when both Γ_A and Γ_B are concave fuzzy soft sets.

PROOF. Suppose that there exist $x, y \in E$ and $a \in I$ and $\Gamma_C = \Gamma_A \tilde{\cup} \Gamma_B$. Then,

$$\gamma_C(ax + (1-a)y) = \gamma_A(ax + (1-a)y) \cup \gamma_B(ax + (1-a)y) \tag{21}$$

Now, since S and T concave,

$$\gamma_A(ax + (1-a)y) \subseteq \gamma_A(x) \cup \gamma_A(y) \tag{22}$$

$$\gamma_B(ax + (1-a)y) \subseteq \gamma_B(x) \cup \gamma_B(y) \tag{23}$$

and hence,

$$\gamma_C(ax + (1-a)y) \subseteq (\gamma_A(x) \cup \gamma_A(y)) \cup (\gamma_B(x) \cup \gamma_B(y)) \tag{24}$$

and thus

$$\gamma_C(ax + (1-a)y) \subseteq \gamma_C(x) \cup \gamma_C(y) \tag{25}$$

□

Theorem 3.17. If $\{\Gamma_{A_i} : i \in \{1, 2, ...\}\}$ is any family of concave fuzzy soft sets, then the union $\tilde{\cup}_{i \in I} \Gamma_{A_i}$ is a concave fuzzy soft set.

Theorem 3.18. Γ_A is a convex fuzzy soft set when $\Gamma_A^{\tilde{c}}$ is a concave fuzzy soft sets.

PROOF. Suppose that there exist $x, y \in E$, $a \in I$ and Γ_A be a convex fuzzy soft set.

Then, since Γ_A is convex,

$$\gamma_A(ax + (1-a)y) \supseteq \gamma_A(x) \cap \gamma_A(y) \tag{26}$$

or

$$U \setminus \gamma_A(ax + (1-a)y) \subseteq U \setminus \{\gamma_A(x) \cap \gamma_A(y)\} \tag{27}$$

we have

$$U \setminus \gamma_A(ax + (1-a)y) \subseteq \{U \setminus \gamma_A(x) \cup U \setminus \gamma_A(y)\} \tag{28}$$

So, $\Gamma_A^{\tilde{c}}$ is a concave fuzzy soft set. □

Theorem 3.19. If $\{\Gamma_{A_i} : i \in \{1, 2, ...\}\}$ is any family of convex fuzzy soft sets, then the intersection $\tilde{\cap}_{i \in I} \Gamma_{A_i}$ is a convex fuzzy soft set.

Remark 3.20. The union of any family $\{\Gamma_{A_i} : i \in I = \{1, 2, ...\}\}$ of convex fuzzy soft sets is not necessarily a convex fuzzy soft set.

Theorem 3.21. Γ_A is a concave fuzzy soft set when $\Gamma_A^{\tilde{c}}$ is a convex fuzzy soft sets. sets.

PROOF. Suppose that there exist $x, y \in E$, $a \in I$ and S be a concave fuzzy soft set.

Then, since S is concave,

$$\gamma_A(ax + (1-a)y) \subseteq \gamma_A(x) \cup \gamma_A(y) \tag{29}$$

or

$$U \setminus \gamma_A(ax + (1-a)y) \supseteq U \setminus \{\gamma_A(x) \cup \gamma_A(y)\} \tag{30}$$

we have

$$U \setminus \gamma_A(ax + (1-a)y) \supseteq \{U \setminus \gamma_A(x) \cap U \setminus \gamma_A(y)\} \tag{31}$$

So, $\Gamma_A^{\tilde{c}}$ is a convex fuzzy soft set. □

Theorem 3.22. S is a concave fuzzy soft set on E iff for every $\beta \in [0, 1]$ and $\alpha \in P(U)$, S^α is a concave set on E.

PROOF. $\Rightarrow$ Assume that S is a concave fuzzy soft set. If $x_1, x_2 \in E$ and $\alpha \in P(U)$, then $\gamma_A(x_1) \supseteq \alpha$ and $\gamma_A(x_2) \supseteq \alpha$. It follows from the concavity of S that

$$\gamma_A(\beta x_1 + (1-\beta)x_2) \subseteq \gamma_A(x_1) \cup \gamma_A(x_2)$$

and thus S^α is a concave set.

$\Leftarrow$ Assume that S^α is a concave set for every $\beta \in [0, 1]$. Especially, for $x_1, x_2 \in E$, S^α is concave for $\alpha = \gamma_A(x_1) \cup \gamma_A(x_2)$.

Since $\gamma_A(x_1) \supseteq \alpha$ and $\gamma_A(x_2) \supseteq \alpha$, we have $x_1 \in S^\alpha$ and $x_2 \in S^\alpha$, whence $\beta x_1 + (1-\beta)x_2 \in S^\alpha$. Therefore, $\gamma_A(\beta x_1 + (1-\beta)x_2) \subseteq \alpha = \gamma_A(x_1) \cup \gamma_A(x_2)$, which indicates S is a concave fuzzy soft set on X. □

4. Conclusion

In the literature, convex fuzzy sets has been introduced widely by many researchers. In this paper, we defined convex soft sets, concave soft sets, convex fuzzy soft sets and concave fuzzy soft sets and give some properties. Also we will try to explore characterizations of convex fuzzy soft sets to optimization in the future. The theory may be applied to many fields and more comprehensive in the future to solve the related problems, such as; pattern classification, operation research, decision making, optimization problem, and so on.

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