

APOS TEORİSİNE YÖNELİK ÇALIŞMALARIN BİR ANALİZİ

AN ANALYSIS OF THE STUDIES ON THE APOS THEORY

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Öz

Bu çalışmada 2000-2019 yılları arasında yazılmış olan APOS (Action-Process-Object-Schema) (Eylem-Süreç-Nesne-Şema) teorisine yönelik bilimsel araştırmaları incelemek amaçlanmıştır. Araştırma betimsel nitelikte olup durum tespiti yapılmıştır. Alanyazın taraması yapılırken uluslararası alanda en çok taranan Web of Science, ERIC, EBSCO ve ULAKBİM veri tabanları tercih edilmiş, Bu veri tabanlarından 2000-2019 yıllarında ulaşılan çalışmalar araştırma kapsamına dâhil edilmiştir. Bu doğrultuda yapılan literatür çalışmasında on iki adet Türkçe ve yirmi dokuz adet yabancı araştırmaya ulaşılmıştır. Ulaşılan bu kırk bir araştırma doküman analizi yöntemiyle yayınlanan üniversite, ülke, yıl, örneklem grubu, örneklemin öğrenim seviyesi ve araştırma metodu kriterlerine göre incelenmiştir. Elde edilen bulgular frekanslar yardımıyla tablolaştırılarak okuyucuya sunulmuştur. Yapılan çalışma sonucunda Türkiye’de bu alanda on iki adet çalışma olduğu, çalışmaların yoğunlukla yurt dışında bulunduğu, yıllara göre çalışma sayılarının benzerlik gösterdiği, araştırmaların çoğunlukla üniversite öğrencileri üzerinde yapıldığı, sınıf düzeyine göre çalışma sayılarının benzerlik gösterdiği ve nitel çalışmaların nicel ve karma araştırmalara göre daha yoğunlukta olduğu sonuçlarına ulaşılmıştır. Çalışmada elde edilen bulgular çerçevesinde APOS teorisine ilişkin önerilerde bulunulmuştur.

Anahtar Kelimeler: Matematik eğitimi, APOS teorisi, zihinsel yapılar, bilişsel beceriler, şema.

Abstract

In this study, it is aimed to examine scientific researches about APOS (Action-Process-Object-Schema) theory written between 2000-2019. The research was descriptive and the situation was determined. While searching the literature, Web of Science, ERIC, EBSCO and ULAKBİM databases which are the most searched internationally were preferred. The studies reached from these databases in 2000-2019 were included in the research scope. In this respect, in the literature study, twelve Turkish and twenty-nine foreign researches were reached. Forty one of these researches were analyzed according to the criteria of university, country, year, study group, education level of the study group and research method published by document analysis method. The findings was presented to the reader were tabulated with the help of frequencies. The studies result in Turkey, where twelve studies in this area, which abroad the intensity of the studies, according to the year in which the similarity of the number of studies is made mostly on college students' research, the similarity of the number of studies on the grade level and according to quantitative and mixed research of qualitative studies more density was reached. In the framework of the findings obtained in the study, suggestions were made about the APOS theory.

Key Words: Mathematics education, APOS theory, mental structures, cognitive skills, Schema.

1. INTRODUCTION

Rapid changes in science world, changing needs of the individual and society, learning and teaching theory innovations and advances in their approaches have directly affected the roles expected from individuals. This change produces information, can use it functionally in life, problem solving, critical thinking, entrepreneur, determined, communication skills, empathy, contributing to society and culture. The curricula that will serve to educate individuals with this qualitative texture have been prepared in a simple and comprehensible structure that aims to gain value and skill, taking into account individual differences rather than a structure that only transmits

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information (MEB, 2018). The APOS Theory was introduced by Piaget in order to define the development of logical thought in children in an attempt to understand the reflection of abstraction the mechanism and extend this idea to more advanced mathematical concepts (Dubinsky, 1991). This theory suggests that an individual must have a mental structure to understand a particular mathematical concept. According to APOS theory, the individual deals with mathematical situations using certain mental mechanisms to construct cognitive structures. The mechanisms mentioned herein are as follows; while internalization, coordination, encapsulation, generalization, reversal, cognitive structures are the actions, processes, objects and schemes that make up the APOS theory.

1.1. APOS Model

The APOS model is one of the models that emerged as a result of this quest. APOS theory argues that there is a close relationship between the nature of mathematical concepts and the development of mathematical concepts in the individual mind (Dubinsky, Weller, McDonald & Brown, 2005). Therefore, the explanations put forward by this theory have both epistemological and psychological value. The APOS Model suggests that an individual must have a mental structure to understand a particular mathematical concept. Mental structures refer to actions, processes, objects and schema necessary to learn the concept. Research based on this theory requires that possible mental structures should be identified for a particular concept and appropriate learning activities should be undertaken to support the formation of these mental structures (Dubinsky & Mc Donald, 2001).

1.2. Mental Structures

APOS theory assumes that a mathematical concept has evolved when trying to transform existing physical or mental objects (Dubinsky, 2010; Weller, Arnon & Dubinsky, 2009). Action, process, object and schema to create the mental structure of the basic mental mechanisms to create the internalization and encapsulation is called. Mental structures of action, process, object and schema form APOS. According to Dubinsky (1991), these mental structures are described as follows:

Action: A reproducible, physical or mental manipulation that transforms objects. The individual's reactions to stimuli can be considered primarily as actions.

Process: A process of actions that takes place entirely in mind. The individual is repeating an action and can be internalized into a mental process while reflecting it. A process is a mental structure that performs the same process as action, but is formed entirely in the individual.

Object: Both expresses the distinction between a process and an object, and is perceived as a set of actions and processes indicating that a process has become an object. New information acquired in action and process states is encapsulated and converted to an object.

Schema: It is a more or less consistent collection of cognitive objects and internal processes to manipulate existing objects in the individual. It can help students to "understand, deal with, organize, or make sense of a perceived problem situation". Typically, the schema contains many actions, processes, and objects that need to be linked and edited to a consistent framework. It is not possible for secondary school students to be at this level.

1.3. Abstraction Processes

This theory adopts the following five types of structures in the reflective abstraction described by Dubinsky (1991):

Internalization: The ability to apply symbols, language, pictures and mental images to

create internal processes as a method to understand perceived phenomena. Operations on objects are internalized into an operation system.

Coordination: Two or more processes are coordinated to form a new process.

Encapsulation: Ability to comprehend a previous operation as an object. **Generalization:** apply the existing schema to wider contexts.

Reversal: The ability of previous internalized processes to reverse thought processes

APOS Theory can be used by a researcher directly in the analysis of data. In finely detailed analyzes, the researcher can compare students' achievements or failures with certain mathematical structures that they may or may not have done with a mathematical task (Dubinsky & Mc Donald, 2001). When literature review is made, it is seen that APOS theory is gaining importance day by day and especially international studies are more intense (Dubinsky & McDonald, 2001; Dubinsky, Weller, Mcdonald & Brown, 2005; Dubinsky, 2010; Kabael, 2011; Kusaeri, 2015; Maharaj, 2013; Moll, Trigueros, Badillo, & Rubio, 2016; Murray, 2002; Parraguez & Oktaç 2010; Pradana, Kriswandani Yunianta, 2016; Tzirias, 2011). At this point, it was seen that the importance of this theorem in the national field was revealed and examined. In the light of all these explanations, the aim of this study is to examine the researches carried out for the APOS theory in terms of the criteria determined according to the institutes, study group, method and universities. Therefore, in this study, it is tried to reach the articles about APOS theory in mathematics education and postgraduate theses. As a result of literature review, 41 articles that can be reached within the scope of mathematics education and this questions have been answered;

1. How is the distribution according to the institution it is prepared?
2. How is the distribution according to the countries it is prepared?
3. How is the distribution by years?
4. How is the study group distribution?
5. How is the distribution according to the grade levels applied in the preparation?
6. How is the method selection of the studies?

2. METHOD

In order to reach the APOS theory studies, the document analysis method, which is designed in the scanning model, was used. Document analysis is the analysis of written materials containing information about the cases or cases to be investigated (Yıldırım & Şimşek, 2013). Screening models are research applications that aim to define a situation that has occurred or has existed in the past (Karasar, 2012). Written documents were analyzed in accordance with the criteria stated for the purposes in accordance with the screening model. While searching the literature, Web of Science, ERIC, EBSCO and ULAKBİM databases which are the most searched internationally were preferred. The studies reached from these databases in 2000-2019 were included in the research scope. In this respect, in the literature study, twelve Turkish and twenty-nine foreign researches were reached. Forty-one of these researches were analyzed according to the criteria of university, country, year, study group, education level of the study group and research method published by document analysis method.

3. FINDINGS

In this study, as a result of the evaluation of the 41 available articles according to quality and quantity; source type of research, method information, participant type, study group,

distribution of articles by years, according to the type of institute where the research was made categories. The distribution of the studies according to the institution is given in Table 1.

Table 1. Distribution of Studies by Institution

Universty-Journal (Turkey)	f	Universty-Journal (Foreign)	f
Eskişehir Osmangazi	2	Warwick University	2
Hacettepe	2	Linear Algebra and its Applications	4
Atatürk	1	Journal of Mathematical Educational Studies in Mathematics	3
Balıkesir	1	Educ Stud Math	3
Anadolu	2	The Mathematics Enthusiast Volume	1
Gazi	1	Missouri University	1
İstanbul Medeniyet	1	ZDM Mathematics Education	1
Journal of Education and Science	1	International Journal of Science and Technology Math. Edu.	4
Turkish Journal of Computer and Mathematics Education	1	Scandinavian Studies in Mathematics Education	1
		For Mathematics Learning	1
		Bandung	1
		PRISMA	1
		Hollins University	1
		Georgia Eyalet University	1
		ALM	1
		FKIP Sriwijay University	1
		FPMIPA Pendidikan University	1
		CHRISTIAN SATYA University	1
Total	12	Total	29

When the distribution of studies by institutions is examined, the maximum number of APOS theory studies belongs to 4 different studies and two different international journals. There is a considerable difference between Turkish and foreign articles. On the other hand, the distribution of the articles according to the countries where they are prepared is given in Table 2.

Table 2. Distribution of articles by country of preparation

Country	f
Turkey	12
Indonesia	7
USA	3
Swedish	1
England	4
Mexican	4
Canada	1
South Africa	1
Virginia	1
Taiwan	1
Colombia	1
France	1

Germany	1
Porto Rico- Mexican	1
Peru- Spain- Mexican	1
Spain	1
Total	41

When Table 2 is examined, it is seen that the total number of articles is 41. Noteworthy articles prepared by different partners in foreign countries Although there is no article published in Turkey in partnership with another country are analyzed in detail. The distribution of articles by publication years is given in Table 3.

Table 3. Distribution of articles by years

Turkey	f	Foreign	f
2006	1	1999	2
2007	1	2003	2
2011	1	2004	1
2014	2	2005	3
2015	2	2007	2
2017	2	2008	3
2018	3	2009	1
		2010	4
		2012	2
		2013	2
		2015	3
		2016	2
		2017	1
		2018	1

When Table 3 is examined, it is observed that the studies on this subject in our country started very late compared to other countries. However, it is obvious that the studies on this subject in our country have increased in recent years and it can be said that the most studies were Conducted in 2018. Table 4 shows the distribution of the articles according to the Study group.

Table 4. Distribution of articles by study group

Study Group (Turkey)	f	Study Group (Foreign)	f
University students	6	University students	13
High School Students	1	High School Students	--
Secondary School Students	3	Secondary School Students	2
Primary School Students	1	Primary School Students	--
Screening Study	1	Theoretical Study	14

When Table 4 is analyzed, it is observed that most of the studies Conducted with university students in our country are aimed, whereas in foreign articles, the study group is mostly about the subject. In contrast to Turkey in foreign countries is observed that more much screening study.

The distribution according to the grade levels applied in the preparation of the articles is given in Table 5.

Table 5. Distribution of articles according to grade levels applied

Level	Turkey	Foreign
5. Grade	1	-
6. Grade	-	1
7. Grade	1	-
8. Grade	1	1
9. Grade	1	-
10. Grade	-	-
11. Grade	-	-
12. Grade	-	-

When the Table 5 is examined, the majority of the grades applied in our country were used in middle school grades, and in the studies conducted in foreign countries, it was applied only once to 6th grade and once to 8th grade students. Table 6 shows the distribution according to the methods used in the studies

Table 6. Distribution according to the methods used

Method	Turkey	Foreign
Quantitative	4	7
Qualitative	7	20
Mixed	1	2

When the data in Table 6 is examined, the method used in the studies conducted in our country and foreign countries is more qualitative. This shows the functionality of the researches on the subject.

4. Discussion, Conclusion and Recommendations

In this study, the researches carried out for APOS theory were examined in terms of the institutions, years study group, methods and criteria determined according to universities. For this purpose, literature was searched and 41 studies were conducted in this field. When the findings of the study are examined, it is concluded that most of the articles conducted in our country are carried out in Eskişehir Osmangazi, Anadolu and Hacettepe Universities, but it can be said that these numbers are very low compared to foreign countries. When the number of articles examined; It was concluded that the highest number of studies conducted in our country was in 2018. It can be said that this situation is related to the effect of the renewed elementary school program on teachers and students, but it will be noticed in Table 3 that the studies related to this issue started quite late in our country compared to foreign countries. Looking at the study group where the articles were performed; While most of the studies conducted in our country are aimed at university students, it is noteworthy that the studies are mostly in the form of screening studies.

If we compare our country and foreign countries according to the grade levels applied in the preparation of the articles, it is seen that 4 of the 12 studies conducted in our country were applied to the secondary school students. When we look at the type of method used in the research, it is seen that most of the article studies conducted in our country use qualitative research type. When the foreign countries are examined, it is seen that qualitative research is used as a method from 29 article studies. This shows that there is not much difference between our country and foreign countries. In this respect, if we make an inter-state comparison with our country in general, it is observed that our country is different from foreign countries in most aspects we have

examined. Compared to foreign countries, the issue of the relevant study in Turkey seems to be very little.

In this study, studies on APOS theory in mathematics education are examined. As a result of this study, 41 studies were reached and these 41 studies were examined according to various criteria. As a result of this study, it can be said that increasing the researches related to APOS theory will be beneficial for mathematics teaching. In addition, such studies are repeated from time to time and included in the studies that will be included in the literature. However, considering the studies conducted, it is thought that it would be beneficial to do these grades since there were no studies conducted in grades 10, 11 and 12. Similarly, it is clear that such studies need to be conducted in prospective teachers.

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