Proceedings of International Conference on Mathematics
and Mathematics Education (ICMME 2019)
Turk. J. Math. Comput. Sci.
11(Special Issue)(2019) 11–15
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# Early Warning Signals of Oxygen-Plankton Dynamics: Mathematical Approach

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Received: 02-08-2019 • Accepted: 18-12-2019

Abstract. Any significant decrease in net oxygen production by phytoplankton is likely to result in the loss of atmospheric oxygen and the global extinction of living beings owing to more than half of the atmospheric oxygen provided by marine phytoplankton. The rate of oxygen production is known to depend on water temperature and hence can therefore be affected by global warming. In this work, it is assumed that oxygen production varies with time under the effect of increasing temperature. This ecological problem is addressed theoretically by a couple of plankton-oxygen dynamics. A nonlinear mathematical model is considered to investigate the effect of temperature on oxygen-plankton dynamics. The model is analysed analytical and numerical ways, based on the behavior and complexity of the system's steady state. From the analysis of the model, it has been observed that as temperature level goes above the critical threshold of oxygen production rate the equilibrium density of plankton population decrease due to a decrease in oxygen concentration. It has also been shown that the system can exhibit sustainable dynamics that can still lead to an environmental disaster, i.e. oxygen depletion and plankton extinction. In this case, extinction takes place after a considerable length of time.

*2010 AMS Classification:* 81T80, 97MXX.

**Keywords:** Global warming, differential equation, extinction, oxygen depletion.

## 1. Introduction

Oxygen depletion has been a challenging environmental phenomenon, with the consequence that oxygen and plankton dynamics were specifically incorporated into the model system to make a number of researchers informed of this issue [2, 3, 6, 10, 12]. In addition, phytoplankton and zooplankton dynamics are considered to address the issue of prey-predator system interaction in a conceptual mathematical model without paying an attention to the oxygen concentration in [8, 9, 11, 14]. There are several papers concerned with oxygen-plankton system. Particularly, a mathematical model of biochemical processes in the lagoon system is examined in [10]. In a furthered research [2], concentrated on the presence of periodic solutions based on the Italian coastal lagoon. The dissolved oxygen content is studied in a multi-component model, with the help of bacterial and environmental pressure on the structure of the lagoon by Hull et al. [6]. In another mathematical study, a plankton-nutrient dynamics system is studied by leaving the oxygen concentration dynamics aside. In addition, the oxygen-algae model is presented to detail oxygen degradation

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This research has been supported by Amasya University Scientific Research Projects Coordination Unit. Project Number: FBM-BAP 18-0337.

under some existing regulation factors [12]. In literature, the production of oxygen in the water and its reliance on rising temperature condition are ignored.

This work describes recent scientific developments in the question of oxygen depletion in marine systems and explores this issue by mathematical modeling and numerical simulations. First, we extend the well-known prey-predator model [9, 14] to include oxygen in order to understand the inherent possible reasons for the problem of oxygen depletion in the marine system. The dynamical properties of the mathematical model is structured by extensive numerical simulations corresponding nonspatial and spatial system. It is observed that a sufficiently large increase in water temperature results in an environmental disaster, i.e. oxygen production is suddenly reduced to zero and phytoplankton and zooplankton densities are zero. Finally, the ecological importance of the obtained results is discussed to detail the underlying properties where the ecological system gives early warning signals when the system approaches its threshold value or values.

## 2. Mathematical Model

In this work, a mathematical model for plankton-oxygen dynamics is considered [17]. Initially, a non-spatial system that applies to a well-mixed ecosystem is considered to understand the general structure of the spatial system. In this system the oxygen dynamics, which, throughout effect, is regulated by its primary source of phytoplankton, which, in turn, is grazed with its predatory zooplankton. Moreover, the oxygen-plankton relationship is believed to pursue the Holling type-II functional response and therefore the process dynamics are controlled by the following set of differential equations.

$$\frac{dc}{dt} = \frac{Au}{c+1} - c - \frac{uc}{c+h_2} - \frac{\nu c v}{c+h_3}, \tag{2.1}$$

$$\frac{du}{dt} = \left(\frac{Gc}{c+h_1} - u\right)\gamma u - \frac{uv}{u+h} - \sigma u, \tag{2.2}$$

$$\frac{dv}{dt} = \left(\frac{\kappa uv}{u+h}\right)\frac{c^2}{c^2+{h_4}^2} - \mu v. \tag{2.3}$$

Here $c$, $u$, and $v$ are levels of dissolved oxygen and densities of phytoplankton and zooplankton populations, respectively, at time $t$, with initial conditions at their steady states with given ecological condition is as follows:

$$c(0) > 0, \quad u(0) > 0, \quad v(0) > 0. \tag{2.4}$$

The positive terms for all of the system components describes the growth parts for the corresponding components. Especially, the positive term of oxygen, $Af(c)$, stems from the primary production of phytoplankton. The negative part of oxygen is for natural depletion in the water body and for the respiration of phytoplankton and zooplankton. The negative term for phytoplankton describes the predation and natural depletion of phytoplankton. Finally, the term loss of zooplankton is the natural depletion, see [17] for further details on plankton-oxygen system.

* The system components-free equilibrium $Q_1 = (0, 0, 0)$ is the extinction case. It is easy to see that this equilibrium always exists, regardless of the value of the parameter.
* Another equilibrium is the zooplankton free $Q_2 = (\check{c}, \check{u}, 0)$ case. When $v = 0$, the system (2.1-2.3) is restricted to the oxygen-phytoplankton system. For this only oxygen-phytoplankton system the system has two positive steady states, i.e., $\check{c}$ and $\check{u}$ given by Eq. (2.5). System steady state values $\check{c}$ and $\check{u}$ are the solutions for the following zooplankton-free system equations. Hence, the equations for isoclines is received as

$$\check{u} = \frac{\check{c}(\check{c}+1)(\check{c}+c_2)}{A(\check{c}+c_2) - \check{c}(\check{c}+1)}, \qquad \check{c} = \frac{c_h(\check{u}\gamma + \sigma)}{G\gamma - \check{u}\gamma - \sigma}. \tag{2.5}$$

* Another system equilibrium is the coexistence one $Q_3 = (\grave{c}, \grave{u}, \grave{v})$. The three component system solutions are the steady state of $\grave{c}$, $\grave{u}$ and $\grave{v}$. In this situation, the production of oxygen is driven by both phytoplankton and zooplankton respiration. The stability of all existing system components is detailed in numerical simulations.

$$\grave{c} = \frac{c_h(\grave{u}\gamma + \frac{\grave{v}}{\grave{u}+h} + \sigma)}{G\gamma - \grave{u}\gamma - \frac{\grave{v}}{\grave{u}+h} - \sigma}, \quad \grave{u} = \frac{\mu h(\grave{c}^2 + {c_4}^2)}{\kappa\grave{c}^2 - \mu(\grave{c}^2 + {c_4}^2)}, \quad \grave{v} = \frac{\grave{c}+c_3}{\nu}\Big(\frac{A\grave{u}}{\grave{c}(\grave{c}+1) - 1 - \frac{\grave{u}}{\grave{c}+c_2}}\Big). \tag{2.6}$$

The system's steady-state is calculated in [17], but only where the case of extinction can be explicitly stated, and then numerical methods need to be invoked.

## 3. Numerical Observations

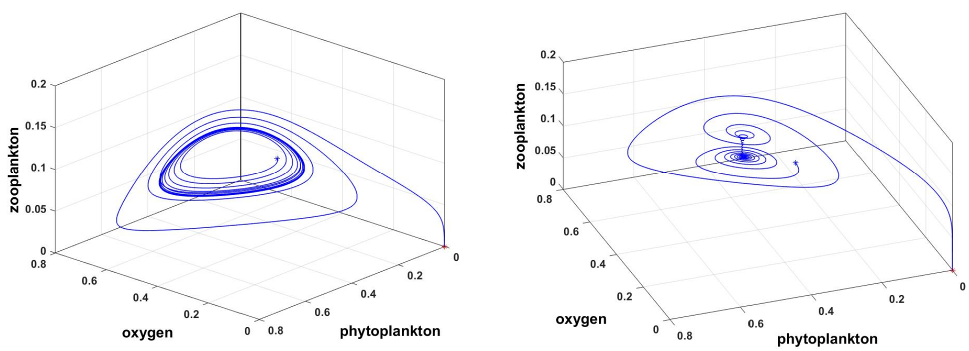
3.1. **Temporal Dynamics.** In this section, the nonspatial system (2.1-2.3) numerical simulations are conducted. In all of the following numerical simulations to utilize from the previously obtained bifurcation parameters in [17], we set parameters to some defined values such as $G = 1.6$, $\gamma = 1.2$, $\sigma = 0.1$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1$, $\nu = 0.01$, $\kappa = 0.7$, $\mu = 0.1$, $h = 0.1$ and varies $A$ and $c_h$ within a certain range. Here, we are interested in the time dynamics of the change in the $c_h$ parameter that accounts for the half saturation value of oxygen production.

Half saturation value of oxygen has a role in the oxygen concentration in water body. In the previous work [17], $c_h$ is considered to be a constant, but here, compared to the previous one, $c_h$ is considered to be a variable of temperature.

In oxygen-plankton model system (2.1-2.3), $c_h$ is the half saturation constant of phytoplankton growth, thereby, here, temperature effect on the phytoplankton growth will be considered. Specifically, we use the following $c_h$ function as follows:

$$c_h = ((c_{h2} - c_{h1})/(t_2 - t_1)) \times (t - t_1). \tag{3.1}$$

Lower limit of $c_h$ function, which is called as $c_{h1}$ corresponds to the stable temperature until a certain time $t_1$ and then $c_h$ function starts to increase until a certain time $t_2$ to the upper limit of $c_h$, which is called as $c_{h2}$ and then again stable. (Figs. 1a-b) shows the concentration of oxygen and the density of plankton versus time obtained for fixing




(a) (b)

Figure 1. Phase plane of oxygen-plankton system in three dimensions, (a) $c_{h1} = 0.6$, $c_{h2} = 0.5$ (b) $c_{h1} = 0.7$, $c_{h2} = 0.5$ and the initial values $c_i = u_i = 0.3$, and $v_i = 0.1$, the system parameters are exactly the same for fixed value of oxygen production rate, i.e., 2.1.

$A = 2.1$ and separate upper limits of $c_h$, called $c_{h1}$. Further increase on $c_{h1}$ leads limit cycle to a plateau which is bounded by decaying oscillations from both sides; see (Figs. 1a-b).

3.2. **Spatial Dynamics.** Spatial structure of oxygen-plankton system is given in this section.

$$\frac{\partial c}{\partial t} = D_c\frac{\partial^2 c}{\partial x^2} + A(1-\frac{c}{c+1})u - c - \frac{uc}{c+c_2} - \frac{\nu c v}{c+c_3}, \tag{3.2}$$

$$\frac{\partial u}{\partial t} = D_u\frac{\partial^2 u}{\partial x^2} + (\frac{Gc}{c+c_h} - u)\gamma\, u - \frac{uv}{u+h} - \sigma u, \tag{3.3}$$

$$\frac{\partial v}{\partial t} = D_v\frac{\partial^2 v}{\partial x^2} + (\frac{\kappa u v}{u+h})\frac{c^2}{c^2+{c_4}^2} - \mu v. \tag{3.4}$$

The above temporal case notations for all system components oxygen, phytoplankton and zooplankton are placed in spatial context. The additional terms for the spatial case are the time $t$ and the location $x$ with the turbulent diffusion coefficient [13]. The specific form of the model Eqs. (3.2-3.4) and some assumptions about the model structure can easily be found in [17] and in the references therein.

With the assist of given initials as in Eqs.(3.5), obtained numerical results produce acceptable results in terms of nature. The initial distribution of the species is patchy for zooplankton with uniformly distributed oxygen and phytoplankton in space:

$$c(x,0) = c_i, \quad u(x,0) = u_i, \quad v(x,0) = v_i + (x - L/2) \times (\tau/L), \tag{3.5}$$

here $c_i$, $u_i$ and $v_i$ are the steady states of the coexistence state with the patch diameter $\tau$. Equations. (3.2-3.4) are solved by finite difference approach by using the Neumann boundary conditions for the specified initial values as in Eqs.(3.5).

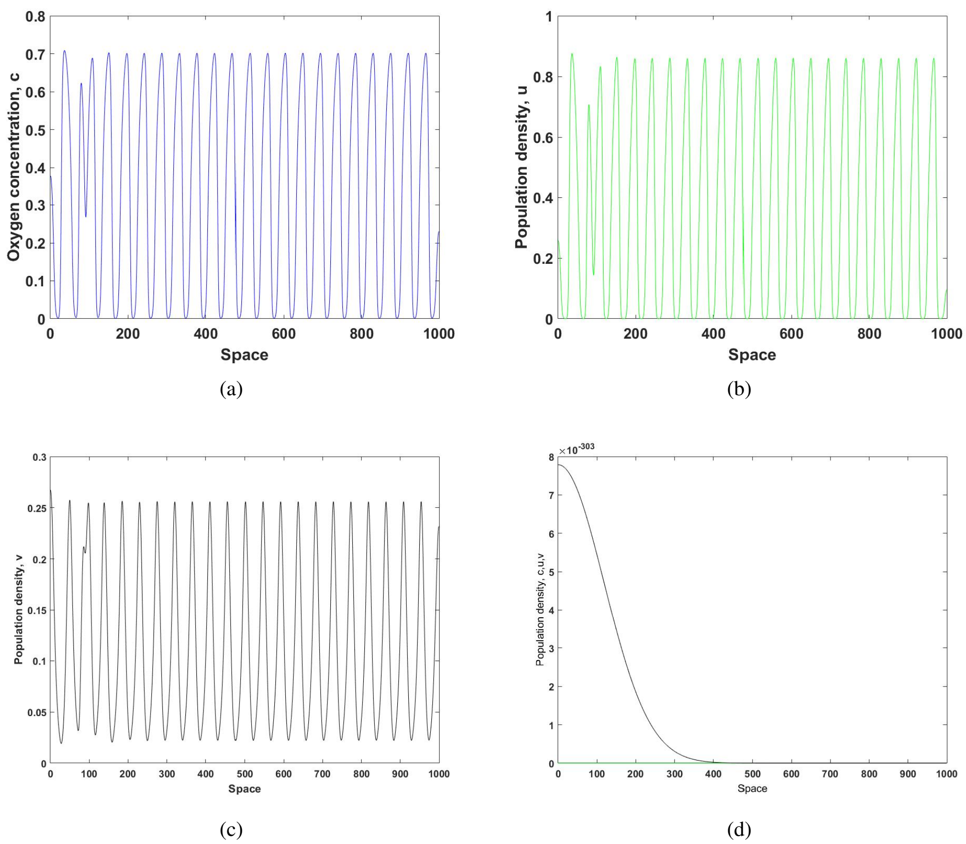



Figure 2. The result of adjusting the variable $c_{h1}$ and setting the oxygen distribution, phytoplankton and zooplankton over space obtained for other parameters defined as (a) $c_{h1} = 0.6$, $c_{h2} = 0.5$. (b)$c_{h1} = 0.7$, $c_{h2} = 0.5$ initial conditions and time difference ($t_d = 1000$) for $t_2 = 2000$, $t_1 = 1000$ and $time = 10000$ are the same for all figures.

(Figs. 2) introduces the numerical observations in spatial system (3.2-3.4) for the non-spatial system (2.1-2.3) for the temporal case given in Fig. 1.

## 4. Discussion

Over several decades, the origins of plankton in marine ecosystems have been a major issue of concern. Existing literature covers considerable features of plankton functioning to reveal the underlying structure of plankton phenomena. However, oxygen-plankton interaction without some external environmental factors has not been focused theoretically before except [17]. The distinction between this study and the previous one is that the critical value of oxygen half saturation concentration can be obtained as a system response to different functional choice. In [17], the functional response is chosen as a ramp function but here this function choice is different. It should be noted that the system has similar periodical behavior as an early warning before extinction.

In this work, the relation between oxygen and plankton components is studied which is essentially structured around the effect of predation and plankton respiration. The behavior of the model has been revealed analytically and through extensive numerical simulations. Three combined ordinary differential equations are known to be a well-mixed system of spatially patchy species distributions. Because in the aquatic system, plankton spatial distribution is highly inhomogeneous and is called plankton patchiness [1, 4, 7] and many external factors are involved in the emergence of plankton patches, such as nutrient availability, temperature, predation, acidification, etc.

Critical thresholds are noted in several complex dynamic systems. In this specific instance, the early warning signal may be the strongest signal for understanding the underlying effects of catastrophic ecological events [5, 15, 16].

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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