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Dark-Bright Optical Soliton Solutions of (3+1)-Dimensional Modified Quantum Zakharov-Kuznetsov Equation

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Abstract. In this paper, the modified $exp\left(-\vartheta(\sigma)\right)$-expansion function method (MEFM) has been used to find exact solutions of (3+1)-dimensional modified quantum Zakharov-Kuznetsov (MQZK) equation. Dark and dark-bright optical soliton solutions of the (3+1)-dimensional MQZK equation have been obtained by using this method. Also, the graphical simulations explicitly exhibit forcefulness of this method.

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1. Introduction

Nonlinear evolution equations (NLEEs) are commonly utilized to interpret a lot of physical events in the fields such as quantum field theory, biology, hydrodynamics, meteorology, optical fibers.

Latterly, many researchers have established to obtain exact solutions of NLEEs a lot of methods such as KP hierarchy reduction and Hirota bilineamethod [6], modified sub-equation extended method [10], trial equation method [4], Sine-Gordon expansion method [1] and many more [7–9]. Herein, MEFM [5] will be used to find new exact solutions of (3+1)-dimensional MQZK equation.

We investigate (3+1)-dimensional MQZK equation [2],

$$s_t + ps_x s^3 + qs_{zzz} + rs_{xxz} + ls_{yyz} = 0, \tag{1.1}$$

where p, q, r and l are real-valued constants. Here, the behaviour of the weakly nonlinear ion acoustic waves in the structure of an uniform magnetic field is controlled by the QZK equation [3].

Herein, our aim is to get new exact solutions of (3+1)-dimensional MQZK equation via proposed method. In Sec. 2, we explain methodology. In Sec. 3, we perform proposed method to (3+1)-dimensional MQZK equation.

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2. Explanation of the Method

Suppose a known nonlinear partial differential equation is been as follows:

$$K(s, s_t, s_x, s_y, s_z, s_{tt}, s_{xx}, s_{yy}, s_{zz}, \cdots) = 0. \tag{2.1}$$

where $s = s(x, y, z, t)$ is an unknown function.

Step 1: Getting the transformation as

$$s(x, y, z, t) = s(\sigma), \sigma = mx + ny + cz - dt,$$

Eq. (2.1) is turned into the following nonlinear equation:

$$L(s, s^{'}, s^{''}, s^{'''}, \cdots) = 0. \tag{2.2}$$

Step 2: Taking the following equation for Eq. (2.3) as solution:

$$s(\sigma) = \frac{\sum_{i=0}^{\gamma} A_i \left[\exp(-\vartheta(\sigma))\right]^i}{\sum_{j=0}^{\delta} B_j \left[\exp(-\vartheta(\sigma))\right]^j} = \frac{A_0 + A_1 \exp(-\vartheta) + \cdots + A_p \exp(\gamma(-\vartheta))}{B_0 + B_1 \exp(-\vartheta) + \cdots + B_q \exp(\gamma(-\vartheta))}, \tag{2.3}$$

where $A_i, B_j, (0 \leq i \leq \gamma, 0 \leq j \leq \delta)$ are constants, such that $A_\gamma \neq 0, B_\delta \neq 0$ and $\vartheta = \vartheta(\sigma)$ is described as;

$$\vartheta'(\sigma) = \exp(-\vartheta(\sigma)) + a \exp(\vartheta(\sigma)) + b. \tag{2.4}$$

There are the following solution families of Eq. (**??**):

Family1: When $a \neq 0, b^2 - 4a > 0$,

$$\vartheta(\sigma) = \ln\left(\frac{-\sqrt{b^2 - 4a}}{2a} \tanh\left(\frac{\sqrt{b^2 - 4a}}{2}(\sigma + E)\right) - \frac{b}{2a}\right).$$

Family2: When $a \neq 0, b^2 - 4a < 0$,

$$\vartheta(\sigma) = \ln\left(\frac{\sqrt{-b^2 + 4a}}{2a} \tanh\left(\frac{\sqrt{-b^2 + 4a}}{2}(\sigma + E)\right) - \frac{b}{2a}\right).$$

Family3: When $a = 0, b \neq 0$ and $b^2 - 4a > 0$,

$$\vartheta(\sigma) = -\ln\left(\frac{b}{\exp(b(\sigma + E)) - 1}\right).$$

Family4: When $a \neq 0, b \neq 0$ and $b^2 - 4a = 0$,

$$\vartheta(\sigma) = \ln\left(-\frac{2b(\sigma + E) + 4}{b^2(\sigma + E)}\right).$$

Family5: When $a = 0, b = 0$ and $b^2 - 4a = 0$,

$$\vartheta(\sigma) = \ln(\sigma + E).$$

where $A_i, B_j, (0 \leq i \leq \gamma, 0 \leq j \leq \delta), E, b, a$ are constants to be obtained later.

Step3: Setting Eq. (2.4) and Eq. (**??**) into Eq. (2.3), a system of can be obtained. We solve this system by using Mathematica to identify the coefficients $A_i, B_j, (0 \leq i \leq \gamma, 0 \leq j \leq \delta), E, b, a$.

3. Application of MEFM

Getting the transformation as

$$s(x, y, z, t) = s(\sigma), \sigma = mx + ny + cz - dt,$$

Eq. (1.1) demeans

$$-ds + mps^3 + (c^3q + m^2cr + n^2cl)s^{''} = 0. \tag{3.1}$$

By use of balance principle in Eq. (3.2), we obtain

$$\gamma = \delta + 1.$$

If we take $\delta = 1$ so $\gamma = 2$, we have

$$s = \frac{A_0 + A_1 \exp(-\vartheta) + A_2 \exp(2(-\vartheta))}{B_0 + B_1 \exp(-\vartheta)} = \frac{\Upsilon}{\Psi},$$

and

$$s' = \frac{\Upsilon'\Psi - \Upsilon\Psi'}{\Psi^2},$$

$$s'' = \frac{\Upsilon''\Psi^3 - \Psi^2\Upsilon'\Psi' - (\Psi''\Upsilon + \Psi'\Upsilon')\Psi^2 + 2(\Psi')^2\Upsilon\Psi}{\Psi^4},$$

$$\vdots$$

Thus, a system of $e^{-\vartheta(\sigma)}$ can be obtained. We solve this system by using Mathematica to identify the coefficients $A_i, B_j, (0 \le i \le \gamma, 0 \le j \le \delta), E, b, a$.

Case1:

$$A_0 = mA_2, A_1 = -\frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}B_0}{\sqrt{mp}}, B_1 = -\frac{\sqrt{mp}A_2}{\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}},$$
$$d = 4c(c^2q + m^2r + n^2l)\left(-a - \frac{2c(c^2q+m^2r+n^2l)B_0^2}{mpA_2^2}\right), b = -\frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)B_0}}{\sqrt{mp}A_2} \quad . \tag{3.2}$$

According to Eq. (3.2), we obtain dark-bright optical soliton solution for Eq. (1.1) as follows:

$$s_1(x,y,z,t) = \frac{-2aA_2\sqrt{-cK}H\left[\sec h\left[f(x,y,z,t)\right]\right]^2}{\left[\sqrt{-2cK}B_0 - L\sqrt{mp}A_2\tanh\left[f(x,y,z,t)\right]\right]\left[\sqrt{2}H + 2\sqrt{-mpcK}A_2B_0L\tanh\left[f(x,y,z,t)\right]\right]}, \tag{3.3}$$

where

$$K = c^2q + m^2r + n^2l, L = \sqrt{-a - \frac{2cKB_0^2}{mpA_2^2}}, f(x,y,z,t) = L\left[E + mx + ny + cz + 4cK + a + \frac{8c^2K^2 + B_0^2}{mpA_2^2}\right],$$

$H = \left(mpaA_2^2 + 2cKB_0^2\right)$.

Case2:

$$A_0 = -\frac{2c(c^2q+m^2r+n^2l)B_0^2}{mpA_2}, A_1 = \frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}B_0}{\sqrt{mp}}, B_1 = -\frac{\sqrt{mp}A_2}{\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}},$$
$$d = 2c(c^2q + m^2r + n^2l)\left(a + \frac{2c(c^2q+m^2r+n^2l)B_0^2}{mpA_2^2}\right), b = -\frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)B_0}}{\sqrt{mp}A_2}. \tag{3.4}$$

According to Eq. (3.4), we find dark optical soliton solution for Eq. (1.1) as follows:

$$s_2(x,y,z,t) = \frac{-2cKB_0}{mpA_2} - \frac{\sqrt{-2cK}aA_2}{\left(\sqrt{-2cK}B_0 - \sqrt{mp}L\tanh\left[g(x,y,z,t)\right]\right)}, \tag{3.5}$$

where $g(x,y,z,t) = L\left[E + mx + ny + cz - 2cK\left(a + \frac{2cKB_0^2}{mpA_2^2}\right)\right]$.

Case3:

$$A_0 = -aA_2 - \frac{4c(c^2q+m^2r+n^2l)B_0^2}{mpA_2}, A_1 = -\frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}B_0}{\sqrt{mp}}, B_1 = -\frac{\sqrt{mp}A_2}{\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)}},$$
$$d = 8c(c^2q + m^2r + n^2l)\left(a + \frac{2c(c^2q+m^2r+n^2l)B_0^2}{mpA_2^2}\right), b = -\frac{2\sqrt{2}\sqrt{-c(c^2q+m^2r+n^2l)B_0}}{\sqrt{mp}A_2}. \tag{3.6}$$

According to Eq. (3.6), we find dark optical soliton solution for Eq. (1.1) as follow

$$s_3(x,y,z,t) = \frac{\frac{-4cKB_0^2}{mpA_2} + aA_2\left[-1 + \frac{mpaA_2^2 + 4cKB_0^2 + 2\sqrt{-2mpcK}A_2B_0L\tanh\left[h(x,y,t,z)\right]}{\left(\sqrt{-2cK}B_0 - \sqrt{mp}A_2L\tanh\left[h(x,y,z,t)\right]\right)^2}\right]}{B_0 + \frac{mpaA_2^2}{2ckB_0 + \sqrt{-2mpcK}A_2L\tanh\left[h(x,y,z,t)\right]}},$$

where $h(x, y, z, t) = L\left[E + mx + ny + cz - 8cKt\left(a + \frac{2cKB_0^2}{mpA_2^2}\right)\right]$.

In Figure 1, we plot 2D and 3D surfaces of Eq. (3.3), which show the dynamics of solutions with proper paremeters. Also, in figure 2, we draw 2D and 3D surfaces of Eq. (3.5), which demonstrate the vitality of solutions with suitable parameters. Let's make physical interpretation of the obtained solutions of Eq. (1.1). If the solution is in terms of tanh function, soliton is called dark soliton. But if the solution is in terms of sech function, soliton is called bright soliton. In the view of such information,the solution in Eq. (3.3) is dark-bright soliton solution and the solution in Eq. (3.5) is dark soliton solution.

Remark: The solutions of Eq. (1.1) have been checked by using Mathematica Release 9. To our knowledge, the solutions of Eq. (1.1) are new and are not found in the previous literature.

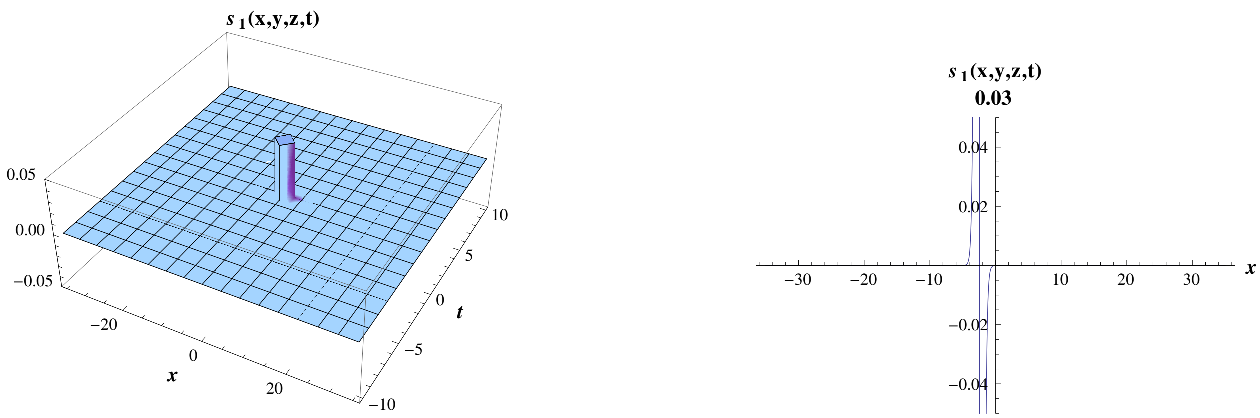

Figure 1. The 3D and 2D surfaces of Eq. (3.3) for $a = -0.2$, $A_2 = 2$, $B_0 = 1$, $p = 4$, $q = 5$, $r = 3$, $l = 1$, $m = 2$, $n = 3$, $c = -1$, $y = 0.01$, $z = 0.02$, $E = 0.5$, $-35 < x < 35$, $-10 < t < 10$ and $t = 0.03$ for 2D surface.

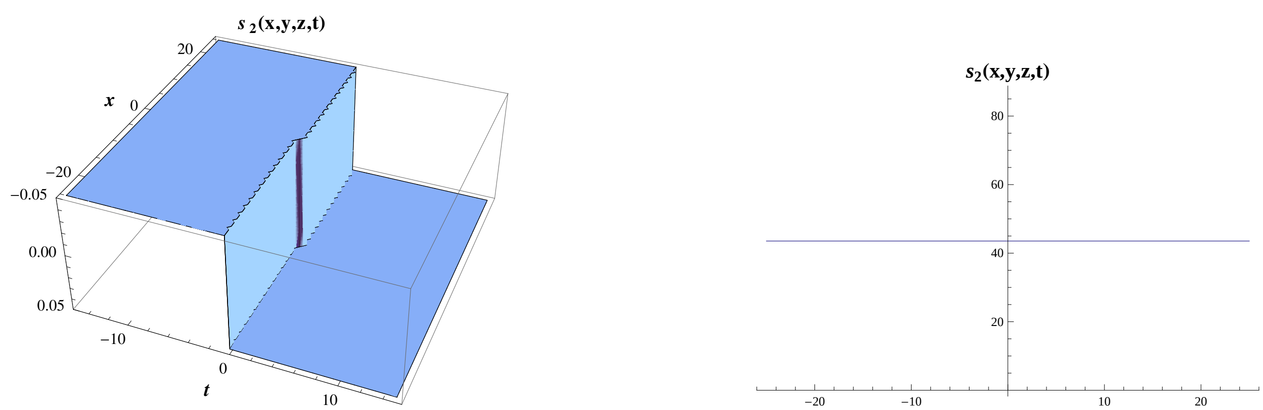

Figure 2. The 3D and 2D surfaces of Eq. (3.5) for $a = -0.4$, $A_2 = 5$, $B_0 = 3$, $p = 2$, $q = 4$, $r = 1$, $l = 2$, $m = 2$, $n = 2$, $c = -3$, $y = 0.03$, $z = 0.01$, $E = 0.2$, $-25 < x < 25$, $-15 < t < 15$ and $t = 0.02$ for 2D surface.

4. Conclusion

In present work, we use MEFM to find exact solutions of (3+1)-dimensional MQZK equation. Then, we plot 2D and 3D surfaces of dark and dark-bright optical soliton solution of this equation via Mathematica.

In the light of these datas, it has been concluded that MEFM is highly reliable and powerful in the sense that finding exact solutions. Thus, it can be said that this method has an important position to obtain exact solutions of NLEEs.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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