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Environmental Change Effect on Oxygen-Plankton System: Mathematical Approach

Yadigar Şekerci Firat

Department of Mathematics, Faculty of Science and Arts, Amasya University, 05189, Turkey.

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Abstract. Oxygen loss in a water body is a significant environmental issue, also accountable for the extinction of marine fauna. Therefore, oxygen production by marine phytoplankton photosynthesis is thought to hold the key to the underlying structure of oxygen dynamics in the marine ecosystem. However, the oxygen concentration is not only determined by primary production. This oxygen concentration also depends on its consumption such as biochemical reaction in the water body, consumption by marine animals, water-air reaeration, etc. Plankton respiration is one of the these factors that play an important role in water body oxygen concentration. Therefore, in this work, this issue is addressed theoretically by considering the oxygen-phytoplankton-zooplankton model to make an insight into system dynamics under the effect of changing environmental condition on phytoplankton growth rate. The difference between this work and the literature is the specific paremeter value of B, suggesting that the changing environment has an effect on the growth rate of phytoplankton. A nonlinear mathematical model is considered to investigate the effect of temperature on oxygen-plankton dynamics with Holling Type II function as respiration of plankton species. The model is studied in both theoretical and numerical forms, based on the existence and behavior of the system's steady state. Moreover, it is noticed from the simulation, oxygen depletion can arise if the temperature exceeds a certain critical level. Interestingly, in a certain parameter range, our model shows the formation of spatial patterns that are qualitatively the same to those observed in field observations.

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Keywords: Differential equation, oxygen consumption, respiration, growth rate.

1. Introduction

Marine ecosystems are of considerable interest for several reasons, in particular, because of their impact on the atmospheric oxygen production [15]. At least one-half of the overall photosynthetic production on Earth is carried out by phytoplankton, while more than a quarter of the Earth's surface is occupied by aquatic ecosystems and more than half of the oxygen production consumed by living beings is regulated by marine phytoplankton [5,9,13]. Phytoplankton is, therefore, one of the main factors in sustaining the life cycle and affecting the global carbon cycle by consuming half of the carbon dioxide that influences the global climate on the planet [6].

Marine ecosystem dynamics are influenced by external factors such as wind speed, sunlight, salinity, eutrophication, climate, and so on. In addition, the temperature is thought to be one of the fundamental controlling variables for the

Email addresses: yadigarsekercii@gmail.com (Y. Şekerci Fırat)
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concentration of oxygen in the water body, which is why it is responsible for shifts impacting primary production in the aquatic environment [11]. Any shift in the temperature of the surrounding water influences the phytoplankton photosynthetic process due to its limited growth and photosynthetic speed [2, 8, 11, 14, 18]. In fact, temperature plays a role in the metabolism of aquatic organisms. Warmer water leads to higher metabolism and respiration due to an increase in cell division under the influence of rising temperatures [1, 7].

Production of oxygen by marine phytoplankton is consumed in some chemical processes and in respiration by marine animals and biota. In this work, the consumption term of oxygen is taken into account as respiration by marine plankton as it was detailed in [19]. The difference between this study and [19], the environmental situation change affecting phytoplankton growth rate dynamics, is discussed here. Net oxygen production is determined by phytoplankton primary production difference from oxygen consumption. In eutrophic zone, consumption of oxygen is greater than production and then oxygen concentration of the water goes down. This situation determines the fate of living organisms in water [3, 12]. The above problem is addressed through mathematical modelling.

2. Mathematical Model

A mathematical model of oxygen and plankton dynamics is considered in this section [19]. First, a non-spatial system that applies to a well-mixed ecosystem is introduced. The oxygen dynamics, which, in turn, is controlled by its main producer of phytoplankton, which, in turn, is grazed by its predatory zooplankton, is described in the following model:

$$
\begin{aligned}
\frac{dc}{dt} &= Af(c)u - mc - u_r(c,u) - v_r(c,v),\\
\frac{du}{dt} &= g(c,u)u - e(u,v) - \sigma u,\\
\frac{dv}{dt} &= e(u,v)\,\kappa - \mu v.
\end{aligned}
$$

The non-dimensional representation of the model is transformed into the following form:

$$\frac{dc}{dt} = \frac{Au}{c+1} - mc - \frac{uc}{c+h_2} - \frac{\nu c v}{c+h_3}, \tag{2.1}$$

$$\frac{du}{dt} = \left(\frac{Bc}{c+h_1} - u\right)\gamma u - \frac{uv}{u+h} - \sigma u, \tag{2.2}$$

$$\frac{dv}{dt} = \left(\frac{\beta uv}{u+h}\right)\frac{c^2}{c^2+{h_4}^2} - \mu v. \tag{2.3}$$

Here c, u, and v are the concentration of oxygen, the density of phytoplankton and zooplankton, respectively, at the time of t. The positive terms for all components of the process describe the growth aspects. The first negative part of oxygen is for natural loss, the second term $Af(c)$ represents the level of oxygen production per unit of phytoplankton mass. The first negative term for phytoplankton describes the predation and the last term of phytoplankton is for natural depletion. Finally, the negative term for zooplankton is for the natural depletion. One can see [19] for more details on the prey-predator model construction.

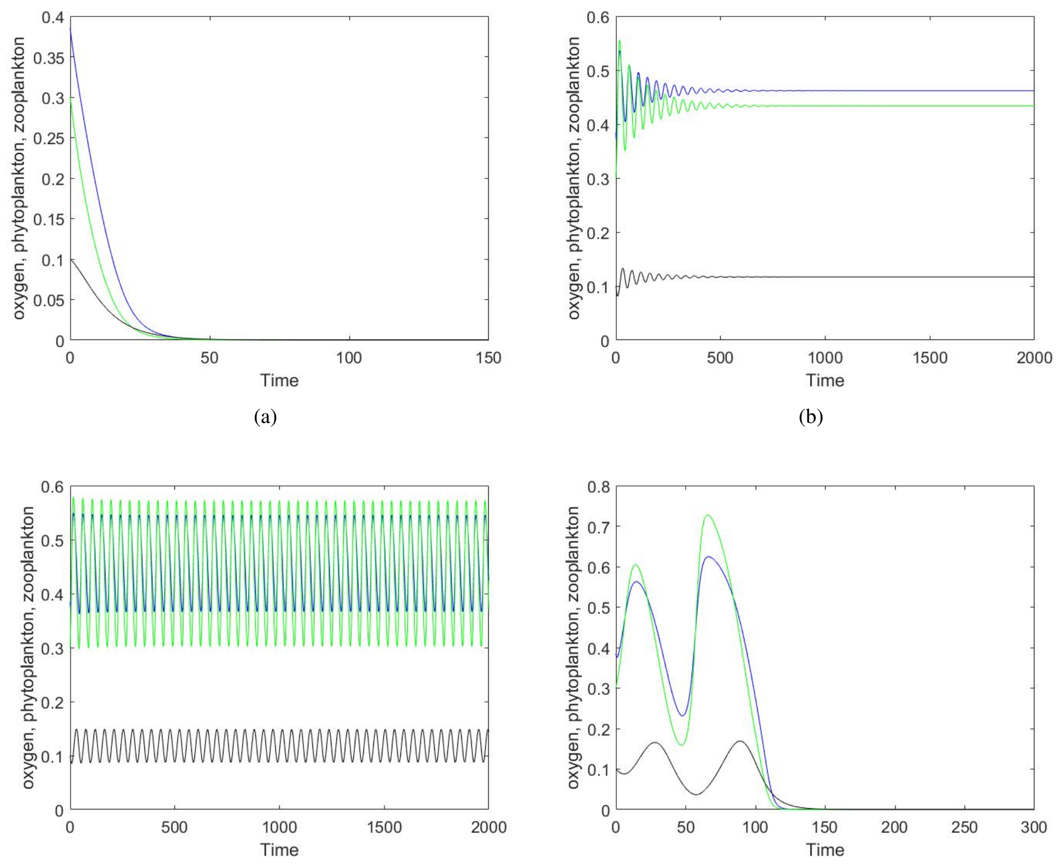
The question here is what kind of relationship occurs between the term respiration and the growth of phytoplankton. In the context of the biological papers, there is a direct link between the concentration of oxygen in the water body and the abundance of phytoplankton owing to photosynthetic oxygen release production as a waste product of the photosynthetic cycle [17]. Phytoplankton needs oxygen like other living organisms in the aquatic environment to be used in its metabolic processes, e.g. development, respiration, etc. [10]. Considering u_r as a function of phytoplankton respiration, therefore u_r can be assumed as the functional response of Holling type II.

In order to build the function of zooplankton respiration, we need to recognize the basic process of its biological viewpoints. Zooplankton respiration relies on shifts in the level of oxygen in the surrounding water [19]. The respiration rates are roughly independent in the oxygen-rich environment, but in the oxygen-poor environment, there are different respiration rates that can be used to explain our choice of zooplankton respiration function [4].

The system's steady state is determined in [19, 20], but only those where the case of extinction can be explicitly stated, and then numerical methods need to be invoked.

3. Numerical Simulations

3.1. **Temporal Dynamics.** In this section, the nonspatial system (2.1-2.3) numerical simulations are conducted. In all of the subsequent numerical simulations, the variables are set at some theoretical values as well $\gamma = 1.2$, $\sigma = 0.1$, , $c_1 = 0.7$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1$, $\nu = 0.01$, $\beta = 0.7$, $\mu = 0.1$, $h = 0.1$ and vary B and A in a certain range. Here, we are interested in the temporal dynamics of the increase in the A factor, which accounts for the environmental changes in our system, as the that term of oxygen is of concern. Fig. 1 displays the concentration of oxygen and the density



(a) (b) (c) (d)

Figure 1. Effect of changes in parameter B, i.e., phytoplankton growth rate. The density of oxygen, phytoplankton and zooplankton against time obtained for other given parameter values (a) $B = 1.45$, (b) $B = 1.76$, (c) $B = 1.8$, and (d) $B = 1.84$. The initial conditions are $c_o = 0.4$, $u_o = 0.3$, $v_o = 0.1$ for given oxygen production rate as $A = 2.02$.

of plankton versus time for the specified parameters. For $A = 2.02$ for a different value of B (Fig. 1a), all system components will be depleted over time. (Fig. 1b), where the state coexistence system has a Hopf bifurcation point and (Fig. 1c) $E_3$is a stable focus the system produces periodic oscillations.

3.2. **Spatial Dynamics.** Here, the spatial extension of the system (2.1-2.3) is focused.

$$\frac{\partial c}{\partial t} = D_c \frac{\partial^2 c}{\partial x^2} + A(1 - \frac{c}{c+1})u - c - \frac{uc}{c+c_2} - \frac{vcv}{c+c_3}, \tag{3.1}$$

$$\frac{\partial u}{\partial t} = D_u \frac{\partial^2 u}{\partial x^2} + (\frac{Bc}{c+c_1} - u)\gamma\, u - \frac{uv}{u+h} - \sigma u, \tag{3.2}$$

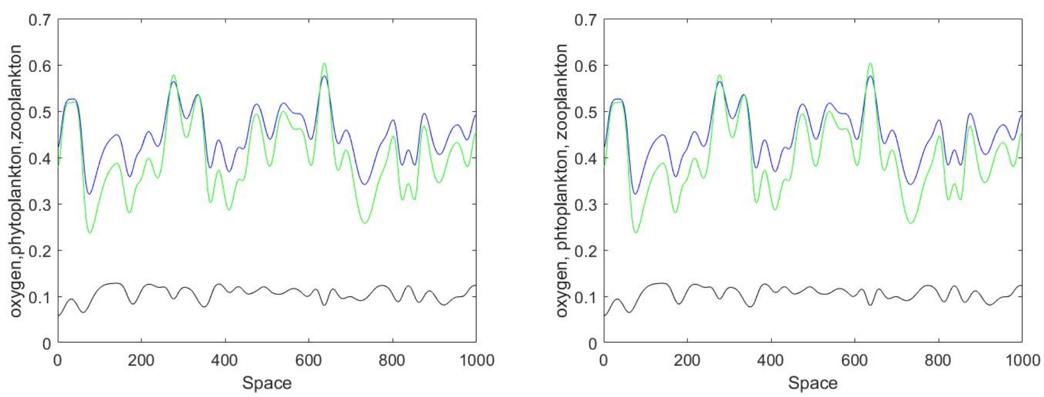
$$\frac{\partial v}{\partial t} = D_v \frac{\partial^2 v}{\partial x^2} + (\frac{\beta uv}{u+h})\frac{c^2}{c^2+{c_4}^2} - \mu v. \tag{3.3}$$

$c = c(x,t)$ is the oxygen concentration and $u = u(x,t)$ and $v = v(x,t)$ are the phytoplankton and zooplankton densities, respectively, at time t and location x with the turbulent diffusion coefficient [16]. The specific form of the model Eqs. (3.1-3.3) and assumptions about the model structure can easily be found in [cf. [19]] and the sources therein.

With the assist of given initials as in Eqs.(3.4), obtained numerical results produce acceptable results in terms of nature. The initial distribution of the species is patchy for zooplankton with evenly distributed oxygen and phytoplankton in space:

$$c(x,0) = c_0, \quad u(x,0) = u_0, \quad v(x,0) = (x - \frac{L}{2})\frac{\tau}{L} + v_0, \tag{3.4}$$

where c_0, u_0 and v_0 are the steady states of coexistence and with the patch size τ as $\tau = 0.01$. Differential equations. (3.1-3.3) are solved finite difference method using zero-flux boundary conditions for given initial values as in Eqs.(3.4).



(a) a (b) b

Figure 2. The impact of changing the B parameter on the distribution of oxygen, phytoplankton and zooplankton over space obtained for other parameters (a) $B = 1.45$, (b) $B = 1.84$ for fixed value of A as $A = 2.08$ and for initial distribution Eqs. (3.4) at *time* = 2000.

(Fig. 2) displays the emergence of the spatial distribution of the system components. In comparison to the non-spatial system species extinction non-spatial case the spatial system shows patchy distribution in (Fig. 2b).

4. Discussion

We studied oxygen, phytoplankton and zooplankton dynamics using a mathematical model that takes into account both the effect of zooplankton predation on phytoplankton and plankton respiration. The model is represented by a system of three combined ordinary differential equations in a non-spatial case and by three corresponding diffusion-reaction PDE's in a spatially specific case. Spatially explicit extension of the system where plankton and oxygen are transferred through turbulent water flows and diffusion-reaction models of phytoplankton dynamics have previously been used to explain this process, e.g. see [16]. Spatiotemporal dynamics have been shown to result in the formation of patchy patterns with different system parameters contributing to the real-world environment.

The difference between this research and the literature is that here the impact of environmental change on the growth rate of phytoplankton is based. In contrast to the previous works [19,20], the dynamic structure of the oxygen-plankton system is investigated in this research by adjusting the B variable. The results presented here improve understanding of the effect of the existing environmental problem i.e. climate change on the oxygen-plankton model, and demonstrate that collapse is unavoidable for all system components at high water temperatures.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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