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Araştırma Makalesi/Research Article

PORTFOLIO OPTIMIZATION WITH LINEAR PROGRAMMING BASED ON TRAPEZOIDAL FUZZY NUMBERS

YAMUK BULANIK SAYILARA DAYALI DOĞRUSAL PROGRAMLAMA İLE PORTFÖY OPTİMİZASYONU

Serkan AKBAŞ * Türkan ERBAY DALKILIÇ**

Abstract

In today's developing financial markets, various complex techniques are used in the creation of portfolios that will provide the best return to the investors. In this study, a portfolio selection model that includes investment data and expert opinions is proposed. This model consists of two stages. In the first stage, the weight of the criteria in the portfolio selection problem was determined by the Constrained Fuzzy Analytic Hierarchy Process method proposed by Enea and Piazza. In the second stage, the model proposed by Lai and Hwang was used to solve the problem of fuzzy linear programming to be formed by using the determined criteria weights. These two methods in the literature use triangular fuzzy numbers (TFNs) in the solutions process of the problem. The methods used in this study were developed for trapezoidal fuzzy numbers (TrFNs) and an alternative method for portfolio selection problems was proposed.

Keywords: Fuzzy Analytic Hierarchy Process (FAHP), Fuzzy Logic, Portfolio Selection, Decision Analysis, Finance

Öz

Günümüzün gelişmekte olan finansal piyasalarında, yatırımcılara en iyi getiriyi sağlayacak portföylerin oluşturulmasında çeşitli karmaşık teknikler kullanılmaktadır. Bu çalışmada, yatırıma ilişkin verilerin ve uzman görüşlerinin de dikkate alındığı bir portföy seçim modeli önerilmiştir. Model iki aşamadan oluşmaktadır. İlk aşamada portföy seçim problemindeki kriterlerin ağırlığı, Enea ve Piazza tarafından önerilen Kısıtlı Bulanık Analitik Hiyerarşi Süreci yöntemiyle belirlenmiştir. İkinci aşamada, Lai ve Hwang tarafından önerilen model, belirlenen kriterlerin ağırlıkları kullanılarak oluşturulan bulanık doğrusal programlama problemini çözmek için kullanıldı. Literatürdeki bu iki yöntemde, problemin çözüm sürecinde üçgen bulanık sayılar kullanılmaktadır. Bu çalışmada, kullanılan bu iki yöntem yamuk bulanık sayılar için geliştirilmiş ve portföy seçim problemleri için alternatif bir yöntem önerilmiştir.

Anahtar Kelimeler: Bulanık Analitik Hiyerarşi Süreci, Bulanık Mantık, Portföy Seçimi, Karar Analizi, Finans

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GENİŞLETİLMİŞ ÖZET

Çalışmanın Amacı: Gelişen teknoloji ile birlikte yatırımcılar bilgisayar yazılımlarından faydalanarak yatırımlarını yönlendirmeye başlamışlardır. Bu yazılımların yatırımcının görüşlerini dikkate almaması, ortak yatırım kümesine sahip bütün yatırımcılar için aynı risk seviyesinde aynı beklenen getiriye ulaşmalarına neden olmaktadır. Bu yüzden bu yazılımlar bir takım bilgisayar uygulamalarının ötesine geçememiştir. Bu soruna çözüm üretmek amacıyla, bu çalışmada yatırıma ilişkin verilerin ve uzman görüşlerinin de dikkate alındığı bir portföy seçim modeli önerilmiştir. Önerilen model, literatürde üçgen bulanık sayıları kullanan iki yöntemin, yamuk bulanık sayılar için geliştirilerek birleştirilmesi ile oluşturulmuştur. Önerilen modelde yamuk bulanık sayıların kullanılma nedeni, yamuk bulanık sayıların optimal çözüm açısından üçgen bulanık sayılara kıyasla daha esnek ve başarılı sonuçlar vermesidir. Önerilen model ile bulanık doğrusal programlama probleminin, bulanık kısıtların ve uzman görüşlerinin kullanılabildiği alternatif bir portföy seçim yöntemi sunulmuştur. Önerilen yöntemin, finansal piyasalardaki belirsizlikler karşısında yatırım yapmayı planlayan tasarruf sahiplerine etkin yatırım yapma konusunda yardımcı olacağı düşünülmektedir.

Araştırma Soruları: Portföy seçim sürecinde finansal oranlar ile uzman görüşlerinin birlikte kullanılmasının etkisi nedir? Kısıtlı Bulanık Analitik Hiyerarşi Süreci yönteminden elde edilen ağırlıkların kesin sayıya dönüştürülmeden kullanılması mümkün mü? Portföy seçim modeline yamuk bulanık sayıların kullanımının katkısı nedir?

Literatür Araştırması: Bu çalışma kapsamında portföy seçimi probleminin çözümünde Enea ve Piazza tarafından önerilen Kısıtlı Bulanık AHP yöntemi kullanılmıştır. Ulusal ve uluslararası literatür incelendiğinde, bu yöntemin kullanıldığı birçok çalışma görülmektedir. Enea ve Piazza, birden fazla proje seçeneği içinden en iyisinin seçilmesi için Bulanık AHP yöntemini kullanmışlardır. Çalışmada, Bulanık AHP'de Genişletilmiş Analiz Yönteminin eksikliklerinden bahsedilmiş ve bu eksikliği giderecek bir yaklaşım önerilmiştir. Bu yaklaşımda, bulanık sayıların aralık değerlerinin azaltılmasıyla belirsizliğin azaltılacağı belirtilmiştir (Enea ve Piazza, 2004). Tiryaki ve Ahlatçıoğlu Bulanık AHP yöntemi ile portföy seçimi problemlerini birleştiren yeni bir yöntem önermişlerdir. Bu yöntem ile bir yatırımcının portföyüne hangi hisse senedinden ne kadar alınması gerektiğinin belirlenmesini kolaylaştırmayı amaçlamaktadırlar. Bunu yapmak için önce Enea ve Piazza tarafından verilen bulanık AHP yöntemini ele almışlardır. Bu yöntemdeki bazı hatalar revize edilerek, Revize Edilmiş Kısıtlı Bulanık AHP yöntemini önermişlerdir (Tiryaki ve Ahlatçıoğlu, 2009). Ahari ve diğerleri, Tahran borsasında bazı ilaç şirketlerinin hisse senetleri arasında sınırlı bir fon tahsis etmeyi planlamışlardır. Çalışmalarında Enea ve Piazza ile Van Laarhoven ve Pedrycz tarafından sunulan iki Bulanık AHP yöntemi kullanmışlardır (Ahari vd., 2011). Bu çalışmanın daha sonraki aşamasında amaç fonksiyonundaki karar değişkenlerinin bulanık sayı olduğu bir doğrusal programlama problemi oluşturulmuştur. Bu doğrusal programlama probleminin çözümünde Lai ve Hwang tarafından önerilen yöntemden yararlanılmıştır. Lai ve Hwang 1992 yılında yapmış oldukları çalışmada doğrusal programlama probleminde amaç fonksiyonundaki karar değişkenlerinin katsayılarının belirsiz olduğu durumu ele almışlardır ve bu durumdaki problemlerin çözülebilmesi için yeni bir yöntem önermişlerdir (Lai ve Hwang, 1992).

Yöntem: Bu çalışmada önerilen algoritma iki aşamadan oluşmaktadır. İlk aşamada, portföy seçim sürecinde kullanılacak kriterlerin ağırlıklarının hesaplanmasında Kısıtlı Bulanık AHP yöntemi ele alınmıştır. Literatürde üçgen bulanık sayılar kullanan yöntem, bu çalışmada yamuk bulanık sayılar için geliştirilmiştir. İkinci aşamada ise, ilk aşamada geliştirilen yöntemden elde edilen bulanık ağırlıkların, amaç fonksiyonundaki fiyat değişkeni olarak kullanıldığı, bir bulanık doğrusal programlama problemi oluşturulmuştur. Bu problemin çözümü için amaç fonksiyonundaki fiyat değişkenlerinin üçgen bulanık sayı olarak kullanıldığı Lai ve Hwang tarafından önerilmiş olan yöntem ele alınmıştır. Bu yöntemin üçgen bulanık sayı odaklı teorik altyapısını, yamuk bulanık sayı kullanımına uygun hale getirebilecek matematiksel modellemeler yapılmış ve sonuç olarak birinci ve ikinci aşamada yamuk bulanık sayılar için geliştirilen yöntemlerin bir arada kullanıldığı bir portföy seçim algoritması önerilmiştir.

Sonuç ve Değerlendirme: Son olarak çalışmada, önerilen modelin etkinliğini irdelemek üzere literatürde bulunan örnek bir portföy seçim problemi ele alınmıştır. Problem önerilen yönteme göre uyarlanmış ve çözülmüştür. Problemin önerilen yöntem ile çözülmesi sonucunda; yatımcının fonunun Anadolu Cam, Trakya Cam, Mardin Çimento, Ereğli Demir Çelik ve İzmir Demir Çelik hisse senetlerine sırası ile %0, %0, %22, %0 ve %78 oranlarında tahsis edilebileceği belirlenmiştir.

INTRODUCTION

In today's developing and liberalized financial markets, many different and complex techniques are used in the creation of portfolios that will provide the best return according to the risk levels that investors can take. The classical mean-variance model, which is considered as the basis of modern portfolio theory, was developed by Markowitz (Markowitz, 1952). According to Markowitz, it is not possible to reduce the risk by merely diversifying. The relationship between the returns is taken into account when making portfolio diversification with the Markowitz model. Fuzzy Logic approach, which is started by Zadeh and has many applications, is one of the techniques widely used in optimal portfolio selection. There are many studies in the literature that use the portfolio selection process and the fuzzy logic approach. Sadjadi et al. address fuzzy linear programming method, which determines the amount of investment in different time periods. They expressed the rate of return and borrowing rates as triangular fuzzy numbers (TFNs). Using fuzzy set theory, they developed a model for the cash amount and profits of investors (Sadjadi et al., 2011). Lukovac et al. proposed a new model for developing a human resources portfolio based on a neuro-fuzzy approach. The purpose of their model is to enable insight into the existing potential and plan assets to improve and promote the employees' potential in a company (Lukovac et al., 2017). In their study, Devran and Deniz compared differences between Markowitz's Modern Portfolio Theory and Traditional Portfolio Theory (Deniz and Okuyan, 2018). Deniz et al. researched diversification benefit of gold, platinum and silver for stock portfolio in Istanbul Stock Exchange (BIST) between April 1999- April 2018 periods (Deniz et al., 2018). Jafarzadeh et al. proposed a new method in their paper. This method combines Quality Function Development (QFD), fuzzy logic, and Data Envelopment Analysis (DEA) to accounts for prioritisation, uncertainty and interdependency (Jafarzadeh et al., 2018). Wang et al. introduced the Sharpe ratio in fuzzy environments and proposed a fuzzy Value-at-Risk ratio in their study. They built a multi-objective model based on these to ratios, to evaluate their joint impact on portfolio selection. Finally, they justified the superiority of algorithm by comparing with existing solvers on benchmark problems and exemplified the model effectiveness by using three case studies on portfolio selection (Wang et al., 2018). In their work, Kim and Kim developed a new model for the optimal Liquefied Natural Gas (LNG) import portfolio. Their model consists of a two-step portfolio model combining the mean-variance (MV) portfolio and the linear programming (LP) model (Kim and Kim, 2018). The purpose of Topaloglu's study is revealing the relationship between financial risks and firm value. He used panel data analysis method in his study (Topaloglu, 2018). Liagkouras proposed a new algorithm for the solution of portfolio optimization problem. He tested the performance of the proposed algorithm to the optimal allocation of limited resources to a number of competing investment opportunities for optimizing the objectives (Liagkouras, 2019). In their work, Bolos et al. developed a modern and innovative management tool based on the artificial intelligence technique and the use of systems with fuzzy logic for companies to substantiate investment decisions in assets purchased from the market (Bolos et al., 2019).

Another technique used effectively in portfolio selection is the Analytic Hierarchy Process (AHP). This method was developed by Saaty to solve many complex problems. All the criteria that affect the stock prices affect the selection of investors in the optimal portfolio selection. Therefore, it is a complex and multi-criteria decision problem for investors to choose the appropriate stocks and decide on which ratios they will form the portfolio. In the AHP, many criteria affecting stock prices are handled in a hierarchy, making the complex structure more regular. In this method, the relative importance of the criteria is determined by the decision makers. Decision makers use linguistic expressions when comparing. The linguistic variables under the uncertain evaluations in their verbal judgments can be expressed in more rational terms with fuzzy numbers. Fuzzy AHP method has been developed by using fuzzy numbers in comparisons due to this feature of fuzzy numbers. Tanaka and Asai used objective function coefficients and right-hand side coefficients of constraints as fuzzy functions in their studies (Tanaka and Asai, 1984). Nakamura solved the multi-objective linear programming models, which are represented by triangular membership functions, by transforming them into fuzzy linear programming models with partial membership functions (Nakamura, 1984). Chang defined a new approach for handling fuzzy AHP, with the use of TFNs for pairwise comparison scale of fuzzy AHP (Chang, 1996). Tiryaki and Ahlatcioglu proposed a new method for group decision making in fuzzy environment. In their paper, they used TFNs for the rating of each stock and the weight of each criterion. They aimed to provide investors with information about ranking and weighting (Tiryaki and Ahlatcioglu, 2005). Chen and Cheng proposed a fuzzy multi criteria decision making methodology in ranking portfolios of information sourcing projects under uncertainty conditions (Chen and Cheng, 2009). Yucel and Guneri expressed the linguistic values as TrFNs to assess the weights of the factors. They obtained the weights by calculated the distances of each factor between Fuzzy Positive Ideal Rating and Fuzzy Negative Ideal Rating (Yucel and Guneri, 2011). In their study, Xiao and Fu proposed a grey-correlation multi-attribute decision-making method based on intuitionistic TrFNs to solve the problem that the attribute weight depends on the various statuses and the attribute values offer multi-attribute decision making in the form of intuitionistic TrFNs (Xiao and Fu, 2015). In their article, Solimanpur et al. presented a new model for optimal portfolio selection using the genetic algorithm and AHP (Solimanpur et al., 2015). Piasecki and Siwek focused on describing the imprecision risk for the portfolio rather than describing its uncertainty. They used the present values of portfolio assets as TrFNs in their study (Piasecki and Siwek, 2018). In their paper, Chatterje et al. used fuzzy AHP for project prioritization in portfolio management (Chatterje et al., 2018).

With the developing technology, investors started to direct their investments by using computer software programs. However, since these programs do not consider the investor's opinions, they provide the same expected return on the same risk level for all investors. With this drawback, investment related existing software packages can be considered somewhat inefficient. In this study, a model including expert opinions has been proposed. The model consists of two stages. In the first stage, the Constrained Fuzzy AHP method proposed by Enea and Piazza was developed for TrFNs and the weights of the criteria were determined. In the second stage, linear programming problem has been established in which the weight of the criteria obtained as TrFNs is used as price variables in the objective function. The model proposed by Lai and Hwang has been developed for TrFNs in order to solve the linear programming problem created by using TrFNs. In this way, as an alternative to the methods in which the expert opinions are used as triangular fuzzy numbers in the literature, a new method using trapezoidal fuzzy numbers is proposed. In order to examine the effectiveness of the model, a sample portfolio selection problem in the literature is discussed. The results obtained from the existing methods and the results obtained from the proposed model were compared.

1. CONSTRAINED FUZZY AHP

The Constrained Fuzzy AHP method focuses on the constraints within the fuzzy AHP in order to take for all available information into consideration. This method is also used to calculate the weights of alternatives in the portfolio selection process. The weights of the alternatives are calculated with the Constrained Fuzzy AHP method using TFNs . The formulas used in the calculations are given in Equation (1-3). Let $S_i = (S_{li}, S_{mi}, S_{ui})$ be the fuzzy score for the i^{th} criterion of triangular fuzzy pairwise comparison matrix, where the indices l, m and u denote its lower, medium and upper respectively. According to the Constrained Fuzzy AHP method proposed by Enea and Piazza, the center value of the fuzzy score related to i^{th} criterion (S_{mi}) calculated by Equation (1) (Enea and Piazza, 2004).

$$S_{mi} = \left(\prod_{j=1}^{n} m_{ij}\right)^{\frac{1}{n}} \Bigg/ \sum_{k=1}^{n} \left[\left(\prod_{j=1}^{n} m_{kj}\right)^{\frac{1}{n}}\right] \qquad (i,j,k = 1,\dots,n) \tag{1}$$

S_{li} can be evaluated using the crisp mathematical programming model:

$$S_{li} = min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Bigg/ \sum_{k=1}^{n} \left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right] \qquad (i,j,k = 1,\dots,n) \tag{2}$$

subject to,

$$a_{kj} \in [l_{kj}, u_{kj}], \quad \forall j > k; \quad a_{jk} = \frac{1}{a_{kj}}, \quad \forall j < k; \quad a_{jj} = 1$$

and similarly, S_{ui} can be evaluated using the crisp mathematical programming model:

$$S_{ui} = max \left[\left(\prod_{j=1}^{n} a_{ij} \right)^{\frac{1}{n}} \right] \Big/ \sum_{k=1}^{n} \left[\left(\prod_{j=1}^{n} a_{kj} \right)^{1/n} \right] \qquad (i, j, k = 1, \dots, n) \tag{3}$$

subject to,

$$a_{kj} \in [l_{kj}, u_{kj}], \quad \forall j > k; \quad a_{jk} = \frac{1}{a_{kj}}, \quad \forall j < k; \quad a_{jj} = 1$$

Tiryaki and Ahlatcioğlu used the Fuzzy Analytic Hierarchy Process in the problem of portfolio selection. They are intended to decide the content of the portfolio that will be created. To do this, they handled the fuzzy AHP method given by Enea and Piazza. And they proposed Revised Constrained Fuzzy AHP method by revising some mistakes in this method

(Tiryaki and Ahlatcioglu, 2009). Ahari et al. planned to allocate a limited funds among the stocks of some pharmaceutical companies in the Tehran stock market, in their study. They used two fuzzy AHP method which proposed by Enea - Piazza and Van Laarhoven – Pedrycz (Ghazanfar Ahari et al., 2011). In his study, Yaghoobi apply the fuzzy analytic hierarchy process (AHP) to the issue of ordering key success factors (KSFs) for software development projects. To do this, Yaghoobi first simplified the constrained fuzzy AHP method, and then from systematic literature reviews, a preliminary list of potential KSFs that influences software development projects was identified and compiled (Yaghoobi, 2018).

2. THE CASE THAT THE PRICE VARIABLES IN THE OBJECTIVE FUNCTION ARE FUZZY

An efficient method to handle the uncertain parameters of a LP is to express the uncertain parameters by fuzzy numbers which are more realistic and create a conceptual and theoretical framework for dealing with imprecision and vagueness.

In their study Lai and Hwang discussed the situation where the coefficients of the price variables in the objective function of the linear programming problem are uncertain (Lai and Hwang, 1992). In their work, they aim to maximize the highest possible value of the uncertain profit, minimize the risk of lower profits, and maximize the possibility of higher profits. If the c_i parameter corresponding to the prices in the objective function is a fuzzy number, the linear programming problem is modeled as given in Equation (4).

$$\max_{x \in X} \sum_{i=1}^{n} \tilde{c}_i x_i \tag{4}$$

$$(x \in X = \{x \mid Ax \leq b \text{ and } x \geq 0\}$$

Here, the objective function coefficients are TFNs, expressed as $\tilde{c}_i = (c_i^m, c_i^p, c_i^o)$ and have a triangular probability distribution as given in Figure 1.

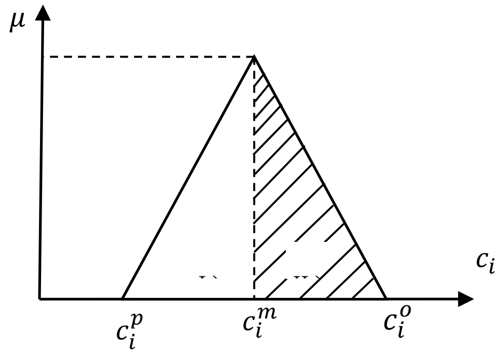


Figure 1. The triangular possibility distribution of $\tilde{c}_i$

c_i^m is the most possible value (possibility = 1 if normalized), c_i^p (the most pessimistic value), and c_i^o (the most optimistic value) are the least possible values.

$$\max_{x\in X} \sum_{i=1}^{n} ((c^m)^T x, (c^p)^T x, (c^0)^T x) \tag{5}$$

$((c^m)^T x, (c^p)^T x, (c^0)^T x)$ is the vector of three objective functions, $(c^m)^T x, (c^p)^T x,$ and $(c^0)^T x$. In order to keep the triangular shape of the possibility distribution, it is necessary to make a little change. Instead of maximizing these three objectives simultaneously, we are going to maximize $(c^m)^T x$, minimize $[(c^m - c^p)^T x]$ and maximize $[(c^0 - c^m)^T x]$, where the last two objective functions are actually relative measures from $(c^m)^T x$, the first objective function (see Figure 2). The three new objectives also guarantee the previous argument of pushing the triangular possibility distribution in direction of the right-hand side.

$$min\, Z_1 = (c^m - c^p)^T x$$
$$max\, Z_2 = c^{mT} x, \tag{6}$$
$$max\, Z_3 = (c^0 - c^m)^T x\,, x \in X.$$

The crisp multi objective linear programming Equation (6) is equivalent to maximizing the most possible value of the imprecise profit. At the same time, we have minimized the inferior side of the possibility distribution. This means minimizing (I) which, in our opinion, is equivalent to "the risk of obtaining lower profit". And, we have also maximized the region (II) of the possibility distribution, which is equivalent to "the possibility of obtaining higher profit".

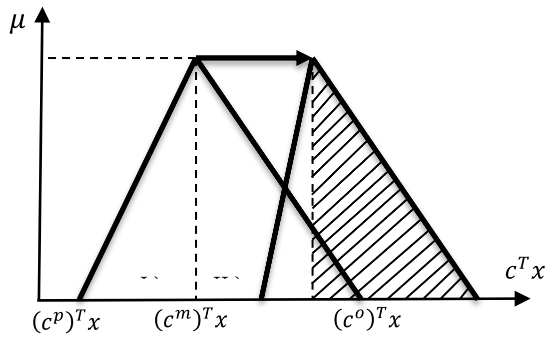


Figure 2. The strategy to solve $max\tilde{c}^T x$

Firstly, the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) of the four objective functions are obtained by Equation (7-9) (Hwang and Yoon, 1981).

$$Z_1^{PIS} = \min_{x\in X}(c^m - c^p)^T x \qquad Z_1^{NIS} = \max_{x\in X}(c^m - c^p)^T x \qquad (7)$$

$$Z_2^{PIS} = \max_{x\in X} c^{mT} x \qquad Z_2^{NIS} = \min_{x\in X} c^{mT} x \qquad (8)$$

$$Z_3^{PIS} = \max_{x\in X}(c^o - c^m)^T x \qquad Z_3^{NIS} = \min_{x\in X}(c^o - c^m)^T x \qquad (9)$$

The linear membership function of these objective functions can now be computed (Equation 10-12) as:

$$\mu_{Z_1} = \begin{cases} 1 & Z_1 < Z_1^{PIS} \\ \dfrac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}} & Z_1^{PIS} \le Z_1 \le Z_1^{NIS} \\ 0 & Z_1 > Z_1^{NIS} \end{cases} \qquad (10)$$

$$\mu_{Z_2} = \begin{cases} 1 & Z_2 < Z_2^{PIS} \\ \dfrac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}} & Z_2^{PIS} \le Z_2 \le Z_2^{NIS} \\ 0 & Z_2 > Z_2^{NIS} \end{cases} \qquad (11)$$

$$\mu_{Z_3} = \begin{cases} 1 & Z_3 < Z_3^{PIS} \\ \dfrac{Z_3 - Z_3^{NIS}}{Z_3^{PIS} - Z_3^{NIS}} & Z_3^{PIS} \le Z_3 \le Z_3^{NIS} \\ 0 & Z_3 > Z_3^{NIS} \end{cases} \qquad (12)$$

This normalization has also been applied by Seo and Sakawa's study (Seo and Sakawa, 1988). Finally, we solve Zimmerman's following equivalent single-objective linear programming model:

$$\begin{aligned} & max\ a \\ & \mu_{Z_i} \ge a, (i = 1,2,3) \\ & x \in X. \end{aligned} \qquad (13)$$

Liu and Gao point out limitations of the existing method to solve fully fuzzy linear programming (FFLP) problem and proposed a modified method to overcome these limitations (Liu and Gao, 2016). The aim of Ebrahimnejad's article is to introduce a formulation of FLP problems involving interval-valued TrFNs for the decision variables and the right-hand-side of the constraints. He proposed a new method for solving this kind of FLP problems based on comparison of interval-valued fuzzy numbers by the help of signed distance ranking (Ebrahimnejad, 2018). Taleshian and Fathali investigated the p-median problem with fuzzy variables and weights of vertices. They showed that the fuzzy objective function also can be replaced by crisp functions in their study (Taleshian and Fathali, 2016). In their study, Dong and Wan developed a new method for the fuzzy linear program in which all the objective coefficients, technological coefficients and resources are TrFNs (Dong and Wan, 2018). Arik and Toksari investigated a multi-objective parallel machine scheduling problem under fully fuzzy environment with fuzzy job deterioration effect, fuzzy learning effect and fuzzy processing times (Arik and Toksari, 2018). In their work, Yu et al. studied on the dual-channel (the traditional channel and the E-commerce channel) supply chain network design (SCND) for Fresh Agri-Product (FAP) under information uncertainty (Yu et al., 2018).

3. AN ALGORITHM BASED ON FUZZY NUMBERS FOR PORTFOLIO SELECTION

Step 1: n being the number of criteria and p being the number of decision makers involved, comparison values (b_{ijp}) relative to each criterion are determined by the decision makers as in Table 1.

Table 1. General form of the comparison matrix of each criterion

	C_1	C_2	$\cdots$	C_n

$\mathbf{C_1}$	$(1\ 1\ 1\ 1)$ ⋮ $(1\ 1\ 1\ 1)$	b_{121} ⋮ b_{12P}	⋯	b_{1n1} ⋮ b_{1nP}
$\mathbf{C_2}$	b_{211} ⋮ b_{21P}	$(1\ 1\ 1\ 1)$ ⋮ $(1\ 1\ 1\ 1)$	⋯	b_{2n1} ⋮ b_{2nP}
⋮	⋮	⋮	$(1\ 1\ 1\ 1)$ ⋮ $(1\ 1\ 1\ 1)$	⋮
$\mathbf{C_n}$	b_{n11} ⋮ b_{n1P}	b_{n21} ⋮ b_{n2P}	⋯	$(1\ 1\ 1\ 1)$ ⋮ $(1\ 1\ 1\ 1)$

A TrFN consist of four parameters indicated as $b^1, b^2, b^3$and b^4 and is expressed as follows:

$$b_{ijp} = (b_{ijp}^1 \quad b_{ijp}^2 \quad b_{ijp}^3 \quad b_{ijp}^4) \tag{14}$$

b_{ijp}:The importance value of m^{th} criteria corresponding to n^{th}criteria, according to p^{th}decision maker.

i, j :Number of criteria $(i = 1, \dots, n, j = 1, \dots, n)$

p :Number of decision maker $(p = 1, \dots, P)$

Step 2: New TrFNs are obtained with pairwise comparisons of n criteria shown in Table 1. For the comparison of C_i and C_j criteria, $b_{ijp} = (b_{ijp}^1 \quad b_{ijp}^2 \quad b_{ijp}^3 \quad b_{ijp}^4)$, where

$$a_{ij}^1 = \left(b_{ij1}^1 * b_{ij2}^1 * \dots * b_{ijp}^1\right)^{\frac{1}{p}}$$
$$\vdots \tag{15}$$
$$a_{ij}^4 = \left(b_{ij1}^4 * b_{ij2}^4 * \dots * b_{ijp}^4\right)^{\frac{1}{p}}, \text{ and}$$

$$A_{ij} = [a_{ij}^1 \quad a_{ij}^2 \quad a_{ij}^3 \quad a_{ij}^4] \tag{16}$$

The new TrFNs, which express the decision maker's opinions, are obtained by repeating the same process n times for each paired comparison, using geometric mean. The obtained new TrFNs are given Table 2.

Table 2. The comparison matrix of each criterion, which consists of the new trapezoid numbers obtained

	$\mathbf{C_1}$	$\mathbf{C_2}$	⋯	$\mathbf{C_n}$
$\mathbf{C_1}$	$(1\ 1\ 1\ 1)$	a_{12}	⋯	a_{1n}
$\mathbf{C_2}$	a_{21}	$(1\ 1\ 1\ 1)$	⋯	a_{2n}
⋮	⋮	⋮	$(1\ 1\ 1\ 1)$	⋮
$\mathbf{C_n}$	a_{n1}	a_{n2}	⋯	$(1\ 1\ 1\ 1)$

Step 3: The weights of the alternatives or criteria are calculated with the proposed method using TrFNs. The formulas used in the calculations are given in Equations (17-20). Let $S_i = (S_{li}, S_{m_1 i}, S_{m_2 i}, S_{ui})$ be the fuzzy score for the i^{th} criterion of trapezoidal fuzzy pairwise comparison matrix, where the indices l, m_1, m_2 and u denote its lower, medium1,medium2 and upper respectively. S_{li} and S_{ui} can be evaluated using the crisp mathematical programming model,

$$S_{li} = min\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Bigg/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right] \quad (i, j, k = 1, \dots, n) \tag{17}$$

$$S_{ui} = max\left[\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}\right] \quad (i,j,k = 1,\dots,n) \tag{18}$$

$a_{in} \in [l_{in}, u_{in}], \forall n > i; a_{ni} = \frac{1}{a_{in}}, \forall n < i;\ \ a_{nn} = 1.$

$S_{m_1 i}$ and $S_{m_2 i}$ are calculated by Equation (19-20).

$$S_{m_1 i} = \left[\left(\prod_{j=1}^{n} m_{1ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{1kj}\right)^{\frac{1}{n}}\right] \quad (i,j,k = 1,\dots,n) \tag{19}$$

$$S_{m_2 i} = \left[\left(\prod_{j=1}^{n} m_{2ij}\right)^{\frac{1}{n}}\right] \Big/ \sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{2kj}\right)^{\frac{1}{n}}\right] \quad (i,j,k = 1,\dots,n) \tag{20}$$

The weights obtained from Step 3 of the algorithm are shown in Table 3.

Table 3. Fuzzy weight of criteria/alternatives

Criteria	Weights
$\mathbf{C_1}$	$(S_{l1}, S_{m_1 1}, S_{m_2 1}, S_{u1})$
⋮	⋮
$\mathbf{C_n}$	$(S_{li}, S_{m_1 i}, S_{m_2 i}, S_{ui})$

Step 4: By using the obtained weights, the linear programming problem given by Equation (21) is modeled.

$$\begin{aligned} & max \sum_{l=1}^{L} w_l \lambda_l + \sum_{t=1}^{T} \beta_t \gamma_t \\ & \lambda_l \le \mu_{Z_l}(x),\ l = 1,\dots,L \text{ (for all objective function)} \\ & \gamma_t \le \mu_{g_t}(x),\ t = 1,\dots,T \text{ (for all fuzzy constraints)} \\ & \sum_{l=1}^{L} w_l + \sum_{t=1}^{T} \beta_t = 1 \\ & g_k(x) \le b_k, k = 1,\dots,K \text{(for deterministic constraints)} \\ & \lambda_l, \gamma_t \in [0,1],\ w_l, \beta_t \ge 0,\ x_m \ge 0, (m = 1,\dots,M) \end{aligned} \tag{21}$$

In Equation (21), w_l and β_t are the weights coefficients that present the fuzzy goals and fuzzy constraints obtained from TrFNs. λ_l and γ_t represents the fuzzy goals and the fuzzy constraint parameters.

Step 5: The four critical parameters of the trapezoid fuzzy weights obtained are pushed to the right as shown in Figure 3, so that the fuzzy goal is maximized.

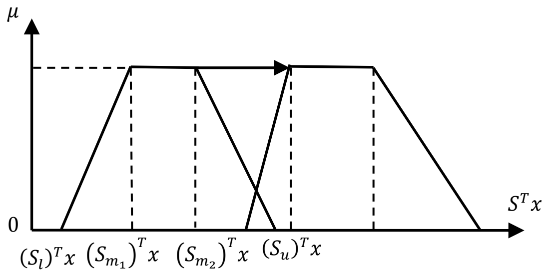


Figure 3. *The strategy to solve* $max\tilde{S}^T x$

The new four objective functions for solving the model are created as in Equation (22):

$$\begin{aligned} &\min Z_1 = \left(S_{m_1} - S_l\right)^T x, \\ &\max Z_2 = S_{m_1}^T x, \\ &\max Z_3 = S_{m_2}^T x, \\ &\max Z_4 = \left(S_u - S_{m_2}\right)^T x, x \in X. \end{aligned} \tag{22}$$

Step 6: In order to solve the equation (22), Zimmerman's fuzzy programming method was used in the normalization process. Firstly, the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) of the four objective functions are obtained by Equation (23-26).

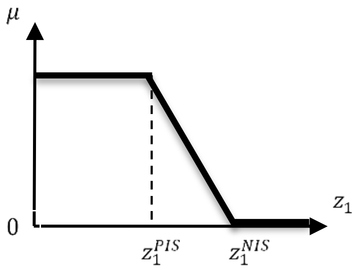
$$Z_1^{PIS} = \min_{x \in X} \left(S_{m_1} - S_l\right)^T x \qquad Z_1^{NIS} = \max_{x \in X} \left(S_{m_1} - S_l\right)^T x \tag{23}$$

$$Z_2^{PIS} = \max_{x \in X} S_{m_1}^T x \qquad Z_2^{NIS} = \min_{x \in X} S_{m_1}^T x \tag{24}$$

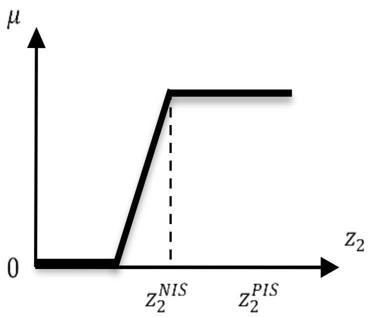
$$Z_3^{PIS} = \max_{x \in X} S_{m_2}^T x \qquad Z_3^{NIS} = \min_{x \in X} S_{m_2}^T x \tag{25}$$

$$Z_4^{PIS} = \max_{x \in X} \left(S_u - S_{m_2}\right)^T x \qquad Z_4^{NIS} = \min_{x \in X} \left(S_u - S_{m_2}\right)^T x \tag{26}$$

Step 7: Then the linear membership function of these objective functions is calculated:



$$\mu_{z_1} = \begin{cases} 1 & z_1 < z_1^{PIS} \\ \dfrac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & z_1^{PIS} \le z_1 \le z_1^{NIS} \\ 0 & z_1 > z_1^{NIS} \end{cases} \tag{27}$$



$$\mu_{z_2} = \begin{cases} 1 & z_2 < z_2^{PIS} \\ \dfrac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}} & z_2^{PIS} \le z_2 \le z_2^{NIS} \\ 0 & z_2 > z_2^{NIS} \end{cases} \tag{28}$$

Step 8: Finally, the results of the distribution of the portfolio are obtained by solving the linear programming model given by Equation (29).

$$\begin{aligned} &\max a \\ &\mu_{Z_i} \ge a, (i = 1,2,3) \\ &x \in X. \end{aligned} \tag{29}$$

4. APPLICATION

In this section, a portfolio selection problem which is present in the literature is discussed. In addition, a solution was obtained by the method based on the TrFNs proposed in this study. In the study, 5 companies were determined using past price movements obtained from BIST; Anadolu Cam, Trakya Cam, Mardin Cimento, Eregli Demir Celik and Izmir Demir Celik. In order to determine the distribution of the portfolio on these five companies, seven criteria were determined as Price/Earnings (P/E), Net Profit/Stockholder's Equity (NP/SE), Net Debt/Market Value (ND/MV), Current Ratio (CR), Market Value/Book Value

(MV/BV), Net Profit/Sales Revenue (NP/SR) and Net Profit/Total Assets (NP/TA). The hierarchical structure of the problem is given in Figure 4 (Ahlatcioglu, 2005).

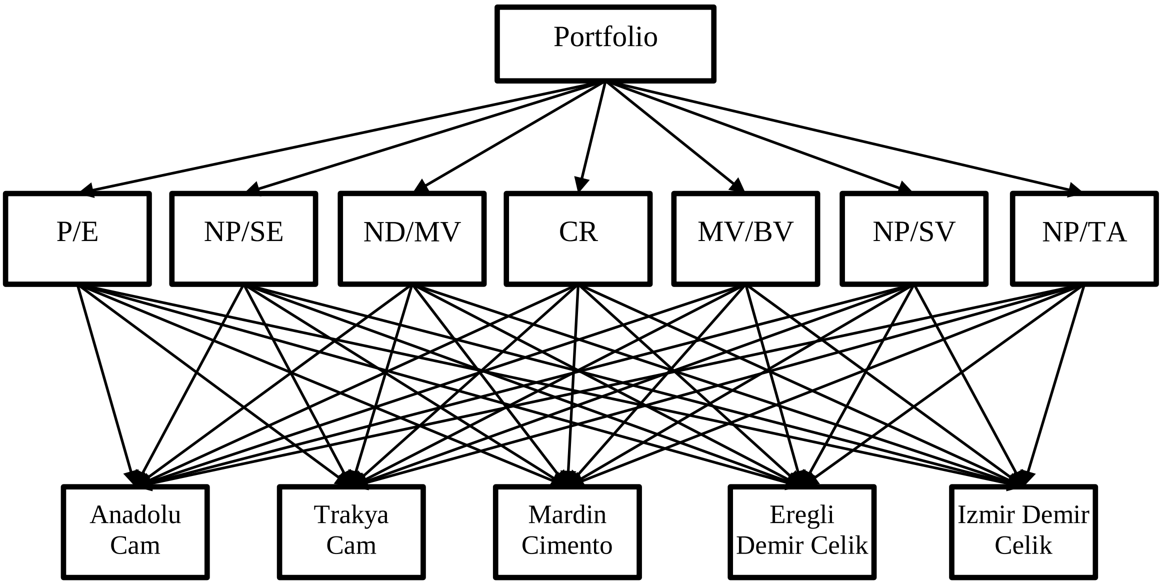


Figure 4. Hierarchy of the problem

Price/Earnings, Net Profit/Stockholder's Equity, Net Debt/Market Value, Current Ratio, Market Value/Book Value, Net Profit/Sales Revenue and Net Profit/Total Assets criteria are represented by C_1, C_2, C_3, C_4, C_5 , C_6 and C_7 respectively. The Current Ratio criterion is usually around 2. In this study, the Current Ratio is defined to be a fuzzy constraint with the value of between 1.4 and 2.5. The ratios of Anadolu Cam, Trakya Cam, Mardin Cimento, Eregli Demir Celik and Izmir Demir Celik are given in Table 4.

Table 4. Stocks and financial ratio

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Anadolu Cam	10.7	11.5	10.0	1.3	12.6	7.6	2.04
Trakya Cam	8.9	12.5	0.2	1.2	18.3	8.6	2.17
Mardin Cimento	7.1	23.4	-26.2	1.9	34.9	20.7	5.83
Eregli Demir Celik	3.6	18.1	-9.0	0.7	18.0	12.7	1.86
Izmir Demir Celik	4.1	30.8	2.8	1.3	9.8	19.9	1.55

In the study of the literature, the importance degrees of the criteria were determined as TFNs by four decision makers and is given in Table 5 (Ahlatcioglu, 2005).

Table 5. Importance degrees or criteria determined by the four decision makers

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
$\mathbf{DM_1}$	(5 7 9)	(7 9 10)	(5 7 9)	(5 7 9)	(7 9 10)	(5 7 9)	(3 5 7)
$\mathbf{M_2}$	(7 9 10)	(5 7 9)	(5 7 9)	(7 9 10)	(7 9 10)	(5 7 9)	(3 5 7)
$\mathbf{M_3}$	(9 10 10)	(7 9 10)	(5 7 9)	(9 10 10)	(7 9 10)	(5 7 9)	(5 7 9)
$\mathbf{M_4}$	(5 7 9)	(9 10 10)	(5 7 9)	(7 9 10)	(9 10 10)	(5 7 9)	(3 5 7)

In this study, these importance degrees are converted to TrFNs as shown in Figure 5.

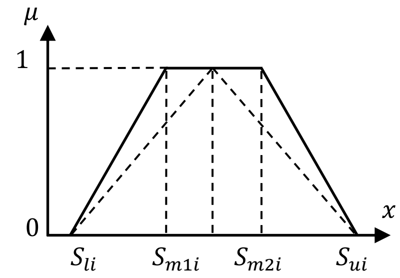


Figure 5. Transform from TFNs to TrFNs

The importance degrees which are TFNs have been converted into TrFNs by keeping the upper and lower bounds of the fuzzy numbers constant and spreading the center to a certain range. The geometric averages of the decision-makers' views, which are converted into TrFNs, are taken. The importance degrees of the criteria obtained are given in Table 6.

Table 6. The importance degrees of criteria given by decision makers transformed into TrFNs

Criteria	Importance Degree
C_1	(6.30 7.69 8.49 9.49)
C_2	(6.85 8.22 8.95 9.74)
C_3	(5.00 6.50 7.50 9.00)
C_4	(6.85 8.22 8.95 9.74)
C_5	(7.45 8.79 9.43 10.00)
C_6	(5.00 6.50 7.50 9.00)
C_7	(3.41 4.93 5.94 7.45)

The seven criteria for the problem are compared with each other according to the purpose of "portfolio selection". The importance degrees of the criteria given in Table 6 were compared with each other in pairs and thus the comparison results are obtained. The same procedure was repeated for each criterion to obtain a fuzzy binary comparison matrix, which was given in Table 7.

Table 7. Fuzzy pairwise comparison matrix for criteria with respect to goal "portfolio selection"

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	(1 1 1 1)	(0.65 0.86 1.03 1.39)	(0.70 1.03 1.31 1.90)	(0.65 0.86 1.03 1.39)	(0.63 0.82 0.97 1.27)	(0.70 1.03 1.31 1.90)	(0.85 1.29 1.72 2.78)
C_2	(0.72 0.97 1.16 1.54)	(1 1 1 1)	(0.76 1.10 1.38 1.95)	(0.70 1.03 1.31 1.90)	(0.69 0.87 1.02 1.31)	(0.76 1.10 1.38 1.95)	(0.92 1.38 1.82 2.86)
C_3	(0.53 0.76 0.97 1.43)	(0.51 0.72 0.91 1.31)	(1 1 1 1)	(0.51 0.73 0.91 1.31)	(0.50 0.69 0.85 1.21)	(0.56 0.87 1.15 1.80)	(0.67 1.09 1.52 2.64)
C_4	(0.72 0.97 1.16 1.54)	(0.70 0.92 1.09 1.43)	(0.76 1.10 1.37 1.96)	(1 1 1 1)	(0.69 0.87 1.02 1.31)	(0.76 1.10 1.38 1.95)	(0.92 1.38 1.82 2.86)
C_5	(0.79 1.03 1.22 1.59)	(0.76 0.98 1.15 1.45)	(0.83 1.18 1.45 2.00)	(0.76 0.98 1.14 1.45)	(1 1 1 1)	(0.83 1.17 1.45 2.00)	(1.00 1.48 1.91 2.93)
C_6	(0.53 0.76 0.97 1.43)	(0.51 0.72 0.91 1.32)	(0.56 0.87 1.15 1.79)	(0.51 0.72 0.9 1.32)	(0.50 0.69 0.85 1.20)	(1 1 1 1)	(0.67 1.09 1.52 2.64)
C_7	(0.36 0.58 0.78 1.18)	(0.35 0.55 0.72 1.09)	(0.38 0.66 0.92 1.49)	(0.35 0.55 0.72 1.09)	(0.34 0.52 0.68 1.00)	(0.38 0.66 0.92 1.49)	(1 1 1 1)

The fuzzy weight of each criterion is calculated by the proposed algorithm and given in Table 8.

Table 8. Fuzzy weight of criteria

Criteria	Weights
C_1	(0.09 0.14 0.16 0.21)
C_2	(0.10 0.15 0.17 0.23)
C_3	(0.09 0.12 0.14 0.21)
C_4	(0.11 0.15 0.17 0.23)
C_5	(0.11 0.16 0.18 0.23)
C_6	(0.09 0.12 0.14 0.21)
C_7	(0.05 0.09 0.11 0.17)

The linear programming model was created by using the obtained fuzzy weights.

$$Z_{P/E(min)} = 10.7x_1 + 8.9x_2 + 7.1x_3 + 3.6x_4 + 4.1x_5, \quad (30)$$

$Z_{NP/SE(max)} = 11.5x_1 + 12.5x_2 + 23.4x_3 + 18.1x_4 + 30.8x_5,$

$Z_{ND/MV(min)} = 10.0x_1 + 0.2x_2 - 26.2x_3 - 9.0x_4 + 2.8x_5,$

$Z_{MV/BV(max)} = 1.3x_1 + 1.2x_2 + 1.9x_3 + 0.7x_4 + 1.3x_5$

$Z_{NP/SR(max)} = 12.6x_1 + 18.3x_2 + 34.9x_3 + 18.0x_4 + 9.8x_5$

$Z_{NP/TA(max)} = 7.6x_1 + 8.6x_2 + 20.7x_3 + 12.7x_4 + 19.9x_5$

$2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5 = 2$

$x_1 + x_2 + x_3 + x_4 + x_5 = 1,$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Price/Earnings, Net Profit/Stockholder's Equity, Net Debt/Market Value, Market Value/Book Value, Net Profit/Sales Revenue and Net Profit/Total Assets Ratio goals are represent by $Z_{P/E}$, $Z_{NP/SE}$, $Z_{ND/MV}$, $Z_{MV/BV}$, $Z_{NP/SR}$ and $Z_{NP/TA}$ respectively. The percentage of investments to be made to the i th stock is expressed as x_i.

The maximum and minimum values for each objective function were determined under the constraints of the model by using WinQSB software. Solutions for each of 6 objective functions are given in Table 9.

Table 9. The maximum and minimum values of the objective functions

	$\mu = 0$	$\mu = 1$	$(\mu = 0) - (\mu = 1)$
$Z_{P/E(min)}$	0.7	3.6	7.1
$Z_{NP/SE(max)}$	30.8	11.5	19.3
$Z_{ND/MV(min)}$	0	-26.2	36.2
$Z_{MV/BV(max)}$	1.9	0.7	1.2
$Z_{NP/SR(max)}$	34.9	9.8	25.1
$Z_{NP/TA(max)}$	20.7	7.6	13.1

A fuzzy multi objective linear programming model (P_1) is generated using fuzzy weights obtained from the Constrained Fuzzy AHP method using TrFNs.

$_1$:

$$max \quad (0.09\ 0.14\ 0.16\ 0.22)\lambda_1 + (0.10\ 0.15\ 0.17\ 0.23)\lambda_2 + (0.09\ 0.12\ 0.14\ 0.21)\lambda_3 + (0.11\ 0.15\ 0.17\ 0.23)\lambda_4 + (0.11\ 0.16\ 0.18\ 0.23)\lambda_5 + (0.09\ 0.12\ 0.14\ 0.21)\lambda_6 + (0.05\ 0.09\ 0.11\ 0.17)\lambda_7$$

$$\lambda_1 \leq \frac{10.7-(10.7x_1+8.9x_2+7.1x_3+3.6x_4+4.1x_5)}{7.1},$$

$$\lambda_2 \leq \frac{(11.5x_1+12.5x_2+23.4x_3+18.1x_4+30.8x_5)-11.5}{19.3},$$

$$\lambda_3 \leq \frac{10-(10.0x_1+0.2x_2-26.2x_3-9.0x_4+2.8x_5)}{36.2},$$

$$\lambda_4 \leq \frac{(1.3x_1+1.2x_2+1.9x_3+0.7x_4+1.3x_5)-0.7}{1.2},$$

$$\lambda_5 \leq \frac{(12.6x_1+18.3x_2+34.9x_3+18.0x_4+9.8x_5)-9.8}{25.1},$$

$$\lambda_6 \leq \frac{(7.6x_1+8.6x_2+20.7x_3+12.7x_4+19.9x_5)-7.6}{13.1},$$

$$\lambda_7 \leq \frac{2.5-(2.04x_1+2.17x_2+5.83x_3+1.86x_4+1.55x_5)}{0.5},$$

$$\lambda_7 \leq \frac{(2.04x_1+2.17x_2+5.83x_3+1.86x_4+1.55x_5)-1.4}{0.6},$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1,\ x_1, x_2, x_3, x_4, x_5 \geq 0.$$

The proposed algorithm is used to solve the P_1 problem, where the price variables in the objective function are TrFNs. The new objective functions such as Equality (31) are created;

$$\begin{aligned} &min\ 0.05\lambda_1 + 0.05\lambda_2 + 0.03\lambda_3 + 0.04\lambda_4 + 0.05\lambda_5 + 0.03\lambda_6 + 0.04\lambda_7\ (Z_1) \\ &max\ 0.14\lambda_1 + 0.15\lambda_2 + 0.12\lambda_3 + 0.15\lambda_4 + 0.16\lambda_5 + 0.12\lambda_6 + 0.09\lambda_7\ (Z_2) \\ &max\ 0.16\lambda_1 + 0.17\lambda_2 + 0.14\lambda_3 + 0.17\lambda_4 + 0.18\lambda_5 + 0.14\lambda_6 + 0.11\lambda_7\ (Z_3) \\ &max\ 0.05\lambda_1 + 0.06\lambda_2 + 0.07\lambda_3 + 0.06\lambda_4 + 0.05\lambda_5 + 0.07\lambda_6 + 0.06\lambda_7\ (Z_4) \end{aligned} \quad (31)$$

PIS and NIS of each objective function given by the Equation (31) are obtained and the results are given in Table 10.

Table 10. Positive and Negative Ideal Solutions of the four objective functions

	Positive Ideal Solution	**Negative Ideal Solution**
Z_1	0	0.197
Z_2	0.605	0
Z_3	0.699	0
Z_4	0.286	0

The linear membership functions for each objective function are obtained as shown in Equation (32-35).

$$\mu_{Z_1} = \begin{cases} 1 & Z_1 < 0 \\ \dfrac{0.197 - Z_1}{0.197} & 0 \le Z_1 \le 0.197 \\ 0 & Z_1 > 0.197 \end{cases} \quad (32)$$

$$\mu_{Z_2} = \begin{cases} 1 & Z_i < 0.605 \\ \dfrac{Z_2}{0.605} & 0 \le Z_2 \le 0.605 \\ 0 & Z_2 > 0 \end{cases} \quad (33)$$

$$\mu_{Z_3} = \begin{cases} 1 & Z_3 < 0.699 \\ \dfrac{Z_3}{0.699} & 0 \le Z_3 \le 0.699 \\ 0 & Z_3 > 0 \end{cases} \quad (34)$$

$$\mu_{Z_4} = \begin{cases} 1 & Z_4 < 0.286 \\ \dfrac{Z_4}{0.286} & 0 \le Z_4 \le 0.286 \\ 0 & Z_4 > 0 \end{cases} \quad (35)$$

The P_2 model is created by using the membership values obtained.

$$\begin{aligned} &max\ \alpha \\ {}_2\!:\ &0.197\alpha + 0.05\lambda_1 + 0.05\lambda_2 + 0.03\lambda_3 + 0.04\lambda_4 + 0.05\lambda_5 + 0.03\lambda_6 + 0.04\lambda_7 \le 0.197\ (Z_1) \\ &-0.605\alpha + 0.14\lambda_1 + 0.15\lambda_2 + 0.12\lambda_3 + 0.15\lambda_4 + 0.16\lambda_5 + 0.12\lambda_6 + 0.09\lambda_7 \ge 0\ (Z_2) \\ &-0.699\alpha + 0.16\lambda_1 + 0.17\lambda_2 + 0.14\lambda_3 + 0.17\lambda_4 + 0.18\lambda_5 + 0.14\lambda_6 + 0.11\lambda_7 \ge 0\ (Z_3) \\ &-0.286\alpha + 0.05\lambda_1 + 0.06\lambda_2 + 0.07\lambda_3 + 0.06\lambda_4 + 0.05\lambda_5 + 0.07\lambda_6 + 0.06\lambda_7 \ge 0\ (Z_4) \\ &7.1\lambda_1 + (10.7x_1 + 8.9x_2 + 7.1x_3 + 3.6x_4 + 4.1x_5) \le 10.7 \\ &-19.3\lambda_2 + (11.5x_1 + 12.5x_2 + 23.4x_3 + 18.1x_4 + 30.8x_5) \ge 11.5 \\ &36.2\lambda_3 + (10.0x_1 + 0.2x_2 - 26.2x_3 - 9.0x_4 + 2.8x_5) \le 10 \\ &-1.2\lambda_4 + (1.3x_1 + 1.2x_2 + 1.9x_3 + 0.7x_4 + 1.3x_5) \ge 0.7 \\ &-25.1\lambda_5 + (12.6x_1 + 18.3x_2 + 34.9x_3 + 18.0x_4 + 9.8x_5) \ge 9.8 \\ &-13.1\lambda_6 + (7.6x_1 + 8.6x_2 + 20.7x_3 + 12.7x_4 + 19.9x_5) \ge 7.6 \\ &0.5\lambda_7 + (2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5) \le 2.5 \\ &-0.6\lambda_7 + (2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5) \ge 1.4 \\ &x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ &x_1, x_2, x_3, x_4, x_5 \ge 0 \end{aligned}$$

The results obtained by solving the P_2 model using the WinQSB program are given in Table 11.

Table 11. The ratios to be invested in the stocks by solving the P3 problem

	Anadolu Cam	Trakya Cam	Mardin Cimento	Eregli Demir Celik	Izmir Demir Celik
Percentage of stocks	0	0	0.222	0	0.778

CONCLUSION

In the case of uncertainty, unplanned investments can cause unexpected losses to the investor. In addition, many portfolio selection methods do not consider expert opinions. Because these methods use only financial data without considering the investor's views, they provide the same expected return for all investors at the same risk level. To handle this problem, adding both financial ratios and experience based expert opinions in to the model has made the proposed method more effective. In the proposed model, the weights of criteria were determined by using the decision makers' opinions. When the studies in the literature were examined, it was seen that TFNs have been widely used in the calculation of the weights of criteria. Instead of using TFNs, this study focused on TrFNs to characterize fuzzy measures of linguistic values. The reason for using the TrFNs was that it was more representative to linguistic estimations in portfolio selection. In terms of optimal solution, it was observed that TrFNs gave more flexible results than TFNs. As a result of the study, it has been determined that investors' funds should be allocated to the stocks Anadolu Cam, Trakya Cam, Mardin Cimento, Eregli Demir Celik and Izmir Demir Celik by 0%, 0%, 22.2%, 0% and 77.8%, respectively (Table 11), using the recommended model for optimal portfolio distribution.

The proposed method can be applied to larger-scale portfolio selection problems in which more criteria and expert opinions are used, and the optimal allocation of stocks can be done out of a variety of sectors.

REFERENCES

AHLATCIOGLU, B. (2005). Fuzzy Approaches to Portfolio Selection, Master Thesis, Marmara University, Institute of Banking and Insurance.

ARIK, O. A. and TOKSARI, M. D. (2018), Multi-objective fuzzy parallel machine scheduling problems under fuzzy job deterioration and learning effects, *International Journal of Production Research*, 56(7), 2488–2505.

BOLOS, M.I., BRADEA, I.A., and DELCEA, C. (2019), A fuzzy logic algorithm for optimizing the investment decisions within companies. *Symmetry*, 11(2), 186.

CHANG, D. Y. (1996), Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, 95(3), 649–655.

CHATTERJEE, K., HOSSAIN, S. A., and KAR, S. (2018), Prioritization of project proposals in portfolio management using fuzzy AHP. *OPSEARCH*, 55(2), 478–501.

CHEN, C.T., and CHENG, H.L. (2009), A comprehensive model for selecting information system project under fuzzy environment. *International Journal of Project Management*, 27(4), 389–399.

DENIZ, D. and OKUYAN, H. A. (2018), The Comparison of The Empirical Results Of Traditional And Modern Portfolio Management: BIST Application, *Journal of Mehmet Akif Ersoy University Economics and Administrative Sciences Faculty*, 5(3), 467–482.

DENIZ, D., SAKARYA, S., and OKUYAN, H. A. (2018), Portfolio Diversification Contribution of Precious Metal: Case of BIST, *Journal of Mehmet Akif Ersoy University Economics and Administrative Sciences Faculty*, 5(2), 366-382.

DONG, J. and WAN, S.P. (2018), A new trapezoidal fuzzy linear programming method considering the acceptance degree of fuzzy constraints violated, *Knowledge-Based Systems*, 148, 100–114.

EBRAHIMNEJAD, A. (2018), A method for solving linear programming with interval-valued trapezoidal fuzzy variables. *RAIRO - Operations Research*, 52(3), 955–979.

ENEA, M. and PIAZZA, T. (2004). Project selection by constrained fuzzy AHP. *Fuzzy Optimization and Decision Making*, 3(1), 39–62.

GHAZANFAR AHARI, S., GHAFFARI-NASAB, N., MAKUI, A., and GHODSYPOUR, S. H. (2011), A Portfolio Selection Using Fuzzy Analytic Hierarchy Process: A Case Study of Iranian pharmaceutical industry. *International Journal of Industrial Engineering Computations*, 2(2), 225–236.

HWANG, C. L., and YOON, K. (1981), *Multiple Attribute Decision Making: Methods and Applications*, Berlin: Springer-Verlag.

JAFARZADEH, H., AKBARI, P., and ABEDIN, B. (2018). A methodology for project portfolio selection under criteria prioritisation, uncertainty and projects interdependency – combination of fuzzy QFD and DEA. *Expert Systems with Applications*, 110, 237–249.

KIM, J. and KIM, J. (2018), Optimal Portfolio for LNG Importation in Korea Using a Two-Step Portfolio Model and a Fuzzy Analytic Hierarchy Process, *Energies*, 11(11), 3049.

LAI, Y.J. and HWANG, C.L. (1992), A new approach to some possibilistic linear programming problems. *Fuzzy Sets and Systems*, 49(2), 121–133.

LIAGKOURAS, K. (2019), A New Three-Dimensional Encoding Multi-Objective Evolutionary Algorithm with Application to the Portfolio Optimization Problem, *Knowledge-Based Systems*, 163, 186-203.

LIU, Q. and GAO, X. (2016), Fully Fuzzy Linear Programming Problem with Triangular Fuzzy Numbers, *Journal of Computational and Theoretical Nanoscience*, 13(7), 4036–4041.

MARKOWITZ, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77–91.

NAKAMURA, K. (1984), Some extensions of fuzzy linear programming. *Fuzzy Sets and Systems*, 14(3), 211–229.

PIASECKI, K., and SIWEK, J. (2018). The portfolio problem with present value modelled by a discrete trapezoidal fuzzy number. *Operations Research and Decisions*, 28(1), 57–74.

SADJADI, S. J., SEYEDHOSSEINI, S. M., and HASSANLOU, K. (2011), Fuzzy multi period portfolio selection with different rates for borrowing and lending, *Applied Soft Computing*, 11(4), 3821–3826.

SEO, F. and SAKAWA, M. (1988), *Multiple Criteria Decision Analysis in Regional Planning: Concepts, Methods and Applications*. Dordrecht: Reidel.

SOLIMANPUR, M., MANSOURFAR, G., and GHAYOUR, F. (2015). Optimum portfolio selection using a hybrid genetic algorithm and analytic hierarchy process. *Studies in Economics and Finance*, 32(3), 379–394.

TALESHIAN, F. and FATHALI, J. (2016), A Mathematical Model for Fuzzy p-Median Problem with Fuzzy Weights and Variables, *Advances in Operations Research*, 2016, 1–13.

TANAKA, H. and ASAI, K. (1984), Fuzzy linear programming problems with fuzzy numbers, *Fuzzy Sets and Systems*, 13(1), 1–10.

TIRYAKI, F. and AHLATCIOGLU, M. (2005). Fuzzy stock selection using a new fuzzy ranking and weighting algorithm. *Applied Mathematics and Computation*, 170(1), 144–157.

TIRYAKI, F. and AHLATCIOGLU, B. (2009), Fuzzy portfolio selection using fuzzy analytic hierarchy process, *Information Sciences*, 179(1–2), 53–69.

TOPALOGLU, E. E. (2018), Determination of the Relationship Between Financial Risks and Firm Value: An Application on Istanbul Stock Exchange Companies, *Journal of Mehmet Akif Ersoy University Economics and Administrative Sciences Faculty*, 5(2), 287-301.

WANG, B., LI, Y., WANG, S. and WATADA J. (2018), A Multi-Objective Portfolio Selection Model with Fuzzy Value-at-Risk Ratio, IEEE Transactions on Fuzzy Systems, 26(6), 3673-3687.

XIAO, Y.Z. and FU, S. (2015), Grey-correlation Multi-attribute Decision-making Method Based on Intuitionistic Trapezoidal Fuzzy Numbers. *Mathematics and Statistics*, 3(4), 95–100.

YAGHOOBI, T. (2018), Prioritizing key success factors of software projects using fuzzy AHP, *Journal of Software: Evolution and Process*, 30(1), e1891.

YU, J., GAN, M., NI, S. and CHEN, D. (2018), Multi-objective models and real case study for dual-channel FAP supply chain network design with fuzzy information, *Journal of Intelligent Manufacturing*, 29(2), 389–403.

YUCEL, A. and GUNERI, A. F. (2011), A weighted additive fuzzy programming approach for multi-criteria supplier selection. *Expert Systems with Applications*, 38(5), 6281–6286.