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alphanumeric journal
The Journal of Operations Research, Statistics, Econometrics and Management Information Systems

Volume 7, Issue 2, 2019

Received: August 05, 2019
Accepted: December 22, 2019
Published Online: December 31, 2019

AJ ID: 2018.07.02.STAT.01
DOI: 10.17093/alphanumeric.629722
Research Article

Comparison of the Global, Local and Semi-Local Chaotic Prediction Methods for Stock Markets: The Case of FTSE-100 Index

Ayşe İşi, Ph.D.*

Lect., Polatlı Vocational School of Social Sciences, Ankara Hacı Bayram Veli University, Ankara, Turkey, ayse.isi@hbv.edu.tr

Fatih Çemrek, Ph.D.

Assoc. Prof., Department of Statistics, Faculty of Science and Letters, Eskisehir Osmangazi University, Eskişehir, Turkey, fcemrek@ogu.edu.tr

* Ankara Hacı Bayram Veli Üniversitesi Polatlı Sosyal Bilimler MYO, Kemal Bıyıkoğlu Cd. No:22 Şehitlik Mahallesi, 06900 Polatlı, Ankara, Türkiye

ABSTRACT

Chaotic prediction methods are classified as global, local and semi-local methods. In this paper, unlike the studies in the literature, it is aimed to compare all these methods together for stock markets in terms of prediction performance and to determine the best prediction method for stock markets. For this purpose, Multi-Layer Perceptron (MLP) neural networks from global methods, nearest neighbour method from local methods, radial basis functions from semi-local methods are used. The FTSE-100 index is selected to represent the stock market and applied the all methods to these data. The prediction performance is measured in term of root mean square error (RMSE) and normalized mean square error (NMSE). As a result of the analysis; it has been determined that the best prediction method for the FTSE-100 index is the semi-local method. While it is possible to make a maximum of 5 days prediction with global and local methods, it has been determined that up to 20 days prediction can be made with the semi-local prediction methods. The results show that semi-local prediction methods are successful in predicting the behavior of stock market.

Keywords: Chaotic Time Series, Chaotic Prediction, Nearest Neighbour Method, Radial Basis Functions Method, Stock Market, FTSE-100 Index

1. Introduction

The stock markets are most important part of the global economy. Any fluctuation in this market effects the financial health of individuals, companies and countries (Hassan and Nath, 2005). Therefore, the examination and prediction of the behaviour of these markets is one of the leading issues by investors and scientists.

Many studies have been carried out to examine the behavior of stock markets. Elridge and Coleman (1993), and Abhyankar et al. (1995) claimed that index data were non-linear; Brock et al. (1991), Mayfield and Mizrach (1992), Vaidyanathan and Krehbiel (1992), and Hanias et al. (2013), Webel (2012), Özdemir and Akgül (2014) showed the chaotic behaviour of stock market.

Due to the inherent non-linearity, non-stationary and also chaotic characteristics of stock market, conventional modelling techniques such as the Box–Jenkins models and the non-linear models are not adequate for stock market forecasting (Kazem et al, 2013). Therefore, chaotic models have been proposed, which seem to be more adequate to explain behaviour of stock market.

The chaotic methods are more preferred in the economy and finance literature, especially in the examination of stock prices and exchange rates. As Hsieh (1991) put it, the most important reason behind this is the ability of chaos theory to potentially describe the fluctuations with random appearance in economy and financial markets.

Chaotic prediction methods suggested in literature are classified as global, local and semi-local. Global methods enable expression of the behaviour of system with a single model by using all the past information in the creation of system, in order to identify the future position of it. Local methods are commonly based on decomposition the phase space into regimes and then fitting a simple (sub) model for each regime (Chan and Tong, 2001). Semi-local methods may combine the smoothness of global predictors with the localised dependence on new information of local predictors (Lillekjendlie et al., 1994).

Even though the general opinion in literature is that local methods show a better prediction performance as compared to global methods (Karunasinghe and Liong, 2006), different results were obtained in various studies. Lillekjendlie et al. (1994), Elshorbagy et al. (2002), Karunasinghe and Liong (2006) claimed that global methods showed better prediction performance whereas Sivakumar et al. (2002), and Guegan and Mercier (2005) found that local, and semi-local methods showed better prediction performance, respectively.

In the prediction studies related to the stock markets, generally chaotic prediction methods are compared with the other statistical methods including Box-Jenkins model, ARCH/GARCH models, neural networks and it has been shown that chaotic prediction methods show better prediction performance (Huang et al, 2010; Kazem et al., 2013; Özdemir ve Akgül, 2014).

In this paper, unlike the studies in the literature, it is aimed to compare all chaotic prediction methods together for stock markets in terms of prediction performance and to determine the best prediction method for stock markets. Multi-Layer Perceptron neural networks from global methods, nearest neighbour method from local methods, and Radial Basis Functions from semi-local method were used for this purpose. The rest of the paper is organized as follow: In Section 2, the chaotic time series prediction methods are briefly presented. In Section 3, we apply these methods to FTSE 100 index data and calculate the criterion. Section 4 reports the prediction results and draws conclusions.

2. Chaotic Prediction Methods

Prediction of chaotic time series is based on reconstruction of chaotic attractors from the observed time series x_t (Takens, 1981). Where d is the embedding dimension and τ is the time delay, Takens states that prediction could be made by creating the $X_t = (x_t, x_{t-\tau}, x_{t-2\tau}, \dots, x_{t-(d-1)\tau})$ vector obtained from the delay coordinates of embedding of attractors in phase space. Dynamics on the attractor define a map as $f: \Re^d \rightarrow \Re^d$ with $x_{t+T} = f(X_t)$ where x_t is the current state and x_{t+T} is the future state. Thus, if an f_i approximation of f is found, f_i can be used as a prediction function. (Xiaofeng and Lai, 1999).

Classification of the methods used in prediction of chaotic time series as global, local and semi-local results from the selection of above-specified f_i prediction function.

2.1. Multi-Layer Perceptron (MLP) Neural Network

Since all the past information in the creation of system is used for prediction, studies on the artificial intelligence applications, which are considered as one of the global methods and have become the most important tool of prediction studies in chaotic time series, started with Lapedes and Farber (1987). Lapedes and Farber are the first researchers to use multi-layer perceptron (MLP) networks among the feedback networks for the prediction of chaotic time series. Afterwards MLP neural networks started to be used in several studies (Elsner, 1992; Smaoui, 1999; Lillekjendlie et al., 1994; Elshorbagy et al., 2002; Karunasinghe and Liong, 2006). The common opinion of studies carried out on MLP neural network is that MLP neural network models the chaotic structure in chaotic time series well. Therefore, MLP neural network has been used in this study as the global prediction approach.

MLP neural networks are feed-forward, back-propagation neural networks. According to the counselling learning method, input and target values are given to the network, and network finds the best weights to minimize the error between the outputs of network and target values.

There are many factors that affect the performance of MLP network. Size of the learning and test set, number of units in the input layer, number of hidden layers and number of units in the hidden layer, activation function used, and learning parameters affect the learning capacity of network significantly.

2.2. Nearest Neighbour Method

As one of the local prediction methods, the nearest neighbour method has been developed by Farmer and Sidorowich (1987). They stated that the local method they created by using the neighbouring points on phase space to make predictions is a more effective method as compared to other methods.

According to the nearest neighbour method, in order to predict the x_{t+T} point, firstly, the k-nearest $X_{t'}$ neighbour of X_t is found. Where $\|.\|$, is Euclidien or maximum norm distance, and $X_{t'}$ neighbours are $t' < t$, the first k to make the minimum $\|X_t - X_{t'}\|$ is defined as the neighbour. Then a local predictor is construct between each $X_{t'}$ point and the related $X_{t'+T}$ point. In the nearest neighbour method, while constructing this local predictor, the nearest neighbour is considered for k=1 only and $\hat{x}_{t+1} = x_{t'+1}$ is determined.

2.3. Radial Basis Functions Method

Suggested for the prediction of chaotic time series, radial basis function method has been developed by Casdagli (1989). He states that the suggested method is a global interpolation technique that has good localization properties.

In the radial basis function method, the data set is divided into two unequal length parts as the learning set that includes NL observations and test set that includes N-NL observations, to show the observed data set $x_1, x_2, \dots, x_N$, as similar to the artificial neural networks. The model is constructed by using the data in the learning set. Then the efficiency of the obtained model is tested by using the data in test set (Chan and Tong, 2001).

Once an Nc set is selected with the centres of $x_j^c: j = 1, \dots, k,\ \ x_j^c \in R^d$, the prediction value is obtained,

$$x_{t+1} = \hat{f}(X_t) = \sum_{j=1}^{k} \lambda_j \phi\left(\left\|X_t - x_j^c\right\|\right) \quad (1)$$

where $\phi(r)$ is a radially symmetric function on $\Re^d$ around the center of x_j^c, λs belong to the X_t learning set and are obtained with the solution of $x_{t+1} = \hat{f}(X_t)$ equation set. While this is not a general rule in determination of the Nc set, the nearest neighbour vectors are generally taken into consideration in the chaotic time series analysis. Therefore, k expresses the number of nearest neighbours.

3. Chaotic Analysis of FTSE-100 Index

3.1. Determination of Chaotic Structure

The most important characteristics of chaotic systems are their sensitive dependence on initial conditions, and their fractal structure (Abarbanel et al., 1990). These characteristics are reviewed using the information obtained from the attractor that occurs in the reconstructed phase space based on observed time series. Attractors are structures that consist of points of a dynamic system in phase space and appear like an object. Therefore, the phase space where these attractors will position must be reconstructed firstly. The most important two parameters for reconstruction of phase space are time delay τ and embedding dimension d.

Data set consists of 4930 observations that include the daily closing values of FTSE-100 index between 20.10.1997 and 28.04.2017. FTSE100 index data (x_t) has been reconstructed with the following equation based on Taken's embedding theorem in the d-dimensional phase space with time delay τ .

$$X_t = \left(x_t, x_{t-\tau}, x_{t-2\tau}, \dots, x_{t-(d-1)\tau}\right),\ \ t = 1,2, \dots, N - (d-1)\tau \quad (2)$$

Chaotic structure of the system is reviewed by calculating the fractal dimension of attractor, and Lyapunov exponents of the system.

3.1.1. Determination of Time Delay and Embedding Dimension

Average Mutual Information method developed by Fraser and Swinney (1986) has been used to determine the optimum time delay (τ). $I(T)$ values that are calculated up to 50 lags vary between 1,442 and 4,594. Since the first minimum value of $I(T)$, 1,464, was obtained in the 47th lag, optimum time delay was determined as τ=47 (Figure 1).

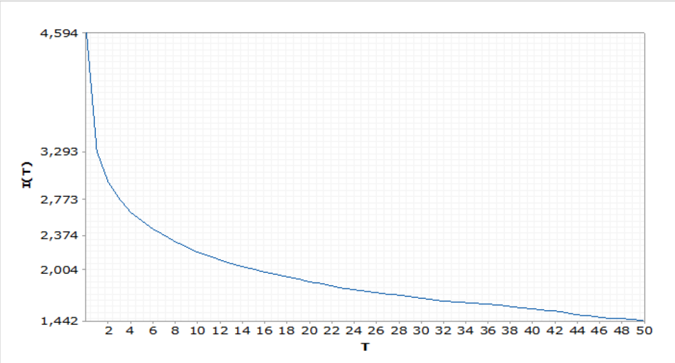

Figure 1. The Average Mutual Information as A Function of Time Delay *I(T)* for *T*=1,2,...,50

FNN method developed by Kennel et al. (1992) was used to determine the embedding dimension. *d* value where the FNN value that is calculated by increasing the embedding dimension values one by one approaches zero is considered as the optimum embedding dimension.

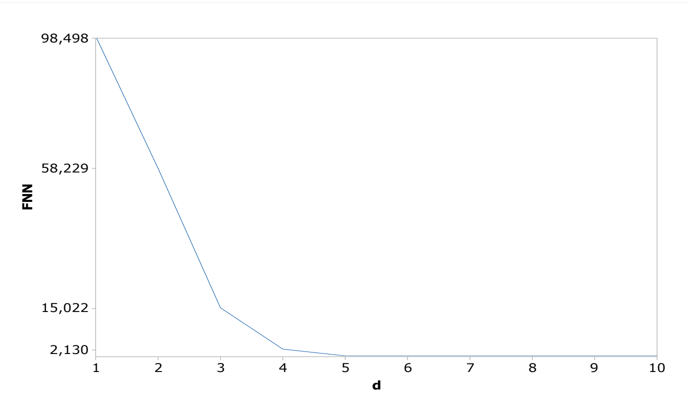

Figure 2. The FNNs as A Function of Embedding Dimension for d=1,...,10

At the end of the analysis performed up to 10 embedding dimensions using the FNN Method, optimum embedding dimension was determined as d=5 (Figure 2).

3.1.2. Determination of Correlation Dimension and The Largest Lyapunov Exponent

Chaos for a time series could generally be defined by observing the attractor, which is the appearance of system in phase space, and calculating the fractal dimension of attractor. However, the most important indicator of whether a time series is chaotic is the Lyapunov exponent developed by the Russian Mathematician Aleksandr Lyapunov. Correlation dimension and the largest Lyapunov exponent are defined as the invariant characteristics of chaotic time series.

Correlation dimension developed by Grassberger and Procaccia (1983 a,b) is a measure of the complexity level of the system (Eckman and Ruelle,1985; Abarbanel et al.,1990). To observe whether chaos exists, the correlation exponent values are plotted against the corresponding embedding dimension values. Limited or saturated correlation exponent value indicates that the system has a chaotic structure and is also deterministic with a sensitive dependence on initial conditions. If the da value increases without bound with increase in the embedding dimension, the system under investigation is thought to be stochastic. When the da value reaches the saturation point based on the increased embedding dimension values, upper integer value of this saturation point is defined as correlation dimension, and indicates the fractal dimension of attractor. (Fraedrich, 1986; Shang et al., 2005).

Correlation exponents that are calculated up to 20 embedding dimensions vary between 0,965 and 2,574 (Figure 3).

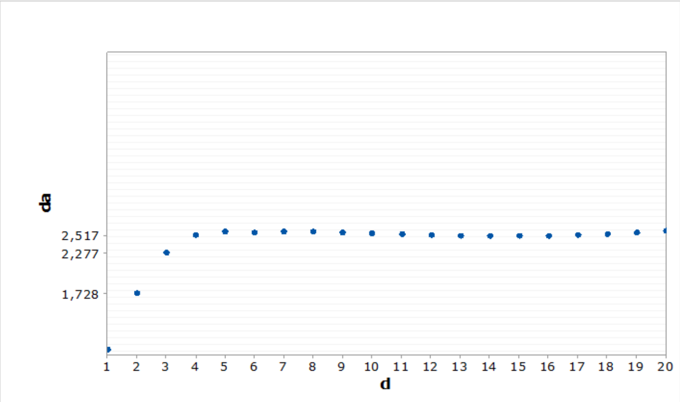

Figure 3. The Correlation Exponents as A Function of Embedding Dimension for *d*=1,2,...20

According to Figure 3, correlation exponent values reach the saturation point around 2.5-2.6 starting from the 5th embedding dimension. Therefore, the correlation dimension has been determined as $d_a = 3$. Furthermore, the fact that correlation exponent values reach the saturation point after a specific embedding dimension shows that FTSE-100 index series has a fractal structure, is deterministic and shows sensitive dependence to initial conditions.

Two near initial conditions on the attractor separate from each other at a distance that grows incrementally depending on time along the orbit. Average growth rate of such distance is called Lyapunov exponent. While orbits move away from each other slowly in periodic systems, such movement is exponentially rapid in chaotic systems. Positive value, zero value and negative value of Lyapunov exponent refer to sensitive dependence (chaos), periodicity (or semi-periodicity) and stable equilibrium, respectively. (Kantz and Schreiber, 2004: 66; Sprott, 2010: 20; Wolf et al., 1985: 286).

The greatest Lyapunov exponent value of FTSE-100 index data was calculated using the algorithm of Kantz (1994) and found as $\lambda_1 = 0{,}01$. Since the greatest Lyapunov exponent value obtained is positive, it is seen that the series shows a sensitive dependence on initial conditions and has a chaotic structure.

3.2. Prediction

While applying the prediction methods, data set has been divided into two so that 90% is learning set and 10% is test set.

In the prediction of chaotic time series, there is a relationship between the maximum prediction length and largest Lyapunov exponent so that:

$$\text{Maximum Prediction Length} = \Delta t_{max} = \frac{1}{\lambda_1} \quad (3)$$

(Abarbanel, 1996; Sprott, 2003). In this study, maximum prediction length was found as 100 days.

In order to observe the performance of prediction methods that are covered in this study based on the prediction length, various prediction lengths including 5, 10, 20, 50 and 100 have been taken into account.

Prediction performances were compared based on the RMSE and NMSE (Normalized Mean Squared Error) criteria. NMSE criterion allows comparison of prediction performance of chaotic prediction models with the performance of stable predictor $\hat{x}_t = \bar{x}$ which indicates the average value of test set, and random walk predictor $\hat{x}_t = \bar{x}_{t-1}$. Main reason for using the NMSE criterion is the belief that it might be useful if the used chaotic predictors show a better performance than the reference predictors.

RMSE and NMSE values have been calculated using the Equation 4.

$$RMSE = \sqrt{\frac{\sum e_t^2}{N}} \quad (4)$$

$$NMSE = max\left(\frac{\sum e_t^2}{\sum(x_t - \bar{x})^2}\ ;\ \frac{\sum e_t^2}{\sum(x_t - x_{t-1})^2}\right) \quad (5)$$

If NMSE≅0 prediction performance is perfect.

If NMSE>1 it is worse that the performance of reference predictors (Lillekjendlie et al.,1994; Karunasinghe and Liong, 2006).

Following criteria were used in the prediction study of FTSE-100 index performed using Multi-Layer Perceptron (MLP) neural network.

Since delayed values of time series are used in the input layer of MLP network, there are 4 units. The study was conducted with 1 and 2 hidden layers. There is only one unit that consists of target values in the output layer. Sigmoid and hyperbolic tangent activation functions were used in the hidden layer and output layer. The learning parameters, learning rate and momentum coefficient were taken as 0.2 and 0.9, respectively.

Firstly, MLP application was carried out with a single hidden layer only. Number of units in the hidden layer were changed between 1 and 15. Study was conducted by changing the activation functions in the hidden layer and output layer for each unit. The best error values were obtained when sigmoid activation function was used both in the hidden layer and output layer.

In MLP study with two hidden layers, the study was again conducted by changing the number of units in the first and second hidden layer between 1 and 15. As in the MLP network with single hidden layer, it was observed that when hyperbolic tangent activation function was used in the output layer of neural network with two hidden layers, errors multiplied approximately by four.

According to the obtained results, the best network architecture of FTSE-100 index included 8 units in the first hidden layer and 7 units in the second hidden layer, and was the 4-8-7-1 architecture model where sigmoid activation function was used both in the hidden layer and output layer.

In prediction studies conducted using the Nearest Neighbour Method, both direct (single step) and iterative (multi step) prediction approaches were taken into consideration. Distances between neighbour points were calculated by considering the Euclidean and Max Norm.

In the review of obtained prediction performances, it was observed that predictions made using the direct method provided better error values as compared to the predictions made using the iterative method, in predictions performed via nearest neighbour method.

The best model obtained in terms of prediction performance was the model in which direct prediction method and Euclidian distance were used.

Prediction studies with radial basis function method were conducted by including different numbers of neighbours (k) and different radial basis functions in the process, in addition to the criteria used in nearest neighbour method. In the case of $k>d+1$, prediction values were individually calculated for k=6, 7 and 8. Among the Radial Basis functions, Linear, Cubic, Thin Plate Spline, Gaussian and Multiquadratic functions were used.

Since the error values grew extremely for many models used as the prediction length increased in prediction studies conducted using the radial basis function method, no predictions were made. In general, it was observed that as the number of nearest neighbours (k) increased, calculable prediction length increased too.

It was observed also in the radial basis function method that predictions made with direct method demonstrated a much better performance as compared to predictions made with iterative method.

The best prediction performances for k=6 were obtained when linear radial basis function and Max Norm distance were used. The best prediction performances for k=7 and 8 were obtained when linear radial basis function and Euclidean distance were used.

RMSE and NMSE values for all models are given in Table 1 for the comparison of prediction performances based on the prediction length of selected models.

Prediction Length	MLP		NN		RBF (k=6)		RBF (k=7)		RBF (k=8)	
	RMSE	NMSE	RMSE	NMSE	RMSE	NMSE	RMSE	NMSE	RMSE	NMSE
5	58,979	1,160	39,819	0,627	53,347	1,126	35,460	0,585	38,495	0,586
10	113,814	2,754	47,857	1,229	42,051	0,949	38,283	0,786	32,875	0,580
20	190,917	7,680	57,205	1,383	103,292	4,510	66,747	1,883	42,65	0,769
50	216,595	7,428	70,626	1,441	125,671	4,562	75,607	1,651	95,388	2,628
100	340,862	17,323	98,606	1,435	--	--	182,484	4,915	198,174	5,797

Table 1. RMSE and NMSE Values

Results

In this paper, it is compared the global, local and semi-local chaotic prediction methods in terms of prediction performance for stock markets. In order to observe the performance of prediction methods that are covered in this study based on the prediction length, various prediction lengths including 5, 10, 20, 50 and 100 have been taken into account. When the prediction results given in Table 1 are evaluated; according to the RMSE values, radial basis function method and nearest neighbour method generally provide smaller error values in short-term, and long-term predictions, respectively.

According to NMSE values, it is again seen that prediction errors of the predictions made using the radial basis function method are smaller than those of other methods. Even though the maximum prediction length of FTSE-100 index data was determined as 100 days, it was seen that prediction can be made for a maximum of 20 days, and making predictions using these methods would not be suitable for predictions after 20 days.

Considering all the prediction results obtained for FTSE 100 index data based on the RMSE and NMSE criteria, it was found that the worst prediction performance was obtained with MLP neural network whereas the best prediction performance was gained using the Radial Basis function method which is calculated for k=8.

When the chaotic prediction methods applied on FTSE 100 index data are assessed in terms of prediction performance, it is seen that semi-local prediction method demonstrates a better prediction performance as compared to global and local prediction methods. While it is possible to make a prediction of index values for a maximum of 5 days using the nearest neighbour method, it was seen that prediction can be made for up to 20 days using the radial basis function method which is among the semi-local prediction methods. It is clear that semi-local prediction methods can provide successful results in the explanation and prediction of stock market behaviours by combining the advantages of global prediction methods and local prediction methods.

Acknowledgement

This study was supported by the Scientific Research Project Unit of Eskisehir Osmangazi University (Project number: 2016-1178).

References

Abarbanel, H. (1996). Analysis Of Observed Chaotic Data. New York: Spinger-Verlag.

Abarbanel, H. D., Brown, R., Kadtke, J. B. (1990). "Prediction in chaotic nonlinear systems: methods for time series with broadband fourier spectra". Physical Review A, 41(4), 1782-1807.

Abhyankar, A., Copeland, L. S., Wong, W. (1995). "Nonlinear dynamics in real-time equity market indices: evidence from the United Kingdom". The Economic Journal, 864-880.

Brock, W. A., Hsieh, D. A., Lebaron, B. D. (1991). Nonlinear Dynamics, Chaos and Instability: Statistical Theory and Economic Evidence. MIT Press.

Casdagli, M. (1989). "Nonlinear prediction of chaotic time series". Physica D: Nonlinear Phenomena, 35(3), 335-356.

Chan, K.S., Tong, H. (2001). Chaos: A Statistical Perspective, New York: Springer-Verlag.

Eckmann, J. P., Ruelle, D. (1985). "Ergodic theory of chaos and strange attractors". Reviews of Modern Physics, 57(3), 617-656.

Eldridge, R. M., Coleman, M. P. (1993). "The British FTSE-100 Index: Chaotically Deterministic or Random". Working Paper, School of Business, Fairfield University.

Elshorbagy, A., Simonovic, S. P., Panu, U. S. (2002). "Estimation of missing streamflow data using principles of chaos theory". Journal of Hydrology, 255(1), 123-133.

Elsner, J.B. (1992). "Predicting time series using a neural network as a method of distinguishing chaos from noise". Journal of Physics A: Mathematical and General, 25(4) 843–850.

Farmer, J.D., Sidorowich, J.J. (1987). "Predicting chaotic time series". Physical Review Letters, 59(8), 845-848.

Fraedrich, K. (1986). "Estimating the dimensions of weather and climate attractors". Journal of The Atmospheric Sciences, 43(5), 419-432.

Fraser, A. M., Swinney, H. L. (1986). "Independent coordinates for strange attractors from mutual information". Physical Reviews A, 33, 1134-1140.

Grassberger, P., Procaccia, I. (1983a). "Characterization of strange attractors". Physical Review Letters, 50(5), 346-349.

Grassberger, P., Procaccia, I. (1983b). "Measuring the strangeness of strange attractors". Physica D: Nonlinear Phenomena, 9(1-2), 189-208.

Guegan, D., Mercier, L. (2005). "Prediction in chaotic time series: methods and comparisons with an application to financial intra-day data". The European Journal Of Finance, 11(2), 137-150.

Hanias M., Magafas L., Konstantaki P. (2013). "Nonlinear analysis of S&P index". Equilibrium Quarterly Journal of Economics and Economic Policy, 8(4), 125-135.

Hassan M.R., Nath, B. (2005). "Stock market forecasting using Hidden Markov Model: A New Approach". Proceedings of the 5th International Conference on Intelligent Systems Design and Applications, 192–196.

Hsieh, D.A. (1991). "Chaos and nonlinear dynamics: Application to financial markets". Journal of Finance, 46(5), 1839–1877.

Huang, S. C., Chuang, P. J., Wu, C. F., Lai, H. J. (2010). "Chaos-based support vector regressions for exchange rate forecasting". Expert Systems with Applications, 37(12), 8590-8598.

Kantz, H. (1994). "A robust method to estimate the maximal Lyapunov exponent of a time series". Physics Letters A, 185(1), 77-87.

Kantz, H., Schreiber, T. (2004). Nonlinear Time Series Analysis. UK: Cambridge University Press.

Karunasinghe, D.S., Liong, S.Y. (2006). "Chaotic time series prediction with a global model: artificial neural network". Journal of Hydrology, 323(1), 92-105.

Kazem, A., Sharifi, E., Hussain, F. K., Saberi, M., Hussain, O. K. (2013). "Support vector regression with chaos-based firefly algorithm for stock market price forecasting". Applied Soft Computing, 13(2), 947-958.

Kennel, M.B., Brown, R., Abarbanel, H.D.I. (1992). "Determining embedding dimension for phase-space reconstruction using a geometrical construction". Physical Review A, 45(6), 3403-3411.

Lapedes, A., Farber, R., (1987). Nonlinear Signal Processing Using Neural Networks: Prediction and System Modelling. Technical Report, Los Alamos National Laboratory, Los Alamos, NM.

Lillekjendlie, B., Kugiumtzis, D., Christophersen, N. (1994). "Chaotic time series part II: System identification and prediction". Modeling, Identification And Control, 15, 225-243.

Mayfield, E. S., Mizrach, B. (1992). "On determining the dimension of real-time stock-price data". Journal of Business and Economic Statistics, 10(3), 367-374.

Özdemir, S. D., Akgül, I. (2014). "Hisse senedi piyasalarının kaotik yapısı ve yapay sinir ağları ile öngörüsü: İMKB-100 örneği". İktisat İşletme ve Finans, 29 (336), 31-58.

Shang, P., Li, X., Kamae, S. (2005). "Chaotic analysis of traffic time series". Chaos, Solitons and Fractals, 25(1), 121-128.

Sivakumar, B., Jayawardena, A. W., Fernando, T. M. K. G. (2002). "River flow forecasting: use of phase-space reconstruction and artificial neural networks approaches". Journal of Hydrology, 265(1), 225-245.

Smaoui, N. (1999). "An artificial neural network noise reduction method for chaotic attractors". International Journal of Computer Mathematics, 73(4), 417-431.

Sprott, J. C. (2003). Chaos and Time Series Analysis. Oxford Press.

Sprott, J. C. (2010). Elegant Chaos: Algebraically Simple Chaotic Flows. World Scientific.

Takens, F., (1981). in Dynamical Systems and Turbulence, Warwick Vol. 898 of Lecture Notes in Mathematics, edited by D.A. Rand, L. S. Young, Springer, Berlin.

Vaidyanathan, R., Krehbiel, T. (1992). "Does the S&P 500 futures mispricing series exhibit nonlinear dependence across time?". Journal of Futures Markets, 12(6), 659-677.

Webel, K. (2012). "Chaos in German stock returns-new evidence from the 0–1 test". Economics Letters, 115(3), 487-489.

Wolf, A., Swift, J. B., Swinney, H. L., Vastano, J. A. (1985). "Determining Lyapunov exponents from a time series". Physica D: Nonlinear Phenomena,16(3), 285-317.

Xiaofeng, G., Lai, C. H. (1999). "Improvement of the local prediction of chaotic time series". Physical Review E, 60(5), 5463-5468.